THE WORLD BANK GROUP ARCHIVES

PUBLIC DISCLOSURE AUTHORIZED

Folder Title: Lal, Deepak - Articles and Speeches (1971) - 1v

Folder ID: 1652091

Fonds: Records of Office of External Affairs (WB IBRD/IDA EXT)

Digitized: February 18, 2014

To cite materials from this archival folder, please follow the following format: [Descriptive name of item], [Folder Title], Folder ID [Folder ID], World Bank Group Archives, Washington, D.C., United States.

The records in this folder were created or received by The World Bank in the course of its business.

The records that were created by the staff of The World Bank are subject to the Bank's copyright.

Please refer to http://www.worldbank.org/terms-of-use-earchives for full copyright terms of use and disclaimers.



© 2012 International Bank for Reconstruction and Development / International Development Association or The World Bank 1818 H Street NW Washington DC 20433

Telephone: 202-473-1000 Internet: www.worldbank.org

Lal, Deepax - ARticles and speeches (1871)

DECLASSIFIED WBG Archives



1652091

A1992-007 Other#: 11

212056B

Lal, Deepak - Articles and Speeches (1971) - 1v



ADJUSTMENTS FOR TRADE DISTORTIONS IN PROJECT ANALYSIS

by

Deepak Lal

Research Seminar

A seminar will be held on Friday, October 1, at 4 p.m. in Room Cloo6 on "Adjustments for Trade Distortions in Project Analysis."

Deepak Lal

Chairman: P. D. Henderson

Montek Singh Ahluwalia and Helen Hughes

surpe following appointments will be effective on Octobe

Copies of the paper are available in Room B1215, Extension 4004.

domestic welfare, given existing resources, technology and foreign trade possibilities. As a result it has been suggested by a number of writers (2, 4, 6, 11, 137 that in making future investment decisions, "shadow" prices, which reflect the true social costs and benefits of inputs and outputs be used rather than the distorted market prices. Thereby the country would be able to develop along the lines of its comparative advantage, which are obscured by the varying, inoptimal and often "ad hoc" controls on foreign trade. A number of ways in which these trade distortions can be taken into account in project analysis have been suggested in the literature /1, 2, 3, 6, 9, 11, 12, 13/. Some attempts have been made to try and relate these differing methods to show what differences there are, if any, in the assum ptions on which they are based /1, 2, 3, 8/. In my view, however, these treatments are not completely satisfactory, in pinpointing (a) the nature of the

adjustments for trade distortions which should be made in project analysis; (b) the particular assumptions underlying the different procedures which have been suggested; (c) the differences in using the procedures in practice. This paper seeks to first set out in terms of simple comparative static, trade theoretic, general equilibrium models, the nature of the adjustments which are required (Section 1); secondly, to show the implicit or explicit assumptions the alternative procedures make about the adjustment process and/or various structural features of the economy (Section II); thirdly, to evaluate the usefulness in practice of the alternative methods (Section III).

I. Some Simple Theory

In this paper we are concerned with project analysis in the presence of trade distortions. As such we assume away all other distortions in the economy, which is thus assumed to correspond to the perfectly competitive neo-classical paradigm in all other respects.

In the absence of all distortions (including trade distortions) simple welfare economics tells us that, resource allocation based on market prices would be optimal. Moreover, the prices of goods and factors would equal and equate the marginal social cost (NSC) of producing and marginal social value (NSV) of using the relevant goods/factors.

In general, distortions in factor and/or commedity markets (which includes the markets for foreign goods and services) drive a wedge between the NSV and NSC of a good/factor. The market price (P) of the relevant good/factor will then equal either the NSV or NSC (or neither, in some cases of rationing!) of the good, but will not equal and equate both MSV and MSC. The first best solution in such cases is always to

correct the distortion at its source, so that the relationship

P = MSV = MSC holds for all goods/factors. Resource allocation would

then be optimal at market prices and project evaluation with a system

of 'shadow' pricing would be redundant. However, especially in

developing countries, for various political and/or administrative

reasons, it may be infeasible at least in the short run, to achieve the

first best solution, and project evaluation using 'shadow' prices

may be required, as a second best method, to move the economy in the

direction of optimal resource-allocation.

Non-optimal trade controls result in two broad sets of distortions in the domestic price structure. These are distortions in relative prices within the traded goods sector and distortions in the relative price of traded to non-traded goods. To demonstrate this, and to pin-point the ensuing adjustments which have to be made to market prices, to obtain the 'shadow' prices for project evaluation in the presence of these distortions, we consider two highly simplified, trade-theoretic general equilibrium models.

^{1/} In thinking about the problems discussed in this paper I have been much influenced by Max Corden's: The Theory of Protection (OUP, 1971); and the simple two and three good models which follow are based on this work.

Case 1 - Two Traded goods produced and Consumed

We first consider the case where there are just two goods, an importable M and an exportable X, being produced domestically with fixed stocks of capital K and labor L. Making the small country assumption, the foreign (border') prices of the two goods P_{xf} , P_{mf} are fixed and given. If there is free trade, then (from Samuelson's theorem on the correspondence between factor and commodity prices) the relative prices of the two factors K and L are uniquely determined. If even a single domestic money price of the two commodities (X, M) or two factors (K, L), or the exchange rate (which converts domestic money prices into foreign money prices) is now given, all the other domestic money prices will be uniquely determined.

Moreover, a change in the foreign exchange rate would have no real effects on the economy as it would affect only the absolute level of domestic money prices, without effecting the relative 'border' price structure. Furthermore, a balance of payments deficit in this model cannot be cured by exchange rate changes, as the expenditure switching effects of a de(re)valuation are non-existent, because a de(re)valuation alters the domestic prices of X and M by equiproportionate amounts. The only cure for a balance of payments deficit would be expenditure reduction. Furthermore, in this case, the optimal pattern of production and of trade will be uniquely determined by the given world ('border') prices of the two commodities.

The domestic market prices of goods and factors would also be their "shadow" prices, and investment decisions based on them would be optimal. Now suppose that a tariff of this imposed on the importable. The exchange rate remains fixed. This will change the domestic relative price of the two commodities (X, M) from their border "relative" price, which will induce changes in domestic production and consumption, and in the domestic factor price ratio. Moreover, assuming that the government maintains internal balance of appropriate fiscal and monetary policy, total domestic expenditure (measured at the domestic relative prices existing in the free trade situation) will have fallen. Both imports and exports will shrink. All this is shown in the standard two factor - two commodity international trade theory diagram in Fig. 1.

Suppose we are now asked to evaluate the relative desirability of a marginal investment project for producing X or M, in the tariff distorted situation. At the existing domestic market prices, there is nothing to choose between the two projects. However, the tariff has introduced a distortion which does not enable us to maximize feasible welfare. It has, as it were, introduced a wedge between the MSC of producing and the MSV of using a unit of foreign exchange. The former is given by the domestic resource cost of a unit of exports, the latter by the domestic price (value) of a unit of imports to consumers. Valuing goods and factors in domestic currency, one unit of export earns, say, $P_{\rm Xf}$ in foreign currency which converted at the official exchange rate, e, yields a domestic value of $eP_{\rm Xf}$. But this understates the benefit from the $P_{\rm Xf}$ units of foreign exchange obtained by exporting.

For if the foreign currency price of M is Pmf, this will enable imports of Pxf/Pmf units, whose domestic value, given the tariff of t%, is e(1+t) Pxf. The domestic social value of one unit of exports therefore is e(1+t) Pxf. If we now evaluate the project, in terms of domestic currency, taking the shadow 'price' of the X good as e(1+t) Pxf The M good's domestic price is e(1+t)Pmf, and this is also its 'shadow' price/, we will find that production of X relative to M is more profitable at the existing domestic factor prices. Production decisions taken in line with this shadow price will move the economy towards the optimal production point, P in Fig. 1. This factor e(1+t), is sometimes identified with the shadow exchange rate (S.E.R.). It is an exchange rate in the sense that the tariff has resulted in a non-unified exchange/rate. There are two different effective exchange rates which apply to imports and exports /that is rates which convert domestic money prices into foreign money prices of the two goods e for export, and e(1+t) for imports. Optimality requires a unified exchange rate. Hence the price of exports must also be multiplied by e(1+t) to get the right investment decision, or alternatively all foreign exchange values have to be multiplied by this S.E.R., e(1+t). This will restore the correct 'shadow' relative price structure of the two goods to the 'border' price one.

But equivalently, we could have taken the value of M net of its tariff rate (t), and we would have got the same result. Whether we choose to use an exchange rate of e(1+t) or e, to convert foreign prices into domestic prices is irrelevant, in this model, as long as we get the correct relative valuation of the two goods which is

given by the relative 'border' prices. If we had decided to use foreign currency as our numeraire we would have just taken the foreign currency prices of the two goods as our "shadow prices", and converted the domestic factor prices into foreign exchange (fe) equivalents. Note that in this case to make the right choice between X and M, it is irrelevant whether we use the existing factor prices and convert them at the official exchange rate of (e) to get their foreign exchange equivalents, or at the SER of e(1 + t).

If in the post-protection situation we have the following cost conditions,

$$A_{lm}W + A_{kml}R = eP_{mf}(l + t)$$

$$A_{lx}W + A_{kx}R = eP_{xf}$$
(1)

where $A_{i,j}$ is the input of the ith factor (i = K, L) in the Jth industry (J = X, M) and W and R and the wage and rental rates.

Assuming fixed coefficient,

In free trade we have

where starred values represent the free trade factor prices.

