A Symmetric Version of the Eurostat-OECD Method

Robert Hill

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1. Symmetric Treatment of Prices and Quantities

$$P_{Aj}^{global} = P_{Aj}^{region} \left[\prod_{a=1}^{N_A} \left(\frac{P_{Aa}^{unfixed}}{P_{Aa}^{region}} \right)^{1/N_A} \right]$$

$$Q_{Aj}^{global} = Q_{Aj}^{region} \left[\prod_{a=1}^{N_A} \left(\frac{Q_{Aa}^{unfixed}}{Q_{Aa}^{region}} \right)^{1/N_A} \right]$$

2. Strong Factor Reversal Test

$$\frac{P_{Bk}^{global} \times Q_{Bk}^{global}}{P_{Aj}^{global} \times Q_{Aj}^{global}} = \frac{\Sigma_{i=1}^{I} p_{Bk,i} q_{Bk,i}}{\Sigma_{i=1}^{I} p_{Aj,i} q_{Aj,i}}$$

3. Comparison with CAR

$$Q_{Aj}^{global} = Q_{Aj}^{region}\left(\frac{\Sigma_{a=1}^{N_A} Q_{Aa}^{unfixed}}{\Sigma_{a=1}^{N_A} Q_{Aa}^{region}}\right)$$

4. Generalization of GEKS Least Squares Result

$$\text{Min}_{\ln(\lambda)} \sum_{a=1}^{N_A} \left[\ln(P_{Aa}^{global}) - \ln(P_{Aa}^{unfixed})\right]^2$$

subject to

$$P_{Aa}^{global} = \lambda P_{Aa}^{region}.$$

$$\text{Min}_{\ln(\lambda)} \sum_{a=1}^{N_A} \left[\ln(\lambda) + \ln(P_{Aa}^{region}) - \ln(P_{Aa}^{unfixed})\right]^2$$

5. Modelling the Covariance Matrix

The least squares problem above assumes that the covariance matrix Σ is the identity I.

But some countries have smaller between-region Paasche-Laspeyres spreads than others.

So we could instead assume that $\Sigma = \sigma^2 D$, where D is a diagonal matrix such that:

$$D_{Aa} = \frac{1}{K} \sum_{k=B}^{K} \left[\frac{1}{N_k} \sum_{j=1}^{N_k} \left| \ln \left(\frac{P^L_{Aa,kj}}{P^P_{Aa,kj}} \right) \right| \right] .$$

6. Symmetric Treatment of Regions

$$\frac{P_{Bk}^{global}}{P_{Aj}^{global}} = \left(\frac{P_{Bk}^{region}}{P_{Aj}^{region}}\right) \left[\frac{\Pi_{b=1}^{N_B}\left(\frac{P_{Bb}^{unfixed}}{P_{Bb}^{region}}\right)^{1/N_B}}{\Pi_{a=1}^{N_A}\left(\frac{P_{Aa}^{unfixed}}{P_{Aa}^{region}}\right)^{1/N_A}}\right]$$

Four countries in Region A

Two countries in Region B

$$\frac{P_{B1}^{global}}{P_{A1}^{global}} = \left(\frac{P_{B1}^{region}}{P_{A1}^{region}}\right) \times$$

$$\frac{\left[\left(\frac{P_{B1}^{unfixed}}{P_{B1}^{region}}\right)\left(\frac{P_{B2}^{unfixed}}{P_{B2}^{region}}\right)\left(\frac{P_{B1}^{unfixed}}{P_{B1}^{region}}\right)\left(\frac{P_{B2}^{unfixed}}{P_{B2}^{region}}\right)\left(\frac{P_{B1}^{unfixed}}{P_{B1}^{region}}\right)\left(\frac{P_{B2}^{unfixed}}{P_{B2}^{region}}\right)\left(\frac{P_{B1}^{unfixed}}{P_{B1}^{region}}\right)\left(\frac{P_{B2}^{unfixed}}{P_{B2}^{region}}\right)\right]^{1/8}}{\left[\left(\frac{P_{A1}^{unfixed}}{P_{A1}^{region}}\right)\left(\frac{P_{A1}^{unfixed}}{P_{A1}^{region}}\right)\left(\frac{P_{A2}^{unfixed}}{P_{A2}^{region}}\right)\left(\frac{P_{A2}^{unfixed}}{P_{A2}^{region}}\right)\left(\frac{P_{A3}^{unfixed}}{P_{A3}^{region}}\right)\left(\frac{P_{A3}^{unfixed}}{P_{A3}^{region}}\right)\left(\frac{P_{A4}^{unfixed}}{P_{A4}^{region}}\right)\left(\frac{P_{A4}^{unfixed}}{P_{A4}^{region}}\right)\right]^{1/8}}$$

$$\mathbf{A1-B1}: \left(\frac{P_{B1}^{region}}{P_{A1}^{region}}\right)\left(\frac{P_{B1}^{unfixed}/P_{B1}^{region}}{P_{A1}^{unfixed}/P_{A1}^{region}}\right)=\frac{P_{B1}^{unfixed}}{P_{A1}^{unfixed}}$$

$$\mathbf{A1}-\mathbf{B2}-B1: \left(\frac{P_{B1}^{region}}{P_{A1}^{region}}\right)\left(\frac{P_{B2}^{unfixed}/P_{B2}^{region}}{P_{A1}^{unfixed}/P_{A1}^{region}}\right)=\left(\frac{P_{B2}^{unfixed}}{P_{A1}^{unfixed}}\right)\left(\frac{P_{B2}^{region}}{P_{B1}^{region}}\right)$$

$$A1-\mathbf{A2}-\mathbf{B1}: \left(\frac{P_{B1}^{region}}{P_{A1}^{region}}\right)\left(\frac{P_{B1}^{unfixed}/P_{B1}^{region}}{P_{A2}^{unfixed}/P_{A2}^{region}}\right)=\left(\frac{P_{B1}^{unfixed}}{P_{A2}^{unfixed}}\right)\left(\frac{P_{A2}^{region}}{P_{A1}^{region}}\right)$$

A1-**A2**-**B2**-B1

A1-**A3**-**B1**

A1-**A3**-**B2**-B1

A1-**A4**-**B1**

A1-**A4**-**B2**-B1