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Implementation of the Parallel Computations for Linking the Regions within the 2011 ICP

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Contents

1. Introduction	2
2. Linking Methods for the 2011 ICP	2
Linking Basic Heading PPPs Across Regions	2
Linking Across Regions Above Basic Heading	2
a. Method of linking at Basic Heading in detail	3
i. Brief Description	3
ii. Steps to implement method (with a fictitious example)	6
b. CAR method of linking above Basic Heading.....	9
i. Brief Description	9
ii. Steps to implement method (with a fictitious example)	11
c. Eurostat – OECD Method of linking above Basic Heading.....	19
i. Brief Description	19
ii. Steps to implement method (with a fictitious example)	19
3. Linking Singleton Countries and Regions (possible options)	21
4. Linking Countries with double participation (possible options)	23
References.....	27

1. Introduction

The ICP computational procedures should be transparent, well documented and the ICP global results should be reproducible. Starting with the same input data, each researcher should be able to reproduce the official ICP results. This was not the case in the ICP 2005. Therefore the GO ICP decided that all computational procedures using for the calculations of the global ICP 2011 results should be documented in detail and parallel calculations (beside the official calculations making by the GO ICP) should be carried out by several players independently. The differences in the results should be carefully investigated, the reasons – detected and the optimal way – found. To obtain this aim, the computational procedures using for linking at all aggregation levels should be described in details and the procedures should be checked on some examples with imaginary data. The present document attempts to carry out these tasks.

2. Linking Methods for the 2011 ICP

The linking methods recommended by the ICP TAG are based on the global core approach. All the economies participating in the ongoing ICP round are required to collect, in addition to their respective regional lists of goods and services, prices for a global core list of products. The general linking schemas at the basic heading (BH) and aggregated levels are described below.

Linking Basic Heading PPPs Across Regions

The same method as that used to link BH-PPPs in the 2005 ICP was recommended by the TAG for the 2011 round, although the linking process will involve all countries in 2011 rather than just the “ring countries”. The following steps will be used to compute the global PPPs for each BH:

Step 1: All relevant global core product prices provided by a country are divided by the regional basic heading PPP of the country, thus converting all global core prices from that region into a common regional numeraire;

Step 2: Converted prices resulting from Step 1 are processed through a single weighted CPD run involving data from all regions, thus generating the inter-regional linking factors for the basic heading (inter-regional BH-PPPs), expressed in the World numeraire.

Step 3: Fixity of regional PPPs in the World comparison is ensured by multiplying each country’s regional basic heading PPP by the inter-regional linking factor.

Linking Across Regions Above Basic Heading

The Country-Approach with Volume Redistribution (CAR-Volume) was adopted by the TAG ICP. This involves estimating simultaneously the linking factors for volumes for all countries in the regions, according to the following steps:

Step 1: All basic heading PPPs from all 180+ countries are used to carry out unrestricted GEKS-Fisher aggregation above the basic heading level.

Step 2: Regional Volumes (real expenditure) totals are obtained by summing up volume from individual countries for each region, using the results from Step 1.

Step 3: The regional Volumes (real expenditure) totals from Step 2 are distributed among the countries in the regions according to country's shares in regional results to uphold regional fixity.

The details of the linking methods are presented below.

a. Method of linking at Basic Heading in detail

i. Brief Description

Weighted CPD method was recommended by the TAG meeting for linking at the BH level.

Input data sets, to obtain the global results with regional fixity, are the following :

- Country's prices (with indications on the importance) for the ICP Core list products in regional numeraires => to calculate between-regional PPP
- Regional BH-PPPs => to obtain the country's BH-PPPs in the global comparison with the Regional fixity

The original CPD method was proposed by Robert Summers (1973). It uses the stochastic approach to price indices. The model underlying the traditional CPD and presented in the context of the inter-region linking (as it was proposed by E.Diewert and is described in the ICP Manual, Chapter 11) is the following

$$(1) p_{ij} = \kappa \alpha_j \beta_i v_{ij} \quad \begin{array}{l} i = 1, 2, \dots, n \text{ (n - no. of items)} \\ j = 1, 2, \dots, r \text{ (r - no. of Regions)} \\ l = 1, 2, \dots, c_j \text{ (c}_j \text{ - no. of countries in a Region j)} \end{array}$$

$$\alpha_1 = \beta_1 = 1$$

where κ is a constant¹, α_j is a parameter for Region j (between regional PPP), β_i is a parameter for product i (average world price) and v_{ij} is a random error term. As the model is concerned with price ratios, there are only $n+r-1$ parameters to estimate. Equation (1) is needed to determine the absolute levels of the prices. When both $i=1$ and $j=1$ then product 1 in Region 1 becomes the 'reference' product (with price = κ in numeraire for Region 1), all prices being measured relatively to its price, Region 1 therefore acts as the reference Region for the between-regional PPPs.

The original CPD model assumes that all products are equi-representative or equi-important in all countries. It is not very realistic. Therefore the concept of representativity (the focus on different price levels for representative and non-representative products) was introduced in the ICP 2005 (the CPRD method was recommended). The experience showed that the concept of representativity was not very understandable for the countries. Therefore it was replaced by the

¹ Most presentations of the CPD model have no constant term, in which case normalization can be achieved simply by letting $\alpha_1 = 1$ without requiring $\beta_1 = 1$. However, when a third type of variable, representativity, is introduced into the model, an additional constraint has to be imposed anyway, in which case the approach adopted here is more convenient and symmetrical. For a simple exposition of regression with dummy variables, see David Huang (1970). He remarks, p. 166 that: "The rule of thumb is that, whenever there are two or more dummy systems, drop one variable from each system (preserving the constant, say) for OLS estimation." When representativity is introduced into the CPD, there are three or more dummy systems, depending on whether interaction terms are included.

concept of importance (the focus on different weights / shares for important and less important products). Respectively the weighted CPD was recommended.

Weighted CPD: CPD with different weights for Important and less important products

Weighted CPD uses some explicit weights. Representative items receive some higher weight than non-representative items. For example, the weights '2' and '1'; or 3 and 1; or some other appropriate weights can be used².

The unweighted CPD derives estimators of regression parameters (1) through the minimization of the squares in logarithmic terms using a standard LSQ procedure

$$\sum_{i=1}^n \sum_{j=1}^r \sum_{l=1}^{c_j} [\ln(p_{ilj}) - \alpha'_j - \beta'_n - k']^2 \quad \text{with } \alpha'_1 = 0; \beta'_1 = 0$$

The CPD weighted approach suggests that each price corresponding to a product in a given country in a region be given a prespecified weight in the least squares estimation. Suppose \mathbf{w} is a set of weights for representative (w^{repr}) and non-representative ($w^{\text{non-repr}}$) items to be given to the price p_{ij} , then the weighted least squares (WLS) approach applied to (1) can be presented as the following³:

$$\sum_{i=1}^n \sum_{j=1}^r \sum_{l=1}^{c_j} w [\ln(p_{ilj}) - \alpha'_j - \beta'_n - k']^2 \quad \text{with } \alpha'_1 = 0; \beta'_1 = 0$$

It is declared often that the advantage of the stochastic CPD approach is that it allows to derive standard errors for the CPD estimates of the purchasing power parities (PPPs), common prices, etc. However it is not easy to implement these standard errors in the analysis - there are numerous problematic points⁴. In effect, standard errors of the CPD parameters do not use in the ICP practice and respective indicators do not produced by the Kit explicitly.

Therefore, to do further considerations more transparent and understandable for a broad circle of users, the weighted CPD method is presented below in a more traditional index⁵ form as a specific kind of the GK method in geometric (logarithmic) terms⁶:

² Note: the weights "1" and "0" are applicable for the EKS method (1 = for asterisked items *; 0 – for non-asterisked items) but not for the CPD method because the items with "0-weights" are non-priced items and they are eliminated from the calculations.

³ See, D.S. Prasada Rao "The CPD method: a stochastic approach to the computation of PPP in the ICP", the SSHRC Conference on Index Numbers and Productivity Measurement, 30 June – 3 July, 2004, Vancouver http://siteresources.worldbank.org/ICPINT/Resources/Country-Product-Dummy_Method.doc

⁴ It is unclear – How the stochastic estimations should be practically used:

- What are criteria for for high / low standard errors?
- What one should do if standard errors are high but the results looks as plausible or vice versa?
- What should be done if the standard errors for PPPs are low but these are high for the international average prices?