If we use (1) then, using SER of e(1 + t), we get the 'social'costs

$$A_{lm}W + A_{km}R = eP_{inf}(l + t)$$

 $A_{lx}W + A_{kx}R < eP_{xf}(l + t)$

If we deflate both the W and R terms by (1+t) in (1'), we still get the production of X as more profitable than M, or alternatively if we use the values W and R in both industries as in (2) we still get X as more profitable than M. Alternatively working in foreign currency, we would have from (2).

$$\Lambda_{lm} \frac{\vec{W}}{e} + \Lambda_{km} \frac{\vec{R}}{e} > P_{mf}$$

$$A_{lx} \frac{W^* + A_{xf}}{e} \frac{R^* = P_{xf}}{e}$$

and again X is relatively more profitable than M.

In this simple model, where only traded goods are produced and consumed, therefore, the only adjustment required to get the right investment decisions, for moving the economy to the optimal production point P, in Fig. 1, is to correct the distortion in the relative price of the two traded goods. This can be done by 3 equivalent methods.

- (1) using domestic currency as the numeraire and using the effective import exchange rate, (that is the rate which converts the foreign currency price of imports into domestic prices, e(1+t)) as the S.E.R., for valuing all foreign currency transactions. This is the method suggested by a number of writers, UNIDO / 137, Harbergee / 67, Schydlowsky, / 127.
- (2) Using foreign currency as the numeraire, and taking the foreign currency 'border' prices as the shadow prices of the traded goods, and deflating the domestic factor prices by either the S.E.R.

or official exchange rate to get its(fe) equivalents. This will be the Little- Mirrlees procedure [11], in this case.

(3) working out the domestic resource cost per unit of foreign exchange earned/saved by producing another unit of X/M, and comparing the two ratios with (in this case) the official exchange rate. From (1) for M this is

$$\frac{\text{Alm}^{W + AR}}{\text{Pmf}} > e$$

for X, it is

$$\frac{\text{A W + A R}}{\text{lm} \quad \text{km}} = e$$

and again X will be preferred. This is the procedure suggested by Bruno [h] and Kruger [9].

Case 2: Two Traded, One Non-Traded Good Produced and Consumed

as there have been no non-traded goods in the model. The introduction of these redically changes the effects of exchange rate alterations on resource allocation, and introduces another distortion which is caused by the protective structure. In addition to the distortion of the relative prices within the traded good sector (which was the only distortion we had in Case I), we now have a further distortion as between the relative price of traded to non-traded goods. To see this, and the relevant adjustments necessary for project evaluation, we expand the previous

model, by including a non-traded good N, which is domestically produced and consumed. We maintain our assumptions of the absence of intermediate inputs and the lack of any domestic distortions apart from the tariff on M. (There are no distortions in foreign trade given our assumption of fixed and constant terms of trade). We now observe the economy, with domestic prices, EP_{xf} , EP_{mf} (1 * t), and P_n of the three goods X, M, and N, in the post-protective situation, with an exchange rate e. The economy is in internal and external balance, with W and R as the domestic money wage and rental rates of the two factors of production K, and L (Row 1, Table 1).

We now have the following production relationships.

$$A_{lm}W + A_{km}R = eP_{mf}(1+t)$$

$$A_{lx}W + A_{kx}R = eP_{xf}$$

$$A_{ln}W + A_{ln}R = P_{n}$$
(3)

At the existing domestic market prices, we are indifferent whether a marginal increase in domestic resources is invested in X, M or N.

However, as we have noted in Case I, valuation at domestic prices, understates the relative social benefit from producing X to M, and the adjustment discussed for Case I, is necessary to correct this; either of the three methods outlined will give the correct investment decisions, comparing X to M. What of the comparison of investments in N and M or N and X? This depends crucially upon what we expect will happen in the future to the protective structure.

(a) First let us assume that the existing protective structure will remain unchanged. As 'ex hypothesi' a marginal investment project will not change the W's and R's in (3), the MSC's of production are given by the costs at market prices, on the LHS of (3), and the only correction we need to make is for the value of the outputs (the RHS of (3) to reflect the MSV's of the 3 goods. In domestic currency, the MSV of M is ePmf (1+t), the MSV of X, (for the reasons given in Case 1 above) is ePmf (1+t), the MSV of N is Pm. Thus, in this case, the only adjustment which is required is, still, just the correction for the distortion between the MSV and MSC of using and producing a unit of foreign exchange. It is the same as in Case 1 above, and the use of an S.E.R. of e (1+t) on the lines of Case 1, will again give the right answer - the production of X, at 'shadow' prices, will be more profitable, relative to both M and N.

If on the lines of the LM procedure, we were using <u>foreign</u> <u>currency</u> as our numeraire, the value of the two traded outputs would be given by P_{mf} and P_{xf} . What about the fe equivalents of the MSC's and MSV of the two factors and the non-traded good N? First consider the factors, say labor. We know that the value marginal product of labor in all the industries is equal to the wage. In the M industry this means that

$$\frac{SL}{SM} P_{mf} = \frac{W}{e(1+t)} \qquad - (4)$$

the IHS is the value marginal product of labour in terms of foreign currency, and we thus have on the AHS, the price of labour (the wage) in terms of foreign currency. Similarly the price of capital in terms of foreign currency will be d/e(1+t). Second, consider the valuation of N in terms

of foreign currency. We know that the correct relative MSV's of M and N in domestic currency are given by $eP_{mf}(1+t)/P_n$ The foreign currency value of the numerator is P_{mf} . Hence the foreign currency value of the denominator must be $P_n/e(1+t)$, if the relative foreign currency values of M and N, are to reflect their correct relative MSV's. We could equivalently have derived the 'shadow' price of N, in terms of foreign currency by following the more general LM procedure of valuing the inputs of the N good in terms of fe equivalents. Thus from Equations (3) and (4), we have the 'shadow' price of N, equal to

$$\frac{A_{\ln W}}{e(1+t)} + \frac{A_{\ln R}}{e(1+t)} = \frac{P_n}{e(1+t)}$$

Thus, we get the result, that, to get the correct MSV's and MSC's in terms of foreign currency, the values of the domestic factors (W, R) and goods (Pn) must all be deflated by e(1 + t), which is the SER to be used to convert foreign goods values into domestic currency in the domestic currency as numeraire method. The two methods (UNIDO/HS, and LM) are therefore equivalent in this case too, as they are in Case 1, and involve nothing more than a change in numeraire.

What of the third method (B/K)? It can provide us no answer, when we are comparing investment projects which produce N with those producing X or M. Though as before, it will be equivalent to LM and UNIDO/HS, when comparing production of X or M.

(b) Let us next assume that we expect that the protective structure will be removed in the future. What will be the relevant 'shadow' prices we should use, in evaluating current investment projects for producing X, M or N?

Clearly, the relevant shadow prices will now be the prices of the goods and factors in the free trade situation.

To determine these prices, we consider what would happen to the prices of goods and factors in our model economy with the removal of the tariff on M. The resulting changes are best considered in two distinct stages. In the first stage, we assume that all other domestic prices, the exchange rate, and domestic expenditure, remain unchanged. With a reduction in the price of M by t% (the tariff rate), the relative domestic prices of the 3 goods X, M and N will change. With a fall in the price of M relative to both X and N, there will be a shift in domestic consumption from X and N towards M, and in domestic productive resources from M to X and N. Now consider the markets for M, X and N. In the market for M there will be excess demand, whilst in the markets for X and N there will be excess supply. Unless the excess demand for M is matched by an equivalent excess supply of X (an exceptional circumstance) normally, there will tend to be a balance of payments deficit, given by the difference between the excess demand for M and excess supply of X. What is more, from Walras' law, this net excess demand for traded goods must be exactly equal to the excess supply of the non-traded In the next stage therefore, to restore equilibrium it will be necessary to cure the balance of payments deficit. To do this, it is necessary to cure the net excess demand for tradeables which is equivalent to curing the excess supply for the non-traded good N. This requires a fall in the relative price of the non-traded to the

two tradeable goods. This change can be brought about by two alternative adjustment mechanisms (or a combination of both). The first is with the exchange rate fixed, but with the domestic money price of N flexible. (This is the 'classical' adjustment mechanism). The other is with the price of N fixed, but with the exchange rate flexible. In the first case, the price of N, will fall from P_n to P_n, with the domestic prices of X and N given by eP_{xf} and eP_{mf}, at the fixed exchange rate e. As a result, the domestic prices of the three goods, in the free trade situation will be different than those in the protective situation (see Table 1, Pow 1, 2), which will result in resource allocation effects which will lead to changes in factor prices; let these free trade factor prices with the price of N flexible and the exchange rate fixed, be W and N*.2/

If the adjustment mechanism is via exchange rate flexibility, then at the new equilibrium free trade exchange rate e, the domestic prices of X and M will be e^*P_{xf} and e^*P_{mf} (e^*) e), and the price of the non-traded good will be the same as in the protection situation, P_n . (See Table I, Row 3). Again, as the relative prices of the three goods are different in the free trade and protection situation, the resulting resource allocation effects will lead to a change in factor prices, say to W^{**} and R^{**} . It is shown in Appendix I, that in general $\frac{W^*}{R^*} \neq \frac{W^{**}}{R^{**}}$, nor is the necessary fall in the price of N,

^{2/} It is shown in Appendix I, that the change in the price of N will be equal to the change in the price of M, if the demand and supply elasticities for N are equal. In such a case, it will be relatively easy to guess the price Ph, in the free trade situation. Moreover, in this case there will be no change in the relative factor price ratio.

(with the exchange rate fixed) equal to the required devaluation (change in the exchange rate from e to e*), with the price of N fixed.

Thus the relative prices of goods and factors which we should use to evaluate a marginal investment project will differ, according to which assumption we make about the adjustment process for changing the relative price of traded to non-traded goods.

Though the relative rankings of the 3 goods in the free trade as compared with the protection situation will be the same.

Assuming, that, we know or can guess at the price changes (in W, R and e or P_n), and assuming the same adjustment mechanism, then either of the two alternative methods of using, domestic currency as the numeraire and a 'shadow' exchange rate, or foreign currency as the numeraire and the LM method, will give the same ranking of the relative social profitability of investments in the three industries M, X and N. The third method (Bruno) again using the relevant P_n, W, and R's, will be able to rank the two traded goods, by comparison with the relevant exchange rate (e in Row II, and e* in Row III) but will be unable to say anything about the relative desirability of investments in the non-traded good N.

This model can be expanded to include non-traded inputs, and traded inputs. Appendix II, considers the changes in a model with

Two Traded Goods Produced and Consumed and One Non-Traded Good Produced and Used as an Intermediate Input. As is shown, therein, this case is similar to Case 1. The intermediate good merely serves as an indirect means of using the domestic factors capital (K) and labour (L). Domestic

relative factor prices are again uniquely determined by the fixed and given relative commodity prices, which are given by "border" prices.

A tariff on M, again, merely distorts the domestic relative prices of traded commodities. Given this distortion, the relative factor prices are still uniquely determined, independently of domestic demand conditions, by the domestic prices of traded goods alone.

With the removal of the tariff the relative factor prices will change. Then, assuming the exchange rate is fixed, the price of N will be uniquely determined. Alternatively if the price of N is fixed, the necessary exchange rate change will again be uniquely determined by the production relationships alone. All these points are proved in Appendix 2.

The adjustments in project analysis which are necessary in this case will be similar to those in Case 1 above. Once again exchange rate changes will not have real resource allocation effects. Balance of payments disequilibrium can only be cured by expenditure reduction, and not expenditure switching.

We could go on to consider more complex and general models with traded and non-traded intermediate goods, and complex systems of taxes and subsidies on exports. However, for our purpose of comparing alternative procedures for project selection, the above two models are sufficient to bring out the essential points which we consider in the the next section.

Alternative Procedures - Theory

We now compare the various procedures which have been suggested.

for project evaluation in an economy with sub-optimal trade controls.

1) The UNIDO, Harberger, Schydlowsky (UHS) (10, 6, 12, 13) shadow exchange rate. These procedures only correct for the distortions in relative prices within the traded goods sector. Taking the existing relative price of non-traded to traded goods as given, their aim is to correct for the distortions in the relative prices of traded goods caused by the existence of non-unified exchange rates. Thus implicitly, it is our Case 12, (A), with the protective structure unchanged (which as we saw above, reduces to Case 1, in terms of estimating shadow prices), which is the relevant model for these procedures.

^{3/} See UNIDO p. 85-86, Harberger p. 241, Schydlowsky p. 2-4.

The SER being derived as the MSV of foreign exchange in the protection situation which, in our simple model, is e(1 + t). The general formula for this case is provided in Bacha and Taylor (1), and it is the weighted sum of dementic prices of traded goods, divided by a similar weighted sum of world prices, the weights in each case being the marginal changes in imports and exports induced by the project" (1, p. 205).

2) The Bacha and Taylor (BT) (1), 'equilibrium'exchange rate - The relevant model for this procedure is our Case II (3), with the protective structure removed and equilibrium maintained by variations in the exchange rate and with the price of the non-traded $\gcd(P_n)$ fixed. This procedure takes account of the distortions within the traded good sector and those between the traded and non-traded goods sector, caused by the sub-optimal trade controls.

The equivalence of the BT formula for the equilibrium exchange rate with the devaluation rate derived for Case II (B), (in Appendix 1) is shown in Appendix 3. From our discussion in the

preceeding section of this case, it is clear that it is not sufficient to just calculate this equilibrium exchange rate, we also have to determine the new relative factor prices (W**, R**) in the new free trade situation. Without calculation, and use of these, in conjunction with the 'equilibrium' exchange rate, the resulting project evaluation rules would be incorrect. These points are obscured in the discussion by Bacha and Taylor.

3) The Bruno - Kruger (BK) /1, 9 7 test - computes the domestic resource cost of the net foreign exchange earned/saved by a project. It is thus only applicable to projects whose outputs are traded (or for non-traded goods which are close substitutes for traded ones). If the net foreign exchange saved/earned (taking account of direct and indirect inputs) by the project (in foreign currency) is F dollars and if the total (direct and indirect) resource cost is D rupees the Bruno ratio is $\frac{RsD}{s_k R}$, and this is clearly like an exchange rate, which converts domestic currency into foreign currency. It gives the exchange rate at which the project would be acceptable. If the economy was in equilibrium in free trade (with no distortions domestically or in foreign trade), and the market exchange rate were e*, then projects could be selected by using D = e* as an investment criterion for tradeable goods. The D and F terms being valued at market prices. If, however, as is our concern, we want to evaluate projects in an economy with sub-optimal trade controls, there is the problem of what prices to use in determining D, [F, given the small country assumption being still determined by given world prices 7,

and what cut-off exchange rate (e) to use to select projects. This clearly depends upon what alternative assumptions we make about (i) whether there will be trade liberalization in the future and (ii) whether with trade liberalization, external balance will be maintained by exchange rate flexibility or domestic price flexibility, as discussed in Case 2, in Section I above.

Firstly, if it is assumed that the protective structure will remain unchanged, then clearly Case 2 (A) is the relevant model, and we will, because of the distortion within the traded good sector, have $\frac{D_{x}}{F_{m}}$ for exports less than $\frac{D_{m}}{F_{m}}$ for imports, the D_{i} naturally being evaluated at the domestic prices in the protection situation. Comparisons of $D_{\rm X}/F_{\rm X}$ and $D_{\rm m}/F_{\rm m}$ with the official exchange rate are clearly irrelevant in this case, for choosing projects. The relevant comparison would seem to be the effective exchange rate for exports $D_{\!_{\! K}}/F_{\!_{\! K}}$. If, however, the supply curve for experts is upward sloping Dx/Fx will rise towards Dn/Fm with an increase in production of exports. So that, in each case we would have to recalculate the marginal Dx/Fx, with which the project to be appraised must be compared. A rough and ready method would take an average between D_m/F_m and D_χ/F_χ as the cut-off exchange rate for selecting projects. Note that both these alternative methods of determining the cut-off rate will give different answers for project selection from those derived by using the UHS shadow exchange rate. The latter, would multiply the F component by the SER, say II. The criterion for acceptance would be IIF - D > 0, or equivalently $\frac{D}{F} \leqslant \overline{11}$. The Bruno and UHS methods would thus only give identical answers if II was taken as the cut-off exchange rate for the Bruno test. But from our discussion of the UHS, we know that $II = D_m/F_m$,

so that, for the two procedures to be equivalent the effective import rate would have to be taken as the cut-off rate for the Bruno ratio, in project selection.

Secondly, if it is assumed that the protective structure will be removed and equilibrium maintained by exchange rate changes with the price of N fixed, then the relevant cut-off effective rate is the B&T 'equilibrium' exchange rate (our ê in Appendix 2), and in calculating the D, primary factors should be valued at their prices in the free trade situation, and not at the existing market prices in the protection situation. The procedure would now be equivalent to the BT procedure. Alternatively, if, the exchange rate is inflexible but the domestic price of N is flexible, then the relevant cut-off rate for the Bruno criterion will be the existing official exchange rate e, but it will be necessary to estimate the prices of N, and the primary factors, when the protective structure is removed.

(h) The Little-Mirrlees (LM) Method - takes foreign currency as its numeraire, values tradable inputs and outputs at their border prices (given and constant in our models) and those of non-tradables by breaking them down into tradables and primary factors. The foreign exchange value equivalents of the latter are determined by valuing the marginal product of these factors in terms of foreign exchange. This method is, in principle, the most general of the ones we have been considering. Unlike the Bruno method, it can be used in all the cases we have discussed in Section I. Moreover, unlike the UHS and BT methods its

^{1/} This point is again obscured in Bacha & Taylor's survey (1).

validity is not dependent on the particular assumptions made about trade liberalization.

Thus, firstly, if it is envisaged that the protective structure will remain unchanged (Case 1), it will give the same results as the U.S method. In the simple model of Section I (Case 1), the LM method involved a mere change in numeraire compared with the UHS method. Domestic factor and non-traded good 'shadow' prices would be deflated by t, on the LM method, and traded good prices would be taken at their foreign currency values. On the UHS method, domestic factor and non-traded good 'shadow' prices would be their domestic market prices, and the foreign currency values of traded goods would be multiplied by e(1 + t) the UHS shadow exchange rate.

If it is assumed that trade liberalization will take place, then with the exchange rate fixed and the prices of non-traded goods flexible, the LM procedure would take the foreign prices of tradables as given, and work out the implicit prices of the non-traded goods and factors in the free trade situation, in terms of their foreign exchange equivalents. If with trade liberalization, the exchange rate were to be changed, the LM method would again take the foreign prices of tradables as given, and determine the free trade foreign exchange equivalent prices of the non-tradables and domestic factors at the 'new' exchange rate.

The adjustment mechanism with trade liberalization, which is implicit in LM, however, is of the second sort, namely, with the prices of domestic goods inflexible and a flexible exchange rate (11, p.53,135). The method is therefore similar in its aims to the BT procedure. Unlike the latter, however, the LM procedures provide a way of estimating not

only the relative price of traded to non-traded goods, but also for approximate calculation of the factor price ratio in the free trade situation. To see this it is necessary to briefly consider the estimation of the 'shadow' wage rate in the LM method.

