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CALIBRATING MEASUREMENT UNCERTAINTY IN PPP EXCHANGE RATES

Basic issues

- We know there is uncertainty in estimated PPPs
  - Much of this is methodological
  - Non-sampling errors
- Inherent uncertainty from the variability of relative prices across countries
  - Each commodity parity can be thought of as a draw from a distribution
  - ICP is sampling commodities within basic heads
  - Mean (or something) is the overall parity or PPP
  - Dispersion of parities gets into overall uncertainty
- Also uncertainty about which index
  - Laspeyres/Paasche ratio captures that kind of uncertainty
  - If very big, the choice of index is doing a lot of work
- These two kinds of uncertainty should be linked

Standard errors for PPPs

- Based on earlier work in ICP
  - Used in Deaton and Dupriez (2011)
- Idea is to use the CPD projection as a basic tool of analysis

$$\ln p_{ic} = a_c + \beta_i + \varepsilon_{ic}$$

- Where I treat the ε as random draws to calculate the distribution of various standard statistics
  - Like PPPs of various forms

Take the US as star

- Consider basic heading parities relative to the US for various countries
  - 2005 ICP has 128 parities
- SD of log of parities
  - Canada is 0.25
  - China is 0.77
  - India is 0.81
  - Tajikistan is 1.35
- Uncertainty about overall PPP is larger for TJK than for Canada
  - SD of log of parities is one measure of uncertainty of PPPs

Formalization

- One crude PPP is the geomean, so logPPP is the mean of the log parities in a star system
- Standard error of the log geomean is the square root of 1/*n* times the variance of the log parities

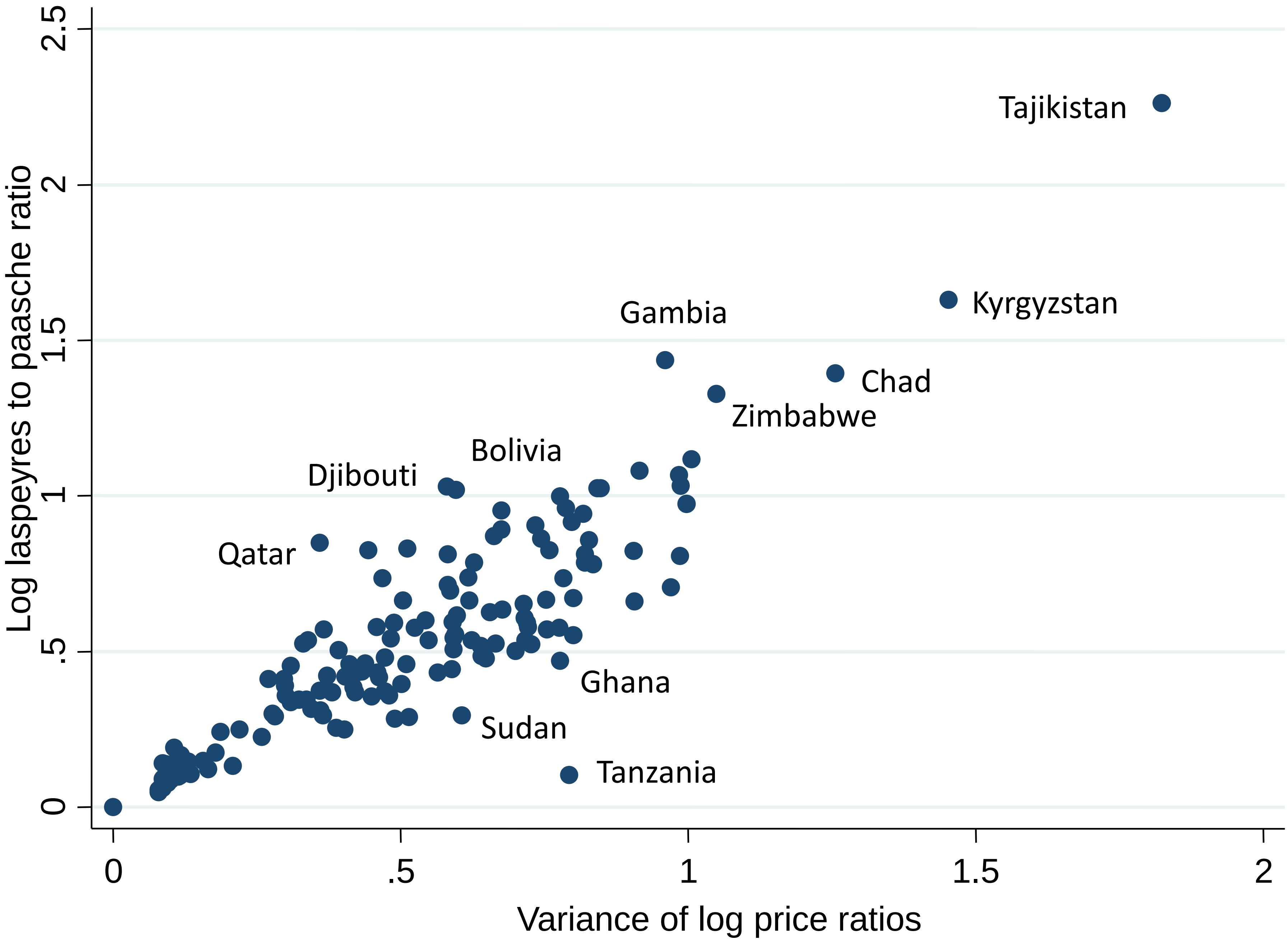
$$\ln P_{cd}^{G} = \frac{1}{n}\sum_{i=1}^{n}\ln\frac{p_{id}}{p_{ic}}$$

$$\ln\left(\frac{p_{id}}{p_{ic}}\right) = \ln p_{id} - \ln p_{ic} = (a_d - a_c) + (\varepsilon_{id} - \varepsilon_{ic})$$

$$s.d.\ln\left(\frac{p_{id}}{p_{ic}}\right) = \sqrt{\sigma_d^2 + \sigma_c^2} \qquad s.e.\ln P_{cd}^{G} = \sqrt{\frac{\sigma_d^2 + \sigma_c^2}{n}}$$

Laspeyres to Paasche ratio

- LP ratio is another aspect of PPP uncertainty
  - Sometimes used as a measure of distance apart of two countries
- What is the relationship between the log of the LP ratio and the variance of the log parities
  - Figure 1 shows this for all countries relative to the US in the 2005 ICP



Tajikistan
Kyrgyzstan
Gambia
Chad
Zimbabwe
Bolivia
Djibouti
Qatar
Ghana
Sudan
Tanzania
Log laspeyres to paasche ratio
Variance of log price ratios
0
.5
1
1.5
2
2.5

Why does this happen?

- I can use the CPD decomposition with the Laspeyres/Paasche ratio to look at this

$$\ln \rho_{dc} = \ln\left[\sum_{i=1}^{n} s_{ic} \exp(\varepsilon_{id} - \varepsilon_{ic})\right] + \ln\left[\sum_{i=1}^{n} s_{id} \exp(\varepsilon_{ic} - \varepsilon_{id})\right]$$

$$\ln \rho_{dc} \simeq \sum_{i=1}^{n} (s_{ic} - s_{id})(\varepsilon_{id} - \varepsilon_{ic}) + \frac{1}{2}\sum_{i=1}^{n} (s_{ic} + s_{id})(\varepsilon_{id} - \varepsilon_{ic})^2$$

$$E(\ln \rho_{cd}) = \sigma_c^2 + \sigma_d^2$$

- To this degree of approximation, and with identical variances by commodity, expectation of Laspeyres Paasche ratio is the variance of the log parities

Standard errors of geomean

- Ireland and Canada 2.5 percent
- India 7.1 percent
- China 6.8 percent
- Gambia 8.7 percent
- Kyrgyzstan 10.7 percent
- Tajikistan 11.9 percent

Better star systems

- Törnqvist and Fishers work in the same way but better

$$\ln P_{cd}^{T} = (a_d - a_c) + \frac{1}{2}\sum_{i=1}^{n}(s_{ic} + s_{id})(\varepsilon_{id} - \varepsilon_{ic})$$

- exactly, from the CPD. So we have at once that the variance of the log Törnqvist is

$$V(\ln P_{cd}^{T}) = \overline{s}^{cd}\,' V_{\varepsilon\varepsilon'}^{cd} \overline{s}^{cd}$$

- To a first order approximation, this is also the variance of the log Fisher

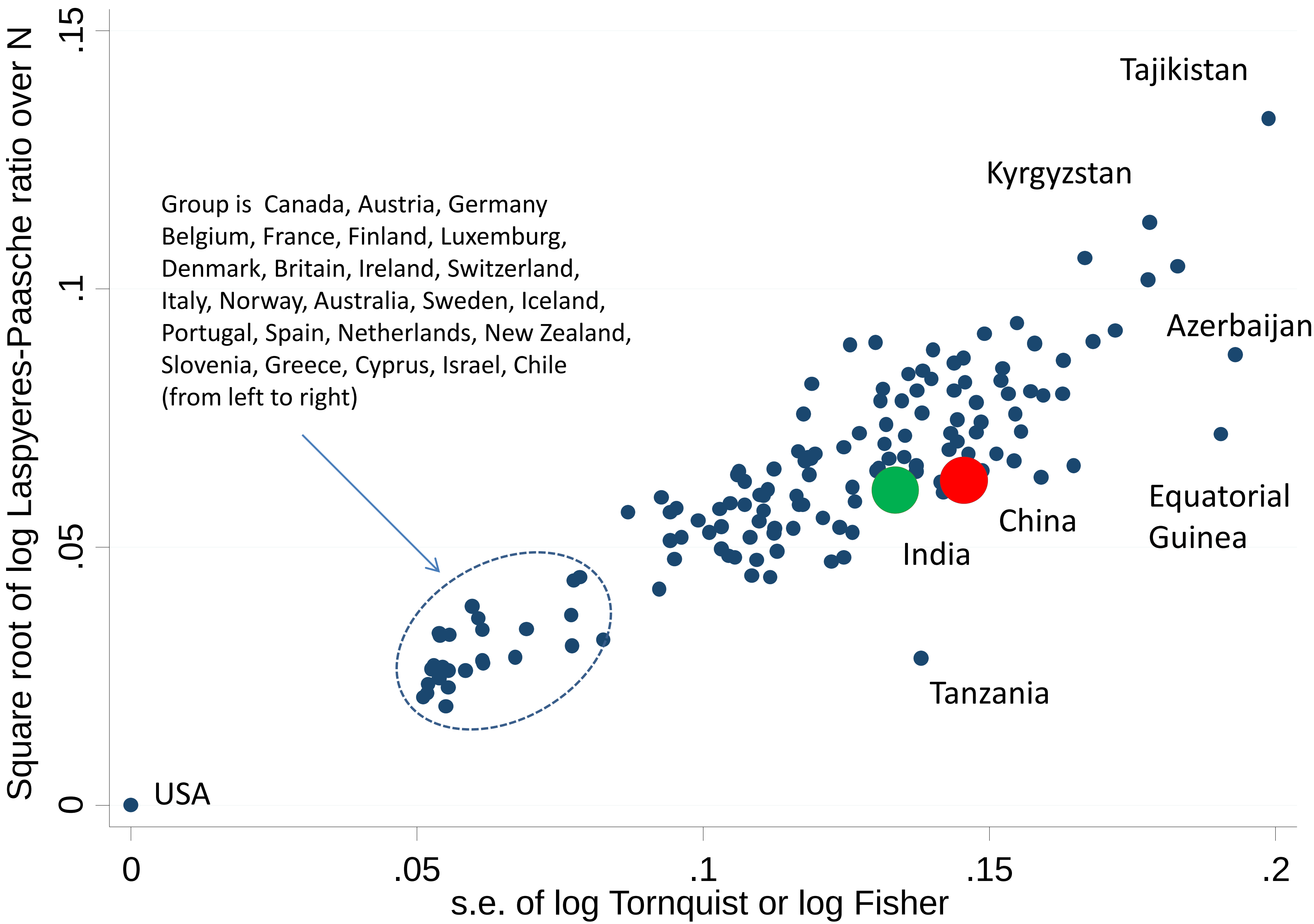
$V(\ln P$

Also related to LP ratio

$$E \ln \rho_{dc} = \sum_{i=1}^{n} \bar{s}_i (\sigma_{di}^2 + \sigma_{ci}^2)$$

$$V(\ln P_{cd}^T) = \sum_{i=1}^{n} \bar{s}_i^2 (\sigma_{di}^2 + \sigma_{ci}^2)$$

- If the budget shares were all equal, the variance of the log Törnqvist would be $1/n$ times the expectation of the log LP ratio
- These variances are larger than those for geomean because of GM theorem


Square root of log Laspyeres-Paasche ratio over N
s.e. of log Tornquist or log Fisher
0
.05
.1
.15
.2
Tajikistan
Kyrgyzstan
Azerbaijan
Equatorial
Guinea
China
India
Tanzania
USA
Group is Canada, Austria, Germany
Belgium, France, Finland, Luxemburg,
Denmark, Britain, Ireland, Switzerland,
Italy, Norway, Australia, Sweden, Iceland,
Portugal, Spain, Netherlands, New Zealand,
Slovenia, Greece, Cyprus, Israel, Chile
(from left to right)

Notes on Figure 2

- The standard errors are *large*
  - 2 s.e. for China and India is around 30 percent
- Much smaller for the group on the left
  - But still substantial, ten percent
- Reminiscent of Richard Stone (1949)
  - "I do not expect a very rapid resolution of the intellectual problems of making welfare comparisons between widely different communities"

Multilateral indexes

- The weighted CPD is calculated as a weighted regression, and its variance matrix comes from standard “outer-product” calculation