Additionally, the estimations of errors depend on the regression specification

⁵ Vice versa, it can be demonstrated that a number of widely used multilateral index numbers for PPPs can be derived using the stochastic approach. See, for example, D.S. Prasada Rao, G. Hajargasht "Stochastic Approach to

$$(2) \quad \pi_i = \left\{ \prod_{r=1}^R \prod_{j=1}^{N(r)} [P_{ijr} / PPP_r]^{q_{ijr}} \right\}^{1/\sum q_{ijr}}; \quad i = 1, 2, \dots, M$$

$$(3) \quad PPP_r = \left(\prod_{i=1}^M \prod_{j=1}^{N(r)} [P_{ijr} / \pi_i]^{q_{ijr}} \right)^{1/\sum q_{ijr}}; \quad r = 1, 2, \dots, N-1 \text{ (} PPP_N = 1 \text{)}$$

π_i is international average price of the item i in the currency of the numeraire Region (in our case, Region R); π_i is an analogue of α_i from the CPD regression,

PPP_r is the PPP of Region r relatively the base region R ($PPP_R = 1$); PPP_r is an analogue of β_r from the CPD regression

R – no. of Regions

$N(r)$ – no. of countries in the region r ,

M – no. of products within a BH

q_{ijr} are weights (imaginary quantities) for product i in country j from a region r ; the appropriate values can be **3** (for representative products) and **1** (for non-representative products).

$\sum q_{ijr}$ is the cumulative value of representativity of item i among all countries in all Regions.

$\sum q_{ijr}$ is the cumulative value of representativity of items priced in the country j / region r .

Average „International price“ of the i th item (π_i) are presented as a ‘implicit quantity’-weighted geometric average of the PPP-adjusted national prices.

PPP for the j th region (PPP_j) are be presented as the geometric average (implicit weighted) deviation of its regional prices from the international prices

Index Numbers for Multilateral Price Comparisons and their Standard Errors”, WP06/2008, Centre for Efficiency and Productivity Analysis (CEPA), The University of Queensland, Brisbane, Australia, 2008

<http://www.uq.edu.au/economics/cepa/docs/WP/WP062008.pdf>

http://www.google.at/url?sa=t&source=web&cd=5&sqi=2&ved=0CEoQFjAE&url=http%3A%2F%2Fwww.wdocs.fce.unsw.edu.au%2Ffce%2FResearch%2FResearchMicrosites%2FCAER%2FWorkshopPapers%2FEMG06%2FEMG06014.ppt&rct=j&q=prasada%20rao%20Geary%20Khamis%20stochastic&ei=AT9BTr3OIpDGswbBw6myBw&usg=AFQjCNEXkw5ez-JHzauxDr0PJTQDmA_P7w

This paper shows that price index numbers from commonly used methods like the G-K, the Ikle, the Rao-weighted and an additive multilateral system are all weighted least squares estimators of the parameters of the country-product-dummy (CPD) model.

The estimation of the parameters of the indices EKS type is described in the following papers by A. Deaton and O. Dupriez

http://www.princeton.edu/~deaton/downloads/deaton_dupriez_purchasing_power_parity_exchange_rates_global_poor_aeja_2011.pdf

http://www.princeton.edu/~deaton/downloads/Global_Poverty_and_Global_Price_Indexes.pdf

⁶ See Sergeev (2005b) and E. Diewert “Weighted Country Product Dummy Variable Regressions and Index Number Formulae”, http://siteresources.worldbank.org/ICPINT/Resources/product_dummy_variable.doc

This system (2) (3) can be efficiently solved by an iterative method. GM of price ratios or simply exchange rates to a country selected as the base can be used as an initial set of unknown PPPs.

As it was indicated above, the weights can be introduced also in the original CPD stochastic concept and the solution can be obtained by the regression techniques. Each researcher can select the computational schema in accordance with own preferences and available tools. The final results are the same in all versions independently on the concrete computational algorithm.

ii. Steps to implement method (with a fictitious example)

Practical steps are demonstrated by an fictitious example from the **ICP Manual, Chapter 14**

http://siteresources.worldbank.org/ICPINT/Resources/270056-1183395201801/icp_Ch14rev.doc

Input data for **3 Regions** (Region I – countries A, B, C, D; Region II – countries E, F, G; Region III – countries H, I, J) X **10 products** (see Table 1) are

- **Country's price data for products from the ICP Global list**
- and
- **Within-regional BH-PPPs**

Table 1. Original Price Data

Product	Region I				Region II			Region III		
	A	B	C	D	E	F	G	H	I	J
1	2*	100		25*	20*	600*		6*	60	
2	5*		12*			900*	450		100	240
3	6*	270	15*			1000*	400	14*	150	200*
4		320	70		180	5000		24		320
5	8*	280		120*	120	2000*	500	20		360
6		210*	60		100		350*	12*	100	
7			50*	140*				40	240	260*
8		120*	12*	100	80	800*		16	50*	
9	2			10*	25	1500	150*			
10					40*		260*		70*	200*
Within-region PPPs										
	1	30	5	13	1	30	6	1	7	16

Table 2 shows the prices converted into each region's numeraire currency. They are obtained simply by dividing the prices in each column of Table 1 by the within-region parity.

Table 2. Prices Deflated by Within-Region PPPs

Product	Region I				Region II			Region III		
	A	B	C	D	E	F	G	H	I	J
1	2*	3.33		1.92*	20*	20*		6*	8.57	

2	5*		2.4*			30*	75		14.29	15
3	6*	9	3*			33.33*	66.67	14*	21.43	12.5*
4		10.67	14		180	166.67		24		20
5	8*	9.33		9.23*	120	66.67*	83.33	20		22.5
6		7*	12		100		58.33*	12*	14.29	
7			10*	10.77*				40	34.29	16.29
8		4*	2.4*	7.69	80	26.67*		16	7.14*	
9	2			0.77*	25	50	25*			
10					40*		43.33*		10*	12.5*

The prices with asterisks * are included in the calculation of the between-regional BH-PPPs. The asterisked (more important) products receive higher conventional weight (like “3”) and non-asterisked (less important) products – lower weight (like “1”). These data are included in the calculation of the system of linear (in logarithmic terms) equations (2) and (3).

The XRs between the regional numeraires can be used as the initial starting values for the between-regional PPPs. After several iterations the between-regional PPPs will be obtained.

The global PPPs with the Regional fixity can be obtained as:

$$\text{PPP „Country / World“} = \text{PPP „Country / Region“} \times \text{PPP „Region / World“}$$

[Regional Comparison] [Global Comparison]

The results (CPD with the weights 3:1) for the example above are presented in Table 3. Region I was selected as the base.

Table 3. Within-regional, between-regional and global BH-PPPs

Country	Region	Within-region PPPs	Between-regional PPPs (weighted CPD; 3:1)	Linked / ‘global’ set of PPPs
		(1)	(2)	(3) = (1) x (2)
A	I	1	1	1
B	I	30	1	30
C	I	5	1	5
D	I	13	1	13
E	II	1	10.79	10.79
F	II	30	10.79	323.7
G	II	6	10.79	64.74
H	III	1	2.67	2.67
I	III	7	2.67	18.69
J	III	16	2.67	42.72

Beside the calculations by the official method it is desirable to carry out the calculations by other methods (with individual country’s prices as well as with average Regional prices⁷). This allows

⁷ In principle, if country’s input data should be used in the official calculations (but not average Regional data) then, maybe, the CAR-PPP method (unrestricted weighted CPD method for all countries with further re-indexation of PPP in accordance with the intra-regional PPPs = Eurostat-OECD approach) would be more practicable and straightforward. In this case, there would not be the problems with the countries priced more products (like China)

to evaluate better the quality of input data and sensitivity of the results to the selected computational method.

First of all, the CPRD method was recommended by the TAG as an alternative method. The extension of the CPD model to include representativity was first proposed by J. Cuthbert and M. Cuthbert (1988). The CPRD can be written as an expansion of the CPD model (1) :

$$(1a) \quad p_{ijl} = \kappa \alpha_i \beta_j \gamma v_{ijk}$$

Variable γ denotes the degree of representativity – an average ratio between price levels for non-representative and representative products. In the theory this should be higher than 1 (positive value of γ_k as a regression coefficient).

The CPRD derives estimators of regression parametrs (1a) through the minimization of the squares in logarithmic terms using a standatd LSQ procedure

$$\sum_{i=1}^n \sum_{j=1}^r \sum_{l=1}^{c_j} [\ln(p_{ijl}) - \alpha'_j - \beta'_n - \gamma' - k']^2 \quad \text{with } \alpha'_1 = 0; \beta'_1 = 0$$

If one wants to use a traditional index approach then the CPRD can be also presented as a specific kind of the GK method in geometric (logarithmic) terms with an additional equation for variable γ which reflect an average ratio between PLI for non-representative and representative products (it is assumed that representative products have generally lower PLI; therefore it is expected that the coefficient γ should be higher than 1)⁸:

$$(4) \quad \pi_i = \left(\prod_{r=1}^R \prod_{j=1}^{N(r)} [(P_{ijr} / \gamma^{Z_{ijr}}) / PPP_r] \right)^{1/n_i}; \quad i = 1, 2, \dots, M$$

$$(5) \quad PPP_r = \left(\prod_{i=1}^M \prod_{j=1}^{N(r)} [(P_{ijr} / \gamma^{Z_{ijr}}) / \pi_i] \right)^{1/m_r}; \quad j = 1, 2, \dots, N - 1 \quad (PPP_N = 1)$$

and single countries would have more options for the inclusion in the global results. The validation and edition of input data for the Global compariosn would be also easier.

⁸ A (more complicated) version of the CPRD with different weights for representative and non-representative (important and less important) products is also possible:

$$\pi_i = \prod_{r=1}^R \left\{ \prod_{j=1}^{N(r)} [(P_{ijr} / \gamma^{Z_{ijr}}) / PPP_r]^{q_{ijr}} \right\}^{1 / \sum_{j=1}^{N(r)} q_{ijr}}; \quad i = 1, 2, \dots, M$$

$$PPP_r = \left(\prod_{i=1}^M \prod_{j=1}^{N(r)} [(P_{ijr} / \gamma^{Z_{ijr}}) / \pi_i]^{q_{ijr}} \right)^{1 / \sum_{i=1}^M q_{ijr}}; \quad r = 1, 2, \dots, N - 1 \quad (PPP_R = 1)$$

$$\gamma = \left\{ \prod_{r=1}^R \prod_{j=1}^{N(r)} \prod_{i=1}^M [(P_{ijr} / PPP_r) / \pi_i]^{Z_{ijr}} \right\}^{1/m_{nr}};$$

q_{ijr} are some weights (imaginary quantities); the appropriate values are **3** (for representative products) and **1** (for non-representative products).

$$(6) \quad \gamma = \left\{ \prod_{r=1}^R \prod_{j=1}^{N(r)} \prod_{i=1}^M [(P_{ijr} / PPP_r) / \pi_i]^{Z_{ijr}} \right\}^{1/m_{nr}};$$

π_i is international average price of the item i in the currency of the numeraire Region (in our case, Region R); π_i is an analogue of α_i from (1)

PPP_r is the PPP of Region r to the base region R ($PPP_R = 1$); PPP_r is an analogue of β_r from (1);

n_i – total no. of prices for item i (sum of X_{ijr} for item i for all countries – no. of countries priced item i)

m_r – total no. of prices for Region r (sum of $Y_{ij} \in N(r)$ for all countries from the Region r)

m_{nr} – total no. of non-representative items within the combined set of prices for all countries (sum of Z_{ijr} for all items for all countries).

The other additional approach (which can be useful for an deep analysis) is the use of the geometric means of regional country prices. This approach treated all Regions in a symmetrical way (one Region – one set of data).

The calculations by different methods allows to evaluate better and more consciously the results by the official methods. The respective summary of the results obtained by different methods for the example above is in Table 4:

Table 4: Between-Regional PPP (Reg I = 1) by different computational methods

	Individual prices of the countries				Average (GM) regional prices			
	Reg I (4 Cou.)	Reg II (3 Cou.)	Reg III (3 Cou.)	C. Non-R ---	Reg I (4 Cou.)	Reg II (3 Cou.)	Reg III (3 Cou.)	C. Non-R ---
CPD (unweighted 1:1)	1.000	11.5447	2.6715	1	1.000	11.5165	2.6606	1
CPD (weighted 2:1)	1.000	11.0811	2.6911	1	1.000	11.0119	2.6357	1
CPD (weighted 3:1)	1.000	10.7937	2.6737	1	1.000	10.7425	2.6128	1
CPD (weighted 10:1)	1.000	10.1487	2.5854	1	1.000	10.1743	2.5476	1
CPRD (unweighted)	1.000	10.5645	2.2306	1.788	1.000	10.3480	2.2905	1.438
EKS 1 (without *)	1.000	11.1137	2.6556	---	1.000	11.1738	2.6069	---
EKS 2 (with *)	1.000	9.8530	2.6651	---	1.000	10.1405	2.5995	---

b. CAR method of linking above Basic Heading

i. Brief Description

The Country-Approach with Volume Redistribution (CAR-Volume) was adopted by the TAG ICP for linking above basic headings. All countries participate simultaneously in the Global calculation. Regional Volumes from the Global comparison are redistributed between the countries in accordance with the Regional Volume shares. The main features:

- Each country treated equally in the Global comparison
- Fixity of regional results is obtained by an indirect (two-stage) approach
- Probably, the differences in the regional methodologies will have some lower impact on the (in)comparability of the World results with fixity

Any aggregated method can be used by this approach.

The GEKS (F) method was recommended by the TAG for the aggregation within the Regions well as for the Global comparison. Firstly, the bilateral Fisher's PPPs⁹ for all pairs of the countries are calculated. At the second stage, all direct and indirect F-PPP are averaged geometrically, to obtain the transitive GEKS-PPPs.

F-PPP

$$P_F(p^k, p^j, q^k, q^j) \equiv [P_L(p^k, p^j, q^k, q^j) \cdot P_P(p^k, p^j, q^k, q^j)]^{1/2}$$

$$= \left[\left(\frac{p^j \cdot q^k}{p^k \cdot q^k} \right) \left(\frac{p^j \cdot q^j}{p^k \cdot q^j} \right) \right]^{1/2}$$

GEKS-F

$$P^j \equiv \prod_{k=1}^K [P_F(p^k, p^j, q^k, q^j)]^{1/K}$$

$$Q^j \equiv p^j \cdot q^j / P^j$$

The countries from different Regions as well as within the Regions can be very different. The bilateral indices for the countries with different price and quantity structures can be unreliable. In this aspect the analysis of Laspeyres-Paasche Spreads (LPS) is very important. The selective EKS – direct F-indices with high LPS (e.g. higher than 1.5) and with PLS less than 1¹⁰ are replaced on the indirect indices via 3rd countries - should be investigated (at least, the experimental calculations should be done).

The Country Approach with Redistribution (CAR) - Unrestricted EKS method – can be used for Volumes (CAR-Volumes) or for the PPPs (CAR-PPPs = Eurostat-OECD approach)

CAR-Volumes approach¹¹ was recommended as the official ICP 2011 method:

The unrestricted global GEKS-PPPs for the aggregates are used to recalculate the country's aggregates in national currencies into Volumes (real expenditure) measured in a world numeraire. Country's Volumes are summed up by the Regions and these Regional Volumes are redistributed in accordance with the country's shares in the Regional comparisons, to keep the regional fixity.

$$\text{Volume "Country in the World"} = \text{Volume "Region in the World"} \times \text{Share "Country / Region"} \\ \text{[Global Comparison]} \qquad \qquad \text{[Regional Comparison]}$$

This procedure can be presented also in the form of Volume indices

$$VI \text{ "Country / World"} = VI \text{ "Region / World"} \times VI \text{ "Country / Region"} \\ \text{[Global Comparison]} \qquad \qquad \text{[Regional Comparison]}$$

⁹ The same can be applied to the Volume / Quantity indices because the F-index is symmetrical relatively variables (prices and quantities). The product of PPP(F) and Q(F) is the expenditure ratio.

¹⁰ In a "normal" case, Laspeyres index (arithmetic mean) is higher than Paasche index (harmonic mean).

¹¹ This method was described in Ch.15 of the ICP Manual as the calculation of the Regional scaling factors by the weighted harmonic mean.

The Volume indices (VI) “Region / World” and “Country / Region” can be considered also as the respective Volume shares.

The respective PPPs (with fixity) are calculated in an indirect way as:

$$\text{PPP} = \text{Nominal expenditure} : \text{Volume}$$

Input data sets, to obtain the global results with regional fixity are the following:

- **Country’s NA data** (the respective shares are used as weights)
- **Global BH-PPPs (with fixity) and Regional BH-PPPs** (to check the regional computations)
- **Regional PPPs and VI /Country’s shares in Regional Volumes for the aggregated headings**

All calculation by the GEKS method are carried out for each aggregated heading separately.

ii. Steps to implement method (with a fictitious example)

Practical steps are demonstrated by an fictitious example from the former Chapter 13 of the ICP Manual:

- **3 Regions** (A – 4 countries; B – 3 countries; C – 2 countries)
- **6 BHs** (4 BHs for Consumption and 2 BHs for Investment)

Input data are presented in Tables 5 and 6:

Table 5: BH expenditures in national currencies

BH	Region A				Region B			Region C	
	Countries				Countries			Countries	
	1	2	3	4	1	2	3	1	2
1	2000	60	1200	9600	380	1000	3000	4500	225
2	500	18	400	2000	70	290	720	1200	67
3	800	35	700	4000	180	315	1260	1500	120
4	300	24	245	3300	70	170	500	800	36
1-4 = C	3600	137	2545	18900	700	1775	5480	8000	448
5	700	31	600	2800	180	430	1080	1500	75
6	700	32	700	2800	130	190	1280	2400	78
5+6 = I	1400	63	1300	5600	310	620	2360	3900	153
Total = GDP	5000	200	3845	24500	1010	2395	7840	11900	601

Table 6: Within-regional and between-regional BH-PPP

Basic heading	Region A				Inter-regional PPP of B on A denominated in currencies of B.1 & A.1	Region B			Inter-regional PPP of C on A denominated in currencies of C.1 & A.1	Region C	
	1	2	3	4		1	2	3		1	2
	Intra-regional PPPs					Intra-reg. PPPs				Intra-reg. PPPs	
1	1	0.6	6	120	2.5	1	2	0.4	9	1	0.5
2	1	0.9	10	100	1.6	1	2.8	0.5	12	1	0.7
3	1	0.5	10	100	3	1	1.4	0.6	15	1	0.4
4	1	0.4	7	110	2.4	1	1.8	0.7	8	1	0.3
5	1	1	12	80	3	1	2.4	0.3	15	1	0.5
6	1	0.8	13	70	3.2	1	1.5	0.5	12	1	0.7
Regional basic heading PPPs linked using the inter-regional PPPs											
1	1	0.6	6	120	--	2.5	5	1	--	9	4.5
2	1	0.9	10	100	--	1.6	4.48	0.8	--	12	8.4
3	1	0.5	10	100	--	3	4.2	1.8	--	15	6
4	1	0.4	7	110	--	2.4	4.32	1.68	--	8	2.4
5	1	1	12	80	--	3	7.2	0.9	--	15	7.5
6	1	0.8	13	70	--	3.2	4.8	1.6	--	12	8.4

The results of Regional comparisons are calculated on the basis of the following data:

Table 7: Input data for regional comparisons

Ch13 ICP Handbook																					
Example (3 Regions=4+3+2 countries)																					
Within-regional BH-PPPs and nominal Shares (%)				A1	A1	A2	A2	A3	A3	A4	A4	B1	B1	B2	B2	B3	B3	C1	C1	C2	C2
Itemcode	Short description	Quan.	Unit	PPP	Sh	PPP	Sh	PPP	Sh	PPP	Sh	PPP	Sh	PPP	Sh	PPP	Sh	PPP	Sh	PPP	Sh
BHC1	BH1-C1	1	Unit	1	40.0	0.6	30.0	6	31.2	120	39.2	1	37.6	2	41.8	0.4	38.3	1	37.8	0.5	37.4
BHC2	BH2-C2	1	Unit	1	10.0	0.9	9.0	10	10.4	100	8.2	1	6.9	2.8	12.1	0.5	9.2	1	10.1	0.7	11.1
BHC3	BH3-C3	1	Unit	1	16.0	0.5	17.5	10	18.2	100	16.3	1	17.8	1.4	13.2	0.6	16.1	1	12.6	0.4	20.0
BHC4	BH4-C4	1	Unit	1	6.0	0.4	12.0	7	6.4	110	13.5	1	6.9	1.8	7.1	0.7	6.4	1	6.7	0.3	6.0
BH11	BH5-I1	1	Unit	1	14.0	1	15.5	12	15.6	80	11.4	1	17.8	2.4	18.0	0.3	13.8	1	12.6	0.5	12.5
BH12	BH6-I2	1	Unit	1	14.0	0.8	16.0	13	18.2	70	11.4	1	12.9	1.5	7.9	0.5	16.3	1	20.2	0.7	13.0
				100		100		100		100		100		100		100		100		100	

The calculations for the Region A at the GDP level are presented in Table 8

Table 8: GEKS results for the region B at the GDP level

The RESULTS by GEKS-method (with weights,%) for GDP

MATRIX of BINARY LASPEYRES's PPPs

(... currency units of row-country per 1 currency unit of column-country)

	A1	A2	A3	A4
A1	1.00000	1.60500	.11674	.01000
A2	.68600	1.00000	.07752	.00673
A3	8.92000	14.06000	1.00000	.09102
A4	101.60000	164.40000	12.12464	1.00000

NUMBER of LASPEYRES-PPPs = 0 in the matrix = 0

INITIAL MATRIX of DIRECT BINARY FISHER's PPPs

(without crucial values for L/P-ratio)

(1st line: PPP = ... currency units of row-country per 1 currency unit of column-country, 2nd line: L/P ratio)

	A1	A2	A3	A4
A1	1.00000	1.52959	.11440	.00992
	1.00	1.10	1.04	1.02
A2	.65377	1.00000	.07425	.00640
	1.10	1.00	1.09	1.11
A3	8.74142	13.46718	1.00000	.08664
	1.04	1.09	1.00	1.10
A4	100.79683	156.23991	11.54157	1.00000
	1.02	1.11	1.10	1.00

NUMBER of the MISSING VALUES in the Fisher's-matrix = 0

FINAL MATRIX of BINARY F-PPPs

	A1	A2	A3	A4
A1	1.00000	1.52959	.11440	.00992
A2	.65377	1.00000	.07425	.00640
A3	8.74142	13.46718	1.00000	.08664
A4	100.79683	156.23991	11.54157	1.00000

Matrix of PPPs by EKS-method

(... currency units of row-country per 1 currency unit of column-country)

	A1	A2	A3	A4
A1	1.00000	1.53744	.11422	.00989
A2	.65043	1.00000	.07429	.00643
A3	8.75511	13.46044	1.00000	.08655
A4	101.15544	155.52026	11.55387	1.00000

The global calculations (unrestricted GEKS) at the GDP level are calculated on the basis of the following data

Table 9: Input data for Global comparisons

Ch13 ICP Handbook																			
Example (3 Regions=4+3+2 countries)																			
Global BH-PPPs and nominal Shares (%)		A1	A1	A2	A2	A3	A3	A4	A4	B1	B1	B2	B2	B3	B3	C1	C1	C2	C2
		PPP	Sh	PPP	Sh	PPP	Sh	PPP	Sh	PPP	Sh	PPP	Sh	PPP	Sh	PPP	Sh	PPP	Sh
BHC1	BH1-C1	1	40.0	0.6	30.0	6	31.2	120	39.2	2.5	37.6	5	41.8	1	38.3	9	37.8	4.5	37.4
BHC2	BH2-C2	1	10.0	0.9	9.0	10	10.4	100	8.2	1.6	6.9	4.5	12.1	0.8	9.2	12	10.1	8.4	11.1
BHC3	BH3-C3	1	16.0	0.5	17.5	10	18.2	100	16.3	3	17.8	4.2	13.2	1.8	16.1	15	12.6	6	20.0
BHC4	BH4-C4	1	6.0	0.4	12.0	7	6.4	110	13.5	2.4	6.9	4.3	7.1	1.7	6.4	8	6.7	2.4	6.0
BHI1	BH5-I1	1	14.0	1	15.5	12	15.6	80	11.4	3	17.8	7.2	18.0	0.9	13.8	15	12.6	7.5	12.5
BHI2	BH6-I2	1	14.0	0.8	16.0	13	18.2	70	11.4	3.2	12.9	4.8	7.9	1.6	16.3	12	20.2	8.4	13.0
		100		100		100		100		100		100		100		100		100	

The global calculations at the GDP level are presented in Table 10

Table 10: Global calculation (GDP)

The RESULTS by GEKS-method (with weights, %) for GDP

MATRIX of BINARY LASPEYRES's PPPs

(... currency units of row-country per 1 currency unit of column-country)

	A1	A2	A3	A4	B1	B2	B3	C1	C2
A1	1.0000	1.6050	.1167	.0100	.3817	.1997	.8798	.0924	.1868
A2	.6860	1.0000	.0775	.0067	.2621	.1348	.6274	.0622	.1175
A3	8.9200	14.0600	1.0000	.0910	3.3681	1.7136	7.7657	.7983	1.5399
A4	101.6000	164.4000	12.1246	1.0000	39.0763	20.8144	89.8901	9.4958	19.7964
B1	2.6520	4.2850	.3070	.0268	1.0000	.5131	2.2850	.2429	.4883
B2	5.0592	7.7900	.5833	.0503	1.9416	1.0000	4.5584	.4605	.9170
B3	1.2188	2.1735	.1425	.0128	.4584	.2380	1.0000	.1124	.2353
C1	11.4600	18.0750	1.3020	.1145	4.3701	2.2495	9.9848	1.0000	2.0418
C2	5.9700	8.7525	.6677	.0587	2.2502	1.1560	5.3181	.5345	1.0000

NUMBER of LASPEYRES-PPPs = 0 in the matrix = 0

INITIAL MATRIX of DIRECT BINARY FISHER's PPPs

(without crucial values for L/P-ratio)

(1st line: PPP = ... currency units of row-country per 1 currency unit of column-country, 2nd line: L/P ratio)

	A1	A2	A3	A4	B1	B2	B3	C1	C2
A1	1.00000	1.52959	.11440	.00992	.37939	.19870	.84962	.08981	.17689
	1.00	1.10	1.04	1.02	1.01	1.01	1.07	1.06	1.12
A2	.65377	1.00000	.07425	.00640	.24733	.13155	.53728	.05865	.11586
	1.10	1.00	1.09	1.11	1.12	1.05	1.36	1.12	1.03
A3	8.74142	13.46718	1.00000	.08664	3.31241	1.71396	7.38189	.78304	1.51864
	1.04	1.09	1.00	1.10	1.03	1.00	1.11	1.04	1.03
A4	100.7968	156.23991	11.54157	1.00000	38.17308	20.35171	83.76126	9.10715	18.36204
	1.02	1.11	1.10	1.00	1.05	1.05	1.15	1.09	1.16
B1	2.63580	4.04320	.30190	.02620	1.00000	.51407	2.23261	.23574	.46584
	1.01	1.12	1.03	1.05	1.00	1.00	1.05	1.06	1.10
B2	5.03272	7.60143	.58345	.04914	1.94528	1.00000	4.37651	.45246	.89064
	1.01	1.05	1.00	1.05	1.00	1.00	1.08	1.04	1.06
B3	1.17700	1.86122	.13547	.01194	.44791	.22849	1.00000	.10612	.21036
	1.07	1.36	1.11	1.15	1.05	1.08	1.00	1.12	1.25
C1	11.13447	17.04892	1.27707	.10980	4.24201	2.21015	9.42357	1.00000	1.95459
	1.06	1.12	1.04	1.09	1.06	1.04	1.12	1.00	1.09
C2	5.65338	8.63135	.65848	.05446	2.14667	1.12279	4.75373	.51162	1.00000
	1.12	1.03	1.03	1.16	1.10	1.06	1.25	1.09	1.00

NUMBER of the MISSING VALUES in the Fisher's-matrix = 0

FINAL MATRIX of BINARY F-PPPs

	A1	A2	A3	A4	B1	B2	B3	C1	C2
A1	1.00000	1.52959	.11440	.00992	.37939	.19870	.84962	.08981	.17689
A2	.65377	1.00000	.07425	.00640	.24733	.13155	.53728	.05865	.11586
A3	8.74142	13.46718	1.00000	.08664	3.31241	1.71396	7.38189	.78304	1.51864
A4	100.797	156.23991	11.54157	1.00000	38.17308	20.35171	83.76126	9.10715	18.36204
B1	2.63580	4.04320	.30190	.02620	1.00000	.51407	2.23261	.23574	.46584
B2	5.03272	7.60143	.58345	.04914	1.94528	1.00000	4.37651	.45246	.89064
B3	1.17700	1.86122	.13547	.01194	.44791	.22849	1.00000	.10612	.21036
C1	11.13447	17.04892	1.27707	.10980	4.24201	2.21015	9.42357	1.00000	1.95459
C2	5.65338	8.63135	.65848	.05446	2.14667	1.12279	4.75373	.51162	1.00000

Matrix of PPPs by EKS-method

(... currency units of row-country per 1 currency unit of column-country)

	A1	A2	A3	A4	B1	B2	B3	C1	C2
A1	1.0000	1.53694	.11484	.00988	.38027	.19808	.84439	.08992	.17719
A2	.6506	1.00000	.07472	.00643	.24742	.12888	.54940	.05850	.11529
A3	8.7075	13.38300	1.00000	.08599	3.31124	1.72480	7.35261	.78296	1.54292
A4	101.2649	155.63819	11.62955	1.00000	38.50821	20.05868	85.50748	9.10553	17.94345
B1	2.6297	4.04169	.30200	.02597	1.00000	.52089	2.22050	.23646	.46596
B2	5.0484	7.75914	.57978	.04985	1.91978	1.00000	4.26287	.45394	.89455
B3	1.1843	1.82017	.13601	.01169	.45035	.23458	1.00000	.10649	.20985
C1	11.1213	17.09272	1.27720	.10982	4.22910	2.20291	9.39072	1.00000	1.97061
C2	5.6436	8.67382	.64812	.05573	2.14609	1.11788	4.76539	.50746	1.00000

These was a general computaional schema of the unrestricted GEKS. In the practice, it is an usual case that not all BHs have price data and some reference PPPs should be used instead. There are two types of reference PPPs – hierarchical and non-chierarchical. Hierarchical reference PPPs are average PPPs within aggregated headings calculated for BHs with existing BH-PPPs – these should be calcualted automatically. Non-hierarchical reference PPPs are calculated as average from BHs from different aggregated headings – these should be programmed / treated case by case. **All reference PPPs should be agreed in advance.** The computaional tool should be flexible and be able to realize any schema of the references.

The cases can occur that **some countries have no NA expenditure data for a whole aggregated heading.** All ad hoc solutions (unweighted GM-PPPs or fictive very small expenditure) should be agreed also in advance.

The schema and respective calculations by the CAR-Volumes approach for the example above are presented in Table 11:

Table 11: Schema of the CAR-Volumes approach

ICP Handbook, Example from former Chapter 13												
CAR-Volume approach (EKS calculations)												
Expenditures in national currencies												
BH	Region A					Region B					Region C	
	Countries					Countries					Countries	
	1	2	3	4		1	2	3			1	2
1	2000	60	1200	9600		380	1000	3000			4500	225
2	500	18	400	2000		70	290	720			1200	67
3	800	35	700	4000		180	315	1260			1500	120
4	300	24	245	3300		70	170	500			800	36
1-4 = C	3600	137	2545	18900		700	1775	5480			8000	448
5	700	31	600	2800		180	430	1080			1500	75
6	700	32	700	2800		130	190	1280			2400	78
5+6 = I	1400	63	1300	5600		310	620	2360			3900	153
Total GDP (NC)	5000	200	3845	24500		1010	2395	7840			11900	601
Within-Reg. GDP-PPP	1	0.65043	8.7551	101.16	Reg.Volume	1	1.9503	0.44676	Reg.Volume		1	0.51162
Within-Reg. GDP Volume	5000	307	439	242	5989	1010	1228	17548	19787		11900	1175
Within-Reg. GDP Vol.Sh. (%)	83.5	5.1	7.3	4.0	100	5.1	6.2	88.7	100		91.0	9.0
Global GDP-PPP	1	0.65064	8.7075	101.26	Reg.Volume	2.6297	5.0484	1.1843	Reg.Volume		11.121	5.6436
Global GDP Volume	5000	307	442	242	5991	384	474	6620	7479		1070	106.5
GDP Volume with fixity	5002	307.6	439.3	242.3	5991	382	464	6633	7479		1071	105.7
Indirect Global PPP - fixity	1	0.65043	8.7551	101.16		2.6467	5.1617	1.1824			11.117	5.6876
Indirect Reg.PPP - fixity	1	0.65043	8.7551	101.16		1	1.9503	0.44676			1	0.51162

c. Eurostat – OECD Method of linking above Basic Heading

i. Brief Description

The Eurostat – OECD Method of linking above Basic Heading is also the CAR method using the unrestricted GEKS. Only the global Regional PPPs instead of regional Volumes are re-indexed here in accordance with the ratios of the PPPs obtained within the regional comparison-

CAR-PPP approach¹² (Eurostat-OECD approach):

$$\text{PPP „Country / World“} = \text{PPP „Country / Region“} * \text{PPP „Region / World“}$$

[Regional Comparison] [Global Comparison]

ii. Steps to implement method (with a fictitious example)

Practical steps are demonstrated by the same fictitious example from the former Chapter 13 of the ICP Manual:

- **3 Regions** (A – 4 countries; B – 3 countries; C – 2 countries)
- **6 BHs** (4 BHs for Consumption and 2 BHs for Investment)

Input data are presented in Tables 5 and 6:

The global unrestricted GEKS calculations are the same as for the CAR-Volume approach.

The regional average (GM) are calculated from the unrestricted global GEKS-PPPs for the aggregates – these regional GM can be considered as the PPP “Region / World”. If one multiply these PPP by the ratios of the country’s GEKS-PPPs to Regional GM from the Regional comparison then the PPP “Country / World” (with Regional fixity) are obtained. The respective Volumes (with fixity) are calculated in an indirect way as:

$$\text{Volume} = I_{\text{exp}} : \text{PPP}$$

The schema and respective calculations by the CAR-PPP approach for the example above are presented in Table 12

¹² This method was described in Ch.15 of the ICP Manual as the calculation of the Regional scaling factors by the unweighted geometric mean.

Table 12: Schema of the CAR-Volumes approach

ICP Handbook, Example from former Chapter 13												
CAR-PPP approach (EKS calculations)												
Expenditures in national currencies												
BH	Region A					Region B				Region C		
	Countries					Countries				Countries		
	1	2	3	4		1	2	3		1	2	
1	2000	60	1200	9600		380	1000	3000		4500	225	
2	500	18	400	2000		70	290	720		1200	67	
3	800	35	700	4000		180	315	1260		1500	120	
4	300	24	245	3300		70	170	500		800	36	
1.4 = C	3600	137	2545	18900		700	1775	5480		8000	448	
5	700	31	600	2800		180	430	1080		1500	75	
6	700	32	700	2800		130	190	1280		2400	78	
5+6 = I	1400	63	1300	5600		310	620	2360		3900	153	
Total GDP (NC)	5000	200	3845	24500	Reg.GM	1010	2395	7840	Reg.GM	11900	601	Reg.GM
Within-Reg. GDP-PPP	1	0.65043	8.7551	101.16	4.8991	1	1.95026	0.44676	0.95512	1	0.51162	0.71527
Within-Reg. PPP (Reg=1)	0.2041	0.1328	1.7871	20.648	1	1.04699	2.04191	0.46776	1	1.39807	0.71527	1
Global GDP-PPP	1	0.65064	8.7075	101.26	4.8941	2.6297	5.0484	1.1843	2.5052	11.121	5.6436	7.9223
Global GDP PPP with fixity	0.99899	0.64978	8.7463	101.053	4.8941	2.6229	5.1154	1.1718	2.5052	11.076	5.6666	7.9223
PPP with fixity (C.A1 = 1)	1	0.65043	8.7551	101.155		2.6256	5.1205	1.1730		11.087	5.6724	
Volumes with fixed PPP	5000	307	439	242	5989	385	468	6684	7536	1073	106	1179
Volume shares - fixed PPP	83.5	5.1	7.3	4.0	100	5.1	6.2	88.7	100	91.0	9.0	100

The within-regional results (PPP and VI) are equal by both approaches: CAR-Volumes and CAR-PPPs.

The differences in the between-regional results obtained by different CAR methods (Volumes or PPPs) are usually small. So, the results obtained by different CAR methods for the Regions and for the countries for the example above are presented in Table 13.

Table 13: CAR-Volume vs. CAR-PPP

(example from former Chapter 13 of the ICP Handbook)

	CAR-Volume		CAR-PPP	
	Volume	World Share	Volume	World Share
Reg.A	5991	40.90	5989	40.73
Reg.B	7479	51.06	7536	51.25
Reg.C	1177	8.03	1179	8.02
World	14646	100	14704	100

	CAR-Volume		CAR-PPP	
	Volume	World Share	Volume	World Share
A1	5002	34.15	5000	34.00
A2	308	2.10	307	2.09
A3	439	3.00	439	2.99
A4	242	1.65	242	1.65
B1	382	2.61	385	2.62
B2	464	3.17	468	3.18
B3	6633	45.29	6684	45.45
C1	1071	7.31	1073	7.30
C2	106	0.72	106	0.72
World	14646	100	14704	100

3. Linking Singleton Countries and Regions (possible options)

There are singleton countries and (semi) singleton Regions in the ICP 2011.

(Semi) singleton Regions are the CIS and the Pacific islands¹³.

CIS Region

Due to double participation of Russia (OECD and CIS Regions) the strong intention to have for Russia fixed results from the OECD comparison, it was decided by the CIS countries that Russia belongs (for fixity) in the ICP 2011 to the OECD-Eurostat region and other CIS countries will be linked to the World comparison via the results with Russia within the CIS comparison (like in the ICP 2005) – see, for example, the Minutes of the EB ICP meeting, Feb'2010, page 6: *“The CIS region will be linked to the OECD through Russia as a bridge country”*.

¹³ It is assumed that the Caribbean countries will be included in the LAC.

This decision means:

- There will be only one set of results for Russia, i.e. the OECD results
- CIS data will not participate in the multilateral calculations for global comparison¹⁴
- CIS countries will be linked to OECD-Eurostat and the World through Russia (CIS comparison) as a bridge country at the BH as well as at the aggregated levels

Of course, it is not very good that the CIS countries are linked to the World via one country (RUS) only but this was own CIS decision, to avoid double results for Russia. It would be better to have an expanded region OECD-Eurostat-CIS but it is doubtful the OECD and Eurostat agree that the Linking factor to the World for the OECD-Eurostat region depends on CIS data.

Pacific Islands

Pacific Island will price very limited list of products. Therefore they will not participate in the multilateral calculations for global comparison and will be linked to the World at high aggregated levels through some ADB countries and Australia as the bridges.

Singleton country Georgia

Georgia is no longer a member of the CIS and a single country in the ICP 2011. It was agreed that Georgia is carried out a **bilateral comparison with Armenia**. As Armenia is a member of the CIS, which is an ICP Region, the bilateral comparison will allow Georgia to be linked through Armenia with the other regions of the 2011 global comparison. In accordance with this approach, Georgian data will not participate in the multilateral calculations for global comparison and Georgia will be linked to the World through Armenian results in the World comparison as a bridge country at the BH as well as at the aggregated levels.

Singleton country Iran

The ADB no longer offers to include Iran under its coordination in the current round (Iran is not the ADB member). A special arrangement has been sought by the Global Office to link Iran to the ICP via another country or region. Turkish Statistical Institute (TurkStat) has accepted to be involved in a bilateral comparison proposal aimed at linking Iran to the global result through Turkey, a participant in the Eurostat – OECD PPP Program. The approach of linking Iran to the ICP is different for different fields:

- For the Household Consumption Price Survey, Iran and Turkey would carry out a bilateral comparison based on a common list of items priced by the both countries.
- For Housing, Government Compensation, Machinery and Equipment, Construction, Health, and Education, Iran would follow the standard ICP methodology and pricing schedule.

Respectively, the linking Iran to the World for HH should be same as for Georgia. Iran will be linked to the World through Turkish results in the World comparison for HH as a bridge country at the BH as well as at the aggregated levels.

¹⁴ It means that only 5 Regions (Eurostat-OECD, Asia, Western Asia, Africa and LAC) will participate in the calculation of the between-regional PPPs.

Concerning the other fields (Housing, Government Compensation, Machinery and Equipment, Construction, Health, and Education) where Iran will collect prices from the ICP Core List only and there will not be a bilateral comparison with Turkey an other approach can be considered:

Housing, Government Compensation, Machinery and Equipment, Construction, Health, Education

Linking at the BH Level

It is possible to link Iran to the World (without Iran) using **the same procedure as for the between-regional PPPs. Iran can be considered as a mini-region.**

This is a kind of two stage CPD procedure. It is assumed that the calculation of between-regional PPPs (for 5 Regions) was made and the global BH-PPPs were obtained. The prices of all countries in the World for the Core List items are recalculated into the World numeraire. These country's price in the World numeraire are combined with Iranian prices in NC and **weighted CPD for two regions (World with country's prices and Iran)** is used for this set of data. World numeraire is 1 and Iranian PPP relatively World numeraire will be obtained from the CPD. All former World PPPs are not changed due to this procedure (fixity is kept).

Linking at the aggregated Levels

If there is an intention to keep the aggregated results of the bilateral comparison Iran – Turkey for HH unchanged in the World comparison then the same procedure as for Georgia should be used. However it seems that this is practicable at the BH level but not at the aggregated levels.

Iran can be treated in the World aggregation in the same way as all other countries. So, it is assumed that Iran has for the aggregation the same set of input data as all other countries:

BH-PPPs (to a World numeraire)

and

BH expenditure in the standard ICP Classification.

The unrestricted GEKS is applied for all involved countries (incl. Iran). This allows to obtain Iranian Volumes (real expenditure) for the analytical categories, aggregates and GDP in the World numeraire. After this in the accordance with the agreed CAR-Volume approach the Regional Volume Totals are redistributed within the Regions in accordance with the country's shares from the Regional comparisons. Iranian Volume is kept as it was obtained from the unrestricted GEKS.

4. Linking Countries with double participation (possible options)

Russia participates in the OECD comparison as well as in the CIS comparison.

Egypt and Sudan participate in the African comparison as well as in the Western Asia comparison.

How these countries with double participation should be treated within the Global comparison?

There is no actual problem with **Russia** because it was officially agreed that Russia will be included for the Global comparison in the OECD Region. The CIS countries will have the link to

the Global comparison via RF only. So fixity of the results within the CIS comparison will be kept in this way.

The situation with Egypt and Sudan need a special treatment because both Regions (Africa and Western Asia) want to include these countries in the Regional results. It is impossible to keep fixity for these countries in both Regional comparisons. Therefore some averages from regional results should be used like it was done for Egypt in the ICP 2005.

The possible procedures at the BH and aggregated levels are described below.

Linking at the BH Level

Option 1

If the agreed version of the **calculation of between-regional BH-PPPs with individual country's data is applied then two sets of price data for Egypt and Sudan should be included in the computations of the between regional BH-PPPs by the weighted CPD:**

- one set of prices should be recalculated in the African numeraire on the basis of the BH-PPPs from the African comparison
- and
- another set of prices – in the ESCWA numeraire on the basis of the BH-PPPs from the Western Asia comparison.

This is logically – both countries participate in both comparisons and all between-regional PPP should be based on input data from both Regional comparison for Egypt and Sudan. *[If the regional average GM prices are used then this approach is especially straightforward]*

Respective two global PPPs with the Regional fixity in both Regions will be calculated for **Egypt** and **Sudan** as the following:

$$\text{PPP1 „Egypt / World“} = \text{PPP „Egypt / Africa“} * \text{PPP „Africa / World“}$$

[Regional Comparison] [Global Comparison]

$$\text{PPP2 „Egypt / World“} = \text{PPP „Egypt / ESCWA“} * \text{PPP „ESCWA / World“}$$

[Regional Comparison] [Global Comparison]

Geometric mean from PPP1 and PPP2 should be considered as global BH-PPP for Egypt:

$$\text{PPP „Egypt / World“} = (\text{PPP1 „Egypt / World“} * \text{PPP2 „Egypt / World“})^{1/2}$$

The same procedure should be carried out also for **Sudan**:

$$\text{PPP1 „Sudan / World“} = \text{PPP „Sudan / Africa“} * \text{PPP „Africa / World“}$$

[Regional Comparison] [Global Comparison]

$$\text{PPP2 „Sudan / World“} = \text{PPP „Sudan / ESCWA“} * \text{PPP „ESCWA / World“}$$

[Regional Comparison] [Global Comparison]

Geometric mean from PPP1 and PPP2 should be considered as global BH-PPP for Sudan:

$$\text{PPP „Sudan / World“} = (\text{PPP1 „Sudan / World“} * \text{PPP2 „Sudan / World“})^{1/2}$$

This is a symmetrical way to include Egypt and Sudan in the Global comparison at the BH level on the basis of input data from both regions.

Option 2

If one does not want to use double sets of prices from Egypt and Sudan in the global comparison then the unrestricted weighted CPD within individual original country's price data should be used. This produces the country's PPPs. The CAR-PPP can be applied to this set. Both regional average PPPs - PPP „Africa / World“ and PPP „Western Asia / World“ - are calculated as the GM with the inclusion of Egypt and Sudan.

Respective two global PPPs with the Regional fixity in both Regions will be calculated for **Egypt** and **Sudan** during the CAR-PPP procedure as the following:

$$\text{PPP1 „Egypt / World“} = \text{PPP „Egypt / Africa“} \times \text{PPP „Africa / World“}$$

[Regional Comparison] [Global Comparison]

$$\text{PPP2 „Egypt / World“} = \text{PPP „Egypt / ESCWA“} \times \text{PPP „ESCWA / World“}$$

[Regional Comparison] [Global Comparison]

Geometric mean from PPP1 and PPP2 should be considered as global BH-PPP for Egypt:

$$\text{PPP „Egypt / World“} = (\text{PPP1 „Egypt / World“} \times \text{PPP2 „Egypt / World“})^{1/2}$$

The same procedure is used for Sudan.

Linking at the aggregated Levels

It is assumed that Egypt and Sudan have the same NA data in both Regions.

Option 1

If the CAR-Volumes approach recommended as the official ICP 2011 method is used then the possible procedure is described below:

- the unrestricted GEKS method produces the country's PPPs,
- the NA aggregates in national currencies are recalculated by the PPPs into Volumes (real expenditure)
- Regional Volumes for both Regions - Volume „Africa“ and Volume „Western Asia“ - are calculated with the inclusion of Egypt and Sudan
- Respective two Volumes with the Regional fixity in both Regions are calculated for **Egypt** and **Sudan**

$$\text{Volume1 „Egypt“} = \text{Volume „Africa“} \times \text{Volume Share „Egypt / Africa“}$$

[Global Comparison] [Regional Comparison]

$$\text{Volume2 „Egypt“} = \text{Volume „Western Asia“} \times \text{Volume Share „Egypt / Western Asia“}$$

[Global Comparison] [Regional Comparison]

$$\text{Volume1 „Sudan“} = \text{Volume „Africa“} \times \text{Volume Share „Sudan / Africa“}$$

[Global Comparison] [Regional Comparison]

$$\text{Volume2 "Sudan"} = \frac{\text{Volume "Western Asia"} \times \text{Volume Share "Sudan / Western Asia"}}{\frac{[\text{Global Comparison}]}{[\text{Regional Comparison}]}}$$

Arithmetic means from Volume1 and Volume2 should be considered as Volume in the Global comparison for Egypt and Sudan:

$$\text{Volume "Egypt"} = (\text{Volume1 "Egypt"} + \text{Volume2 "Egypt"}) / 2$$

$$\text{Volume "Sudan"} = (\text{Volume1 "Sudan"} + \text{Volume2 "Sudan"}) / 2$$

The indirect global PPPs for Egypt and Sudan are derived after this in the standard way as

$$\text{PPP} = \text{National Expenditure} / \text{Volume}$$

Option 2

If the CAR-PPP approach (Eurostat-OECD) is used then the possible procedure is described below:

- the unrestricted GEKS method produces the country's PPPs,
- Both regional average PPPs - PPP „Africa / World“ and PPP „Western Asia / World“ - are calculated as the GM with the inclusion of Egypt and Sudan
- Respectively two global PPPs with the Regional fixity in both Regions will be calculated for **Egypt** and **Sudan** during the CAR-PPP procedure as the following:
- $\text{PPP1 „Egypt / World“} = \frac{\text{PPP „Egypt / Africa“}}{[\text{Regional Comparison}]} \times \frac{\text{PPP „Africa / World“}}{[\text{Global Comparison}]}$
- $\text{PPP2 „Egypt / World“} = \frac{\text{PPP „Egypt / ESCWA“}}{[\text{Regional Comparison}]} \times \frac{\text{PPP „ESCWA / World“}}{[\text{Global Comparison}]}$
- Geometric mean from PPP1 and PPP2 should be considered as global BH-PPP for Egypt:
- $\text{PPP „Egypt / World“} = (\text{PPP1 „Egypt / World“} \times \text{PPP2 „Egypt / World“})^{1/2}$

The same procedure is used for Sudan.

$$\text{PPP „Sudan / World“} = (\text{PPP1 „Sudan / World“} \times \text{PPP2 „Sudan / World“})^{1/2}$$

The global Volumes for Egypt and Sudan are derived after this in the standard way as

$$\text{Volume} = \text{National Expenditure} / \text{PPP}$$

References

- Cuthbert, J., M. Cuthbert 1988. "On Aggregation Methods of Purchasing Power Parities", *OECD Working Paper*, OECD, Paris. <http://www.oecd.org/dataoecd/15/26/2002058.pdf>
- Diewert, W. E. (2004) "On the Stochastic Approach to Linking the Regions in the ICP", Discussion Paper No. 04-16, Department of Economics, University of British Columbia, <http://www.econ.ubc.ca/diewert/icp.pdf>
- Diewert E. (2008) "New Methodological Developments for the International Comparison Program", Joint UNECE/ILO Meeting on Consumer Price Indices (Geneva, 8-9 May 2008). The latest version of this paper can be found on the web-site: <http://www.econ.ubc.ca/diewert/dp0808.pdf>
- Diewert E. (2010) "Approaches to Linking the Regions", paper for the TAG ICP meeting (World Bank, 17-19 February 2010).
- Dikhanov Y. (1994) „Sensitivity of PPP-Based Income Estimates to Choice of aggregation Procedures“ (IARIW Conference; St.Andrews, New Brunswick, Canada on 21-27 August 1994). A 1997 version of this paper can be found on the WB ICP web-site <http://siteresources.worldbank.org/ICPINT/Resources/icppapertotal.pdf>
- Dikhanov, Yuri (2007), "Two Stage Global Linking with Fixity: Method 1 (EKS)". World Bank, Mimeo
- Heston, Alan (1986), "World Comparisons of Purchasing Power and Real Product for 1980". United Nations and Eurostat, New York: United Nations
- Heston A., Summers R., Aten A. (2001) "Price Structures, the Quality Factor, and Chaining". WB / OECPP PPP Seminar (Washington; 30.01-02.02.2001)
- Heston A. ,Aten B. (2002) "Linking Country Groups in International Real Product and Purchasing Power Comparisons", World Bank ICP Conference, Washington, March 11-15, 2002 <http://pwt.econ.upenn.edu/papers/heston-atenwb.pdf>
- Heston A., B. Aten (2009) "Are All Fishers Equal?". http://pwt.econ.upenn.edu/papers/AH_BA_april30.pdf
- Heston A. (2010) "Background Note on Productivity Adjustments as a Special Case of a General Regional Linking Problem", Paper prepared for ICP Technical Advisory Group, (Washington, October 2010). http://siteresources.worldbank.org/ICPINT/Resources/270056-1255977007108/6483550-1257349667891/6544465-1285250669548/05.01_ICP-TAG04_ProductivityAdjustment.pdf
- ICP Manual, Chapters 11-15. World Bank, 2007.
- ICP Manual, Chapter 15 "Linking PPPs and Real Expenditures for GDP and Lower Level Aggregates (Two-stage methods and scalar adjustments)", World Bank, 2007 http://siteresources.worldbank.org/ICPINT/Resources/270056-1183395201801/icp_Ch15rev.doc

Hill R.J., Hill T.P. "Regionalization and its Implications for Price Index Construction: The Case of the International Comparisons Program", School of Economics UNSW, Sydney 2052, Australia, School of Economics Discussion Paper: 2007/28

http://www.docs.fce.unsw.edu.au/economics/Research/WorkingPapers/2007_28.pdf

Hill, Robert (2011), "Linking the Regions in the International Comparisons Program at Basic Heading Level and at Higher Levels of Aggregation". Paper prepared for ICP Technical Advisory Group meeting, (Washington, 18-19 April 2011) – see Doc. 8.02.

Koves P. (1983) "Index Theory and Economic Reality", Akademia Kiado, Budapest.

Kravis, I.B., A.W. Heston, R. Summers (1982), „World Product and Income, International Comparison of Real Gross Product“, Baltimore, John Hopkins Univ. Press

Prasada Rao D.S. (2001) "Weighted EKS and Generalized CPD methods for Aggregation at Basic Heading Level and above Basic Heading Level" – a paper for the Joint WB / OECD Seminar on PPPs (Washington, 30.01-02.02.2001).

Sergeev S. (2001a) "Aggregation methods on the basis of structural international prices" (Joint World Bank - OECD Seminar on PPPs „Recent Advances in Methods and Applications“, Washington, D.C.; 30.01-02.02 2001)

http://siteresources.worldbank.org/ICPINT/Resources/ss_WBOECD_v2004full.doc

Sergeev S. (2005a) "Calculation of the results of the Eurostat GDP comparison with the use of fixity" http://siteresources.worldbank.org/ICPINT/Resources/FIXITY_PPP_vs_ShVol_SS53.DOC

Sergeev S. (2005b), "The use of weights (indication of representativity) within the CPD and EKS methods at the basic heading level", Paper prepared for ICP Technical Advisory Group.

http://siteresources.worldbank.org/ICPINT/Resources/Use_of_Weights.doc

Sergeev, S. (2009a), "Aggregation Methods Based on Structural International Prices", pp. 274-297 in Purchasing Power Parities of Currencies: Recent Advances in Methods and Applications, D.S. Prasada Rao (ed.), Cheltenham UK: Edward Elgar.

http://siteresources.worldbank.org/ICPINT/Resources/ss_WBOECD_v2004full.doc

Sergeev, S. (2009b), "The Evaluation of the Approaches Used for the Linking of the Regions in the ICP 2005", unpublished paper, Statistics Austria, December 2009.

Sergeev, Sergey (2011), "Fixity of Volumes (shares) as the Least absolute deviations (LAD) and Least square deviations (LSQ) procedures." Paper prepared for ICP Technical Advisory Group.

Sergeev, Sergey (2011), "Possible approaches for the linking of the regions at the BH and aggregated levels for the ICP 2011", paper prepared for ICP Technical Advisory Group meeting (Washington, 18-19 April 2011) – see Doc. 8.01.

http://siteresources.worldbank.org/ICPINT/Resources/270056-1255977007108/6483550-1257349667891/08.01_ICP-TAG05_LinkingRegions.pdf