As we are assuming no other distortions apart from trade distortions, two aspects of the LM shadow wage are redundant for our purposes, namely, the distortion due to a higher industrial wage than the social opportunity cost of labor given by the value marginal product of labor in agriculture, and the distortion due to the government's inability to directly legislate the optimum savings ratio. The shadow wage in the LM method for our purposes, therefore, is given by the value marginal product of labor in agriculture (m), whose value in foreign currency is, say, P ... If agricultural output consists of tradables whose 'border' prices are given and constant, P. is determined from the world market in terms of foreign currency. Assuming that there is an elastic supply of labor at this 'shadow' wage, the value of w in terms of foreign currency, in Table 1, is fixed for all the 3 cases (Rows I to 111) considered. The removal of the tariff and the subsequent adjustment process will still change the wage rental ratio from that in the protection situation, but given that the wage rate is constant (in terms of foreign currency) this will be the result entirely of changes in r (the rental rate). Also looking at Row 111, Table 1, we have, in terms of foreign currency, the price of M as P_{mf} , of X as P_{xf} and of L as P_{Lf} (all given by constant 'border' prices). The foreign exchange equivalent of the rental rate r is r /e* From equation (3), Section 11, we have

$$a_{LN}$$
. $P_{Lf} + a_{KN} = \frac{r^{**}}{e^{*}} P_{n/e}$, and

the price of the non-traded good in the free trade situation will be

determined. Given the input/output coefficients (a;) and with PIf, P and P determined by the 'world' market, the only estimate we need to make is of that is, the rental rate in terms of foreign currency in the free trade situation. Now note that r, in our simple neo-classical models, is also the rate of return on investment. Suppose, in guessing $\frac{r^{**}}{\sqrt{x}}$, we guess a number which is less than the true number, this will mean that more investment will be undertaken in the economy than is feasible in equilibrium. The excess of investment will spill over into a balance of payments deficit on the lines of the absorption approach of balance of payments theory. Assuming that consumption cannot be cut (ex hypothesi the wage rate is fixed and is all consumed), the only way in which the balance of payments deficit (actual or incipient) can be cured is with a rise in r, and a consequent cut in investment. Iteratively, therefore, the r which will maintain equilibrium would be determined. This shows why in the LM procedures, once Plf, Pxf and Pmf are determined by 'border' prices, balance of payments deficits can be only cured by changing absorption, by changing the level of investment via changes in r. (The IM, ARI) (See 11., pp. 89, 138, 139). Thus, on the LM procedures, given no domestic distortions, the only 'price' the project evaluator would have to 'guess' is the r in terms of foreign currency (the ARI) in the free trade situation. The PLf, Pxf, Pmf, and aij's (assuming fixed coefficients) would be known directly from the protection situation.

We can contrast the LH procedure with the BT procedure which works in terms of an 'equilibrium' exchange rate. Making the same assumption of an elastic supply of labor at a constant wage in terms of the alternative value marginal product of labor in terms of tradables (whose foreign currency value is FLf, we have (Table 11, Row 111) in the free trade situation,

^{1/} See following page for this footnote.

the price of X, M and N, as e^*P_{xf} , e^*P_{mf} and P, and the factor prices as $W^{36} = e^*P_{Lf}$, and r^{36} . Comparing rows II and III in Table II, it is obvious that given a correct estimate of e^* , and the same values for P_{xf} , P_{mf} , P_{Lf} , the $a_{ij's}$ and r^{36} both the LM and BT procedures, are equivalent. These points are given obscured in the BT survey (1).

4/ (from previous page) It may seem odd that we are assuming that the LM procedure assumes that the wage rate is constant in terms of a constant value marginal product of labor valued in foreign currency. Whereas the normal assumption is of a fixed money wage. The reason why the latter is eliminated from our discussion is because of our perfectly competitive assumptions, which necessitate that labor is paid its alternative value marginal product. In the general IM discussion, it is assumed, both that the money wage paid by the industrial sector (W) is constant, and that the value marginal product of labor in agriculture in terms of foreign currency (M) is constant. The LM, SWR, then is SWR = C - \$ (0-11), where S reflects the premium on savings relative to consumption, and C is the value of 'W' in terms of foreign exchange, and with C> M. With an exchange rate change, say a devaluation, with the money wage, W constant, C must fall, but H will remain the same. In the general IM case, therefore, exchange rate changes, for instance, those accompanying trade liberalization, will involve (or reflect) changes in the SWR (assuming S > 1, and $C \neq M$) and vice versa. (That is, if an exchange rate change is anticipated in the future, the SWR will be lower, (see 11, p. 138). However, as we are assuming that C = M, and as the value of M is assumed constant in terms of foreign currency, for our purposes the SWR cannot change. (For the necessary assumptions about peasant farming implicit in assuming M constant, see Lal (10)). Also from the general SAR formulation it can be seen that, whilst on the one hand, for a balance of payments deficit to be cured, the SWR will tend to be lower as C will be lower because of the necessary exchange rate change, on the other hand, there will also have to be some reduction in domestic absorption, which (assuming consumption cannot be cut) will mean a reduction of investment and hence a rise in r (the LM, ARI), this cet. par. will tend to raise S, and hence raise the SWR. Given M, the net change in the SWR will thus depend upon these two opposing effects of a fall in C and a rise in S. These two opposing tendencies are caused by the two instruments which are normally necessary to cure a balance of payments deficit to achieve internal and external balance, namely, a combination of expenditure switching (the exchange rate change) and expenditure reduction (the cut in investment and rise in r).

(5) It may be noted that so far we have not considered rankings according to effective protective rates (2, 3) as a project selection procedure. This is because, in principle, rankings according to effective protective rates (EPR's) cannot tell us anything about the desirability of investment projects. For instance, consider two industries I and II. An investment of \$100 in both produces value added at world prices (V) of \$10 in I (VI) and of \$8 in II (VII). Value added at domestic prices (V) is $V_I = h0$ and $V_{II} = 10$. The APR (Z's), defined as $Z = \frac{V - V^*}{V^*}$ for the two industries are $Z_I = \frac{h0 - 10}{10} = 3$

$$Z_{II} = \frac{10 - 8}{8} = .25$$

assuming that the free trade exchange rate is \$1 = Rs.1.

Ranking and choosing projects according to EPR's and taking a zero Z, as our benchmark, industry II, would be preferred to I, though the rate of return (R) at world prices, of the two industries is R_I = 10% and R_{II} = 8%, that is, I is preferable to II. Of course, if the value added at world prices V* per unit investment is taken to be the EPR criterion, then it is the LM procedure. Also there are obvious links between EPR's and the SER (see 1), and between EPR's and the B/K ratio (see 3). But as the above example demonstrates, ranking by EPR's cannot be used as an investment criterion.

III.

Alternative Procedures - Practice

We now turn to the practical application of the various procedures. Their practicality and usefulness will depend upon (a) the realism and relevance of the assumptions on which they are based and (b) the practical problem of obtaining the data for making the adjustments which the different methods envisage.

From Section II, it is clear, that except for the LM procedure (and to some extent the B/K ratio flexibly interpreted), the other two procedures, the UHS and BT SER's are only valid on two mutually exclusive assumptions about the future course of the economy. They make diametrically opposed assumptions about future trade liberalization. However, given (i) that trade controls are seldom fixed; (ii) that most developing country governments have at some stage or another actually moved towards trade liberalization; and (iii) that trade liberalization would often maximize feasible welfare, so that it is important to know the impact of projects on potential welfare, it would seem to be more desirable to use the BT SER rather than the UHS SER for project evaluation. (Then for the Bruno test, the relevant cut-off rate would be given by the BT 'equilibrium' exchange rate). The crucial question then is whether it is better/easier to use the BT or LM procedures for project evaluation.

From our discussion in the last section it is clear that, in practice, both procedures require a knowledge of P_{xf}, P_{mf}, P_{Lf}, and the rental rate (r) in terms of domestic or foreign currency in the free trade situation. The 3 foreign prices, are assumed given from 'border' prices; this means 'guessing' or approximately estimating the likely rental rate in the free trade situation. Once this has been done, the LM procedure would immediately give the social returns to the project under consideration, without any further computation. In the case of BT, however, there would have to be the additional step of calculating the 'equilibrium' exchange rate e. The essential simplicity and superiority of LM procedures, in practice, depends upon its cutting through the need to estimate e.

In principle, of course, the two methods are equivalent as it can be seen from Table II, that the LM procedure too, implicitly needs an

estimate of e* to get the rental rate in terms of foreign currency. But it is not essential to calculate e* in the IM procedures, as can be shown as follows:

Write, r*/e*, the foreign currency equivalent of the rental rate in free trade as Pkf, and Pn/e* as pn. Then at the protection exchange rate (e), the values of the domestic relative prices of the 3 goods and two factors in domestic currency with free trade will be given by Now II in Table III. How III of the same table gives the same relative prices in terms of foreign currency. Whilst Row IV (as in Table II, Row III) gives the prices in domestic currency at the BT 'equilibrium' exchange rate. Table III, is thus exactly equivalent to Table II. But now consider Row III. We have Pxf, Pmf and PLf given by 'border' prices. Suppose we can guess Pkf. Then from the equations in Section II, pin is determined, and the relative prices to be used in project evaluation on the LM procedures are determined without any need to estimate ex. Remembering that we have guessed Pkf, what are the results of errors in this guess? First, if the K input in N is large (that is, non-traded goods are relatively capital-intensive), then the pn we have derived will tend to be wrong. If, however, either N is relatively more labor-intensive and/or, the proportion of non-traded goods to traded goods is small (in the limiting case we have only traded goods as in Case I, Section II) the error in the estimate of p_n^{***} as a result of errors in estimating Pkr, will not be very important in practice for evaluating and ranking projects. The second effect of a mistake in estimating Pkf, will be to provide an incorrect cut-off rate for the IRR of projects. This will imply that the volume of investment will be too large (small) relative to the 'equilibrium' level if Pkf is underestimated (overestimated) and this will imply a balance of payments deficit (surplus).

However, as soon as it appears that a particular cut-off rate (P_{kf}) implies an excess (deficiency) of domestic investment, then the cut-off rate will be raised, and thus the correct P_{kf} will be iteratively approximated. Thus, in a relatively open economy, the LM procedures will give a good approximation to the 'true' relative prices to be used in project evaluation without the need to estimate e*. In fact, as Table III shows, if P_{kf} is known, e* is redundant.

The last statement seems to imply that in that case it is equally irrelevant that we do not have an accurate estimate of e* on the BT procedures. This would be true if the LM procedures of identifying and estimating PLf, Pkf and thereby determining p were also followed in BT procedures. In practice, however, this is not likely to be done, and only the prices of X and M will tend to be 'shadow' priced with e (see 2). But in that case, especially in a relatively open economy, a correct estimate of ex becomes essential. For consider Row IV, of Table III, we will now have p, w*, r*,p and p given. To get the correct relative price of the traded goods (X, M) to the domestic good and factors (N, L, K), the whole burden falls on the estimate of e*. However, as can be seen from an examination of the BT formula for calculating ex, (see Appendix III), getting a reasonably accurate estimate of this variable is going to be extremely difficult in an open economy (one with even a relatively small number of traded goods, say, 100!) in which there are complex trade controls, including quotes and multiple exchange rates (factors not taken into account in the BT formula). Further complications arise in calculating the BT rate as the system of trade controls is changed, and hence the weights (imports and export shares of different commodities) and the effective

protective rates which enter the formula change, thereby altering the estimate of e*. All these complications can be cut through by the use of the IM procedures, which therefore, in practice, are likely to be easier to apply than a correct application of the BT procedures. (This conclusion is the reverse of that reached by BT(1)!

Another argument for favoring the use of LM rather than BT procedures is one of diplomacy. Even though in principle the two methods are equivalent, governments are not likely to take kindly to the calculation (and publication!) by the project evaluators, of the 'shadow' exchange rate for their countries (especially if these calculations are done by 'outsiders') as this would be an open acknowledgment that the 'official' exchange rate was wrong! Thus, whilst achieving the same ends for the purposes of project evaluation, the LM procedures are likely to be more palatable.

Finally, it should be noted, that all the methods we have discussed, if applied properly, need estimates of the 'border' prices of tradable cormodities. Thus the common impression that LM procedures require any extra information, beyond that required by other evaluation procedures is completely mistaken. For relatively open economies, the LM methods provide the simplest methods for taking proper account of trade distortions, and enabling countries to choose projects in line with their comparative advantage.

University College London and . IBRD, Washington, D.C.

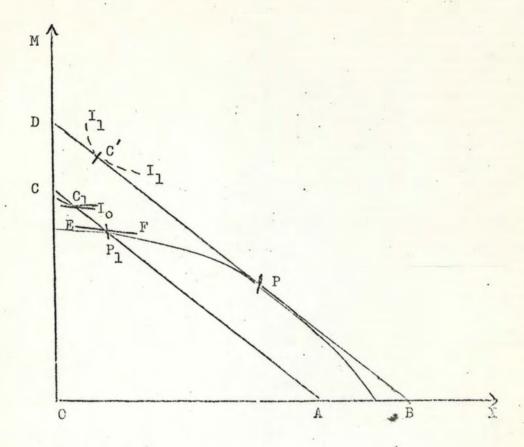
August 1971

REFERENCES

- (1) Bacha, E. and L. Taylor. 'Foreign Exchange Shadow Prices: A Critical Review of Current Theories', QJE, May 1971.
- (2) Balassa, B. 'Project Appraisal in Developing Countries' in Festchrift in Honour of J. Tinbergen (forthcoming).
- (3) Balassa, B. and D. M. Schydlowsky. 'Effective Tariffs, Domestic Cost of Foreign Axchange, and the Equilibrium Exchange Rate.', JPE, May/June 1968.
- (4) Bruno, M. 'The Optimal Selection of Export Promoting and Import Substituting Projects' in Planning the External Sector: Techniques, Problems and Policies (U.N. 1967).
- (5) Corden, W. M. The Theory of Protection (O.U.P. 1971).
- (6) Harberger, A. C. 'Survey of Literature on Cost-Benefit Analysis for Industrial Project Evaluation' in Evaluation of Industrial Projects (UNIDO, New York, 1968).
- (7) Jones, R. W. 'The Structure of Simple General Equilibrium Models!; JPE, December 1965.
- (8) Joshi, V. 'Foreign Trade and Investment Criteria' in Ellis and Scott ed.

 Essays in Honor of Sir Ray Harrod (OUP, 1970).
- (9) Kruger, A. O. 'Some Economic Costs of Exchange Control: The Turkish Case', JPE, October 1966.
- (10) Lal, D. 'Disutility of Effort, Migration, and the Shadow Wage Rate', (mimeo) 1971.
- (11) Little, I. M. D. and J. A. Mirrlees. Manual of Industrial Project
 Analysis, Vol. II (OECD, Paris, 1969).
- (12) Schydlowsky, D. M. 'On the Choice of a Shadow Price for Foreign Exchange', Economic Development Report, No. 108, Development Advisory Service, Harvard, 1968.
- (13) UNIDO Cuidelines for Project Evaluation (mimeo), May 1970.





BD is the foreign price ratio.

P and C'are the production and consumption points with free trade P₁ and C₁ are the production and consumption points with a tariff (given by differences in the slopes of EF and AC).

EF is the domestic price ratio with the tariff.

I₁ I₁ is the indifference curve giving the welfare level at the free trade consumption point.

I o is the indifference curve giving the welfare level at the tariff-distorted consumption point.

OB/OD is the value of domestic expenditure at foreign ('border') prices, in the free trade situation, with I/N as the numeraire.

OA/OC is the value of domestic expenditure at foreign ('border') prices, in the tariff situation, with X/M as the numeraire.

Table I

Goods and Factor Prices

		· Goods		Facto	rs
I.Protection	х	М .	Ν .	L	K
Prices	eP _{xf}	eP _{mf} (1+t)	P _n	W	r
II. Free Trade, Fixed and Pn Flexibles	Exchange R	ate			
Prices	ePxf	eP _{mf}	P_n^*	W*	r*
III. Free Trade Pn Fix	ed				
Rate	e*Pxf	e*Pmf	P _n	w**	r**

Note: e - is the exchange rate

 $\boldsymbol{P}_{\boldsymbol{\mathbb{X}}\boldsymbol{f}}$ - the foreign currency price of $\boldsymbol{\mathbb{X}}$

Pmf - the foreign currency price of M

Comparison of Goods and Factor Prices on the
LM and BT Procedures

		Goods			Factors	
		X	M	N	L	K
ı.	Protection Situation (domestic currency)	ePxf	eP _{inf} (1+t)	$P_{\mathbf{n}}$	eP _{Lf} (=w)	r
II.	Free Trade, LM (foreign currency)	Pxf	P _{mf}	$\frac{P_n}{e^*}$	$^{\mathrm{P}}$ Lf	$\frac{\mathbf{r}}{\mathbf{e}}$
III.	Free Trade, BT (domestic currency)	e*Pxf	e*P _{mf}	$P_{\mathbf{n}}$	e*PLf(=w*).	r**

Table III

				**		
		. <u>X</u>	11	N	L	K
I.	Protection Situation (domestic currency)	e ^p xf	eP _{mf} .(1+t)	P _n	$^{ m eP}_{ m Lf}$	r
II.	Free Trade, Exchange Bate Flexible (domestic currency at protection exchange rate e)	eP _{xf}	eP mf	$eP_n^* = eP_n = \frac{n}{e^*}$	eP _{Lf}	eP _{kf} = e.r*
	Free Trade, Exchange Rate Flexible (foreign currency values)		Pmf	P _n ***	$P_{\mathbf{Lf}}$	P _{kf}
IV.	Free Trade, Exchange Rate Flexible (domestic currency at BT equilibrium exchange rate e*)	e*P xf	e*Pmf	P n	e*P _{Lf} (=v	r [*] e [*] P _{kf} (=r

Where $x = \frac{dx}{x}$, and ϵ_{xy} , is the elasticity of demand for x with respect to price P_y , and η_{xy} is the elasticity of supply of x with respect to price P_y .

To restore equilibrium in the market for N, $\hat{N}_d = \hat{N}_s$, and hence from (6) and (7) we get

$$\hat{P}_{n} = \begin{bmatrix} \frac{\eta_{nm} - \epsilon_{nm}}{\epsilon_{nm} - \eta_{nm}} \end{bmatrix} \hat{P}_{m} \qquad - \qquad (8)$$

From this it follows that for $\hat{P}_n = \hat{P}_m$,

$$\eta_{nn} + \eta_{nm} = \epsilon_{nn} + \epsilon_{nm} - (9)$$

that is the sum of the own and cross elasticity of demand (with respect to M) for N must equal the sum-of the own and cross elasticity of supply (with respect to M) for N.

If we assume that the cross elasticities are zero, we get $\hat{P}_n = \hat{P}_m$, if the elasticities of demand and supply of N are equal.

(B) The change in factor prices, (with price of N flexible, and the exchange rate fixed), with the removal of the tariff, of the on M, can be derived from the production relationships (1) to (3) as follows.

Defining
$$\theta_{ij}$$
 as $\frac{a_{ij} \cdot P_{i}}{P_{j}}$, where $i = L, k$

$$P_{i} = w, r$$

$$j = X, M, N$$

that is input i's distributive share in industry (j) and as before $\hat{x} = \frac{dx}{x}$,

then, totally differentiating (1) through (3), and remembering that from the cost-minimization requirement,

$$\theta_{Lj} \hat{a}_{Lj} + \theta_{Kj} \hat{a}_{Kj} = 0,$$
where $(j = X, M, N)$

yields,

$$\theta_{LM}\hat{W} + \theta_{KM}\hat{r} = \hat{P}_{m} - (10)$$

$$\theta_{LX}\hat{W} + \theta_{KX}\hat{r} = \hat{P}_{x} - (11)$$

$$\theta_{LN}\hat{W} + \theta_{KM}\hat{r} = \hat{P}_{n} - (12)$$

1/ The method of analysis and notation, is based on R.W. Jones: "The Structure of Simple Concret Equilibrium Models." 12E, Dec. 1965

APPENDIX I

Case 2: Two Traded, One Non-Traded Good Produced and Consumed

The production relations are given by

$$a_{LM}W + a_{KM}r = P_m - (1)$$

$$a_{LX}^{W} + a_{KX}^{P} = P_{X} - (2)$$

$$a_{LN}^{W} + a_{KN}^{P} = P_{n} - (3)$$

Where a_{ij} is the coefficient of the ith factor (i = K,L) used in the jth industry (j = M,X,N)

w is the wage rate,

 ${\bf r}$ is the rental rate, ${\bf P}_{\rm m}$, the price of the importable M, ${\bf P}_{\rm x}$, the price of the exportable x, and ${\bf P}_{\rm n}$, the price of the non-traded good N, all in the protection situation.

- (1) With the removal of the tariff t, the domestic price of M changes from to ePmf,

 Pmfe(l+t)/where Pmf is the price in foreign currency, and e is the exchange rate. Assuming that equilibrium is restored with the exchange rate fixed and with the price of N flexible.
- (A) The change in the price of N $(dP_n/P_n) = P_n$ can then be derived as follows: Let the demand (N_d) and supply (N_s) functions for N be given by:

$$N_{d} = N_{d} (P_{n}, P_{x}, P_{m}, Y) - (4)$$

$$N_{s} = N_{s} (P_{n}, P_{x}, P_{m}, Y) - (5)$$

Where Y, is domestic money income.

Then differentiating (4) and (5) totally, dividing through by N_d in (4) and N_s in (5), and noting that d_{px} and dY are both zero, as ex hypothesi P_x and Y do not change we have:

$$\hat{N}_{d} = \epsilon_{nn} \hat{P}_{n} + \epsilon_{nm} \hat{P}_{m} - (6)$$

$$\hat{N}_{s} = \eta_{nn} \hat{P}_{n} + \eta_{nm} \hat{P}_{m} - (7)$$

Subtracting (12) from (10) and noting that, ex hypothesi, P_x is zero we have

$$\begin{bmatrix} \theta_{LM} - \theta_{LN} \end{bmatrix} \hat{\mathbf{w}} + \begin{bmatrix} \theta_{KM} - \theta_{KN} \end{bmatrix} \hat{\mathbf{r}} = \begin{bmatrix} \hat{\mathbf{p}}_{m} - \hat{\mathbf{p}}_{n} \end{bmatrix} - (13)$$

$$\theta_{LX} \hat{\mathbf{w}} + \theta_{KX} \hat{\mathbf{r}} = 0 - (13)$$

This yields the following solutions for w and r.

$$\hat{\mathbf{w}} = \frac{\begin{bmatrix} \mathbf{P}_{m} - \mathbf{P}_{n} \end{bmatrix}}{|\mathbf{e}'|} \mathbf{e}_{KX} - (14)$$

$$\hat{\mathbf{r}} = \frac{\hat{\mathbf{P}}_{m} - \hat{\mathbf{P}}_{n}}{|\mathbf{e}'|} \mathbf{e}_{LX} - (15)$$

Where $|\theta'|$ is the determinant of the coefficients of \hat{w} and \hat{r} in equations (13) and (11')

that is,
$$|\theta'| = \theta_{KX} \left[\theta_{LM} - \theta_{LN}\right] - \theta_{LX} \left[\theta_{KM} - \theta_{KN}\right]$$

$$= (\theta_{LM} - \theta_{LN}) \qquad \left[\text{as } \theta_{LJ} + \theta_{Kj} = 1 \right]$$
where $j = X, H, N.$ (16)

From (14) and (15), the change in relative factor prices $(\hat{w} - \hat{r})$ is

$$(\hat{\mathbf{w}} - \hat{\mathbf{r}}) = \frac{(\hat{\mathbf{p}}_{m} - \hat{\mathbf{p}}_{n}) (\hat{\mathbf{p}}_{KX} + \hat{\mathbf{p}}_{LX})}{|\hat{\mathbf{p}}|}$$

$$= \frac{(\hat{\mathbf{p}}_{m} - \hat{\mathbf{p}}_{n})}{|\hat{\mathbf{p}}_{LM} - \hat{\mathbf{p}}_{LN}|} - (17)$$

as $\theta_{KX} + \theta_{LX} = 1$, and substituting the value of $\left|\theta'\right|$ from (16). From (17) it can be seen that when $\hat{P}_m = \hat{P}_n$ (that is when (9) holds), $\left(\hat{w} - \hat{r}\right) = 0$, and the wage-rental ratio will be the same in the protection and in the free trade situation. The problem of estimating the relative prices in the free trade situation (given the fixed exchange rate e) would be greatly simplified, as the prices of the traded goods would be given by their world prices, and that of the non-traded good by the change in the price of N, that is by the tariff rate.

- (2) Alternatively, the adjustment mechanism could be with the exchange rate variable and the price of N fixed.
- (A) The Change in the exchange rate can then be derived as follows:

Assuming that (i) the cross-elasticities in both production and consumption of X and N are zero, (ii) the income effects of the tariff and exchange rate changes can be ignored, and if the elasticity of supply of exports (X) in the protection situation is η_{XX} elasticity of demand for imports (M) in the protection situation is ϵ_{mm}

The protection exchange rate is e

The free-trade exchange rate is e*

So that
$$\hat{e} = \frac{de}{e}$$

$$= \frac{e^*}{e} - 1$$

The value of imports in the protection situation is M

The value of exports in the protection situation is X

Rise in imports (at the protection exchange rate e) with tariff removal and devaluation is dM

Rise in exports (at the protection exchange rate e) with a devaluation is dX We have

$$\eta_{XX} = \frac{dx}{x} \cdot \frac{e}{(e - e^{x})} = \frac{dx}{x} \cdot \frac{1}{e} - (18)$$

$$\epsilon_{mn} = \frac{dM}{M} \cdot \frac{(1 + t)}{(t - e)} - (19)$$
as
$$\frac{dPx}{Px} = \frac{e^{x} - e}{e} = q$$
and
$$\frac{dpm}{Pm} = \frac{e(1 + t) - e^{x}}{e(1 + t)} = \frac{(t - q)}{(1 + t)}$$

To maintain the balance of payments unchanged dx = dM and hence from (18) and (19)

$$\hat{e} = \frac{t}{1 + \frac{x}{N} \frac{1}{C_{NN}} (1 + t)}$$
 (20)

^{1/} Corden, op. cit. p. 112.

If we assume that initially the balance of payments was in equilibrium with X = M,

then
$$\hat{e} = \frac{t}{\frac{\int xx}{\epsilon_{mm}} (1+t)+1}$$
 - (21)

If we had evaluated the exchange rate adjustment at the free trade elasticities, then the exchange rate adjustment would have been given by:

$$\hat{e} = \frac{t}{1 + \frac{\eta_{x'x}}{e_{mm'}}}$$
 (assuming x = M) - (21')

$$\eta_{xx} = \epsilon_{nm}$$

 $\hat{e} = t/2$ then

The change in relative factor prices, can be derived as follows: Making use of (10), (11), (12), and remembering that now with P_n fixed $P_n = 0$, we get

$$(\theta_{LM} - \theta_{LX}) \hat{W} + (\theta_{KM} + \theta_{KX}) \hat{r} = (\hat{P}_{m} - \hat{P}_{x}) - (22)$$

and
$$\theta_{I,N}$$
 $w + \theta_{KN}$ $\hat{r} = 0$. - (12')

This yields the following values of w and r

$$\hat{W} = \frac{\theta_{KN} \left(\hat{P}_{IM} - \hat{P}_{X} \right)}{\left(\hat{P}_{IM} - \hat{P}_{X} \right)} - (23)$$

and $\hat{\mathbf{r}} = \frac{\theta_{LN} (\hat{\mathbf{P}}_{m} - \hat{\mathbf{P}}_{x})}{10^{111}}$ (24)

where
$$|\Theta''| = \Theta_{KN} (\Theta_{LM} - \Theta_{LX}) - \Theta_{LN} (\Theta_{KM} - \Theta_{KX})$$

= $(\Theta_{LM} - \Theta_{LX})$ - (25)

The change in relative factor prices $(\hat{\mathbf{w}} - \hat{\mathbf{r}})$ is from (23) and (25) given by $\hat{\mathbf{w}} - \hat{\mathbf{r}} = \frac{(\hat{\mathbf{p}}_{m} - \hat{\mathbf{p}}_{x})(\theta_{KM} + \theta_{LM})}{|\mathbf{q}_{LM}|}$

$$\hat{\mathbf{v}} - \hat{\mathbf{r}} = \frac{(\hat{\mathbf{P}}_{m} - \hat{\mathbf{P}}_{x})(\hat{\mathbf{e}}_{KN} + \hat{\mathbf{e}}_{LN})}{|\hat{\mathbf{e}}_{LN}|}$$

and as
$$\theta_{KN} + \theta_{LN} = 1$$
, and using (25)
$$\frac{\Lambda}{W} - \frac{\Lambda}{r} = (\frac{\Lambda}{P} - \frac{\Lambda}{P})/(\frac{\theta}{H} - \frac{\theta}{H}) \qquad -. \qquad (26)$$

$$\frac{1}{W} = \frac{1}{W} + \frac{1}{$$

- (3) We can now compare the changes in relative commodity and factor prices given the two alternative adjustment mechanisms:
- (A) With the exchange rate fixed and Pn variable we have

$$\hat{P}_{n} = \begin{bmatrix} \overline{q}_{nm} - e_{nn} \\ e_{nn} - q_{nn} \end{bmatrix} \hat{P}_{m} - (8)$$
and
$$(\hat{w} - \hat{r}) = \begin{bmatrix} \widehat{P}_{m} - \hat{P}_{n} \\ \overline{\theta}_{LM} - \theta_{LN} \end{bmatrix} - (17)$$

Note that Pm = - t

(B) With Pn fixed and the exchange rate variable we have

and
$$\hat{\mathbf{e}} = \frac{\mathbf{t}}{\eta_{\underline{XX}}} (1 + \mathbf{t}) + 1$$

$$\hat{\mathbf{e}}_{\underline{mm}} (1 + \mathbf{t}) + 1$$

For $\overset{\bullet}{P}_n$ in (A) to equal $\overset{\bullet}{e}$ in (B) clearly from (8) and (20) neglecting the cross elasticities in (8)

$$\frac{1}{\eta_{\text{nn}} - \varepsilon_{\text{nn}}} = \frac{1}{\frac{\eta_{xx}}{\varepsilon_{\text{mm}}} (1+t) + 1 - (27)}$$

If we assume that the demand and supply schedules for imports (M) and exports

(X) are of constant elasticity, and we evaluate, e, using the values of imports and exports at the free-trade exchange rate e, then,

$$\hat{e} = \frac{t}{1 + \eta_{xx}}$$
 (See Cordon p. 111-112) (20')

and hence for $e = P_n$, we have

$$1 + \frac{\eta_{xx}}{c_{iva}} = \eta_{nn} - \epsilon_{nn} \qquad (27')$$

This condition will not in general hold, and hence in general $\hat{P}_n \neq \hat{e}$.

- (D) For $(\mathring{\mathbf{w}} \mathring{\mathbf{r}})$ in the two adjustment situations to be the same, from
- (17) and (26) we have

$$\frac{(\hat{P}_{m} - \hat{P}_{n})}{(\theta_{LM} - \theta_{LN})} = \frac{(\hat{P}_{m}^{\dagger} - \hat{P}_{x}^{\dagger})}{(\theta_{LM} - \theta_{LX})} - (28)$$

From (8), neglecting cross elasticities and making use of the relationship for $(P_m^1 - P_x^1)$ and (20) for e, we get,

$$\frac{(1 + \epsilon_{nn} - \ell_{nn})}{(\epsilon_{nn} - \ell_{nn})(\epsilon_{LM} - \epsilon_{LN})} = \frac{(\ell_{xx} - \epsilon_{mm} (1 + t))}{(\epsilon_{mn} + \ell_{xx})(1 + t)(\epsilon_{LM} - \epsilon_{LX})} - (29)$$

This condition will not hold in general and hence $(\mathring{\mathbf{w}} - \mathring{\mathbf{r}})$ will not be the same for the two adjustment mechanisms.

APPENDIX II

Case 3: Two Traded Goods Produced and Consumed, and One Non-Traded Intermediate Good Froduced

The symbols are the same as in Appendix I.

The production relations are given by:

$$\mathbf{a}_{\mathrm{LM}}^{\mathrm{W}} + \mathbf{a}_{\mathrm{KM}}^{\mathrm{r}} + \mathbf{a}_{\mathrm{JM}}^{\mathrm{p}} = \mathbf{P}_{\mathrm{m}} \tag{1}$$

$$a_{LX}^W + a_{KX}^r + a_{NX}^p = P_x$$
 - (2)

$$\mathbf{a}_{\mathsf{L},\mathsf{N}}^{\mathsf{W}} + \mathbf{a}_{\mathsf{K}\mathsf{N}}^{\mathsf{r}} = \mathbf{P}_{\mathsf{n}} \tag{3}$$

As before, differentiating totally yields

$$\Theta_{LM} + \Theta_{KM} + \Theta_{MM} + \Theta_{MM} = P_{m} \qquad - \qquad (4)$$

$$\theta_{LX}^{\hat{W}} + \theta_{KX}^{\hat{r}} + \theta_{NX}^{\hat{r}} + \theta_{NX}^{\hat{r}} = \hat{P}_{X}^{\hat{r}} \qquad - \qquad (5)$$

$$\Theta_{LN}^{\wedge} + \Theta_{KN}^{\wedge} = P_n \qquad - \qquad (6)$$

Substituting (6) into (4) and (5), we get

$$(\Theta_{LM} + \Theta_{NM} \Theta_{LN}) \hat{\mathbf{w}} + (\Theta_{KM} + \Theta_{NM} \Theta_{KN}) \hat{\mathbf{r}} = \hat{\mathbf{p}}_{m} -$$
 (7)

$$(\Theta_{LX} + \Theta_{NX}\Theta_{LN})\hat{w} + (\Theta_{KX} + \Theta_{NX}\Theta_{KN})\hat{r} = \hat{P}_{X} - (8)$$

Subtracting (8) from (7), and after simplification we get

$$(\hat{\mathbf{w}} - \hat{\mathbf{r}}) = \frac{(\hat{\mathbf{p}}_{m} - \hat{\mathbf{p}}_{x})}{(\hat{\mathbf{e}}_{KX} - \hat{\mathbf{e}}_{KA}) + \hat{\mathbf{e}}_{KN} (\hat{\mathbf{e}}_{NX} - \hat{\mathbf{e}}_{NM})} - (9)$$

Irrespective of the adjustment mechanism for bringing about the equilibrium price of traded to non-traded goods, the relative factor prices in the free trade situation, with the removal of the tariff will be given by (9).

Suppose the adjustment is with the exchange rate fixed and the price of N flexible

Then $P_m = t$, and $P_x = 0$, and

$$(\hat{\mathbf{w}} - \hat{\mathbf{r}}) = \frac{\mathbf{t}}{(\theta_{KX} - \theta_{KM}) + \theta_{KM} (\theta_{MX} - \theta_{MM})}$$
(91)

What will be the change in price of N, P_n in the movement to the free trade position?

Solving for w and r from (7) and (8) we get

$$\dot{\mathbf{A}} = \frac{\mathbf{P}_{m} \left(\mathbf{\Theta}_{KX} + \mathbf{\Theta}_{NX} \mathbf{\Theta}_{KN} \right) - \mathbf{P}_{x} \left(\mathbf{\Theta}_{KM} + \mathbf{\Theta}_{NM} \mathbf{\Theta}_{KN} \right)}{\mathbf{P}_{m} \left(\mathbf{\Theta}_{KX} + \mathbf{\Theta}_{NX} \mathbf{\Theta}_{KN} \right) - \mathbf{P}_{x} \left(\mathbf{\Theta}_{KM} + \mathbf{\Theta}_{NM} \mathbf{\Theta}_{KN} \right)} - \mathbf{P}_{m} \mathbf{P}_{m}$$

and
$$\hat{\mathbf{r}} = \frac{\hat{\mathbf{P}}_{m}(\Theta_{LX} + \Theta_{NX}\Theta_{LN}) - \hat{\mathbf{P}}_{x}(\Theta_{LM} + \Theta_{NM} \Theta_{LN})}{(\Theta_{LX} + \Theta_{NX}\Theta_{LN}) - (\Theta_{LM} + \Theta_{NM}\Theta_{LN})}$$
 - (11)

Substituting (10) and (11) in (6) and after simplification yields,

$$\hat{\mathbf{P}}_{\mathbf{n}} = \frac{\hat{\mathbf{P}}_{\mathbf{m}} \begin{bmatrix} \mathbf{\theta}_{\mathrm{LX}} & \mathbf{\theta}_{\mathrm{KN}} - \mathbf{\theta}_{\mathrm{KX}} \\ \mathbf{\theta}_{\mathrm{LN}} \end{bmatrix} - \hat{\mathbf{P}}_{\mathbf{x}} \begin{bmatrix} \mathbf{\theta}_{\mathrm{LM}} & \mathbf{\theta}_{\mathrm{KN}} - \mathbf{\theta}_{\mathrm{KM}} \\ \mathbf{\theta}_{\mathrm{LN}} \end{bmatrix}}{\begin{bmatrix} \mathbf{\theta}_{\mathrm{LX}} & \mathbf{\theta}_{\mathrm{KN}} - \mathbf{\theta}_{\mathrm{KX}} \end{bmatrix} - \begin{bmatrix} \mathbf{P}_{\mathrm{H}} \mathbf{\theta}_{\mathrm{KX}} - \mathbf{P}_{\mathrm{X}} \mathbf{\theta}_{\mathrm{KM}} \end{bmatrix}}$$

$$= \frac{\mathbf{\theta}_{\mathrm{KM}}}{\frac{\mathbf{\theta}_{\mathrm{LN}}}{\mathbf{\theta}_{\mathrm{LN}}}} \begin{bmatrix} \hat{\mathbf{P}}_{\mathbf{m}} \mathbf{\theta}_{\mathrm{LX}} - \hat{\mathbf{P}}_{\mathbf{x}} \mathbf{\theta}_{\mathrm{LM}} \end{bmatrix} - \begin{bmatrix} \hat{\mathbf{P}}_{\mathrm{H}} \mathbf{\theta}_{\mathrm{KX}} - \hat{\mathbf{P}}_{\mathbf{x}} \mathbf{\theta}_{\mathrm{KM}} \end{bmatrix}}{\mathbf{\theta}_{\mathrm{LN}}} - \hat{\mathbf{\theta}}_{\mathrm{KM}}$$

$$= \frac{\mathbf{\theta}_{\mathrm{KM}}}{\frac{\mathbf{\theta}_{\mathrm{LN}}}{\mathbf{\theta}_{\mathrm{LN}}}} \begin{bmatrix} \hat{\mathbf{P}}_{\mathbf{m}} \mathbf{\theta}_{\mathrm{LX}} - \hat{\mathbf{P}}_{\mathbf{x}} \mathbf{\theta}_{\mathrm{LM}} \end{bmatrix} - \begin{bmatrix} \hat{\mathbf{P}}_{\mathrm{H}} \mathbf{\theta}_{\mathrm{KX}} - \hat{\mathbf{P}}_{\mathbf{x}} \mathbf{\theta}_{\mathrm{KM}} \end{bmatrix}}{\mathbf{\theta}_{\mathrm{LN}}}$$

$$= \frac{\mathbf{\theta}_{\mathrm{KM}}}{\frac{\mathbf{\theta}_{\mathrm{LN}}}{\mathbf{\theta}_{\mathrm{LN}}}} \begin{bmatrix} \mathbf{\theta}_{\mathrm{LX}} - \mathbf{\theta}_{\mathrm{LM}} \end{bmatrix} - \begin{bmatrix} \hat{\mathbf{P}}_{\mathrm{KX}} - \mathbf{\theta}_{\mathrm{KM}} \end{bmatrix}}{\mathbf{\theta}_{\mathrm{LN}}} - \hat{\mathbf{\theta}}_{\mathrm{KM}}$$

$$= \frac{\mathbf{\theta}_{\mathrm{KM}}}{\frac{\mathbf{\theta}_{\mathrm{LN}}}{\mathbf{\theta}_{\mathrm{LN}}}} \begin{bmatrix} \mathbf{\theta}_{\mathrm{LN}} - \mathbf{\theta}_{\mathrm{LM}} \end{bmatrix} - \begin{bmatrix} \mathbf{\theta}_{\mathrm{KX}} - \mathbf{\theta}_{\mathrm{KM}} \end{bmatrix}$$

$$= \frac{\mathbf{\theta}_{\mathrm{KM}}}{\frac{\mathbf{\theta}_{\mathrm{LN}}}{\mathbf{\theta}_{\mathrm{LN}}}} \begin{bmatrix} \mathbf{\theta}_{\mathrm{LN}} - \mathbf{\theta}_{\mathrm{LN}} \end{bmatrix} - \begin{bmatrix} \mathbf{\theta}_{\mathrm{KN}} - \mathbf{\theta}_{\mathrm{KM}} \end{bmatrix}}{\mathbf{\theta}_{\mathrm{LN}}} - \mathbf{\theta}_{\mathrm{KM}}$$

We know $P_x = 0$, and $P_m = t$, then from (12) we have

$$\hat{\mathbf{P}}_{\mathbf{n}} = \frac{\mathbf{t} \left[\frac{\mathbf{\theta}_{\mathrm{LN}}}{\mathbf{\theta}_{\mathrm{LN}}} \, \mathbf{e}_{\mathrm{LX}} - \mathbf{e}_{\mathrm{LM}} \right] - \left[\mathbf{e}_{\mathrm{KX}} - \mathbf{e}_{\mathrm{KM}} \right]}{\mathbf{t} \left[\mathbf{e}_{\mathrm{LN}} - \mathbf{e}_{\mathrm{LM}} \right] - \left[\mathbf{e}_{\mathrm{KX}} - \mathbf{e}_{\mathrm{KM}} \right]}$$
(12')

The change in the non-traded good price will be the same as that of the importable, if,

or
$$\frac{a_{KN}}{\theta_{LN}} = \frac{a_{KN}}{\theta_{LM}} = \frac{a_{KN}}{\theta_{LM}}$$
or $\frac{a_{KN}}{\theta_{LN}} = \frac{a_{KN}}{\theta_{LM}}$
or $\frac{a_{KN}}{\theta_{LN}} = \frac{a_{KN}}{\theta_{LM}}$

$$- (13)$$

That is if the M and N industries have the same factor intensities.

Next consider the adjustment mechanism with the price of N fixed and with the exchange rate variable.

Then in (4) to (6)
$$\hat{P}_n = 0$$

and
$$(\hat{\mathbf{w}} - \hat{\mathbf{r}}) = \frac{(\hat{\mathbf{p}}_{m} - \hat{\mathbf{p}}_{x})}{|\theta|}$$
 (14)

where
$$|\theta| = \theta_{LM} \theta_{KX} - \theta_{KM} \theta_{LX}$$
 - (15)

$$\hat{\mathbf{w}} = \frac{\Theta_{\mathrm{KN}}}{\Theta_{\mathrm{LN}}} \hat{\mathbf{r}}$$

$$\mathbf{\hat{r}} = \frac{\mathbf{e}_{LN}}{\mathbf{e}_{KN}} \mathbf{\hat{w}}$$

Substituting either the value of w or r in (4) and (5), yields

$$\frac{\hat{P}_{m}}{\hat{P}_{x}} = \frac{\left[\theta_{KM} \; \theta_{LN} - \theta_{KN} \; \theta_{LM}\right]}{\left[\theta_{KX} \; \theta_{LN} - \theta_{KN} \; \theta_{LX}\right]} - \frac{1}{(16)}$$

Given that the needed devaluation rate is ê, and that the tariff was t, we have

$$\frac{\hat{P}_{m}}{\hat{P}_{x}} = \frac{(t - \hat{e})}{\hat{e}(1 + t)}$$

Substituting this in (16) and simplifying yields

$$\hat{\Theta} = \frac{t \left(\Theta_{KX} \Theta_{LN} - \Theta_{KN} \Theta_{LX}\right)}{\left(1 + t\right) \left(\Theta_{KM} \Theta_{LN} - \Theta_{KN} \Theta_{LM}\right) + \left(\Theta_{KX} \Theta_{LN} - \Theta_{KN} \Theta_{LX}\right)}.$$
(17)

As is to be expected in this model, as the domestic price of N, has been fixed, this immediately determines the requisite devaluation, that is we correct relative price of traded to non-traded goods, from the domestic

production relationships (the 0's) and the change in the relative commodity price of the two traded commodities. As the relative commodity prices which are fixed from world trade, determine the relative domestic factor prices, and as soon as the domestic money price of the only domestic good is fixed, all money prices, including the exchange rate in the free trade situation are determined.

APPENDIX III

The Equivalence of the BT 'Equilibrium' Exchange Rate and the Devaluation Rate of Appendix I

The BT 'equilibrium' exchange rate is (see 1, p. 216) for three tradable goods (two importables M_1 M_2 and one exportable X) is

$$r^* = \frac{r \left[g v_1 \in x_1 - T_1 u_1 \eta m_1 - T_2 u_2 \eta m_2 \right]}{v_1 \in x_1 - (u_1 \eta m_1 + v_2 \eta m_2)}$$
 (1)

where using the notation in Appendix I.

r = e, the protection exchange rate

 \emptyset = (1 + s), the export subsidy to X

 $T_1 = (1 + t_1)$, the tariff on importable M_1

 $T_2 = (1 + t_2)$, the tariff on importable M_2

 $V_i = \frac{x}{x} = 1$, the share of x in total exports

 $u_1 = \frac{M_1}{M_1 + M_2}$, the share of M_1 in total imports

 $u_2 = \frac{M_2}{M_1 + M_2}$, the share of M₂ in total imports

 $\epsilon_{x_1} = \eta_{xx}$, the elasticity of supply of experts (x)

 $\eta_1 = -\epsilon_{m_1 m_1}$, the elasticity of demand for imports of M_1

Hence (1) using our notation become

$$i^* = \underbrace{e \left[(1+s) \eta_{xx} + (1+t_1) \dot{\epsilon}_{m_1 m_1} \frac{M_1}{M_1 + M_2} + (1+t_2) \dot{\epsilon}_{m_2 m_2} \frac{M_2}{M_1 + M_2} \right]}_{q_{xx} + \dot{\epsilon}_{m_1 m_1} \frac{M_1}{M_1 + M_2} + \dot{\epsilon}_{m_1 m_1} \frac{M_2}{M_1 + M_2}}$$

$$- (1!)$$

The change in the exchange rate (e), evaluated at the free trade elasticities (BT assume constant elasticities) from equation (21') Appendix I, incorporating

two importables, with tariff rates t_1 and t_2 , and an export subsidy of s_1 , and assuming $x = M_1 + M_2$, would be given by

$$\hat{e} = \frac{8 / xx + t_1 + \epsilon_{m_1 m_1} + t_2 + \epsilon_{m_2 m_2} + \epsilon_{m_2 m_2} + \epsilon_{m_1 m_2}}{\sqrt{1 + x_1 + \epsilon_{m_1 m_2} + \epsilon_{m_1 m_2} + \epsilon_{m_2 m_2} + \epsilon_{m_2 m_2} + \epsilon_{m_2 m_2} + \epsilon_{m_2 m_2}}} - (2)$$
Noting that $\hat{e} = \frac{e^*}{e^*} - 1$

or $e^*=e(1+\hat{e})$, we get from (2)

$$e^{*} = \frac{e\left[(1+s)\eta xx + (1+t_{1})\epsilon_{m_{1}m_{1}}\left[\frac{M_{1}}{M_{1}+M_{2}}\right] + (1+t_{2})\epsilon_{m_{2}m_{2}}\left[\frac{M_{2}}{M_{1}+M_{2}}\right]}{\eta xx + \epsilon_{m_{1}m_{1}}\left[\frac{M_{1}}{M_{1}+M_{2}}\right] + \epsilon_{m_{2}m_{2}}\left[\frac{M_{2}}{M_{1}+M_{2}}\right] - (2!)}$$

(2') and (1') are equal, and hence e* = r*.

^{1/} See Corden's equation (5.1) p. 113, op. cit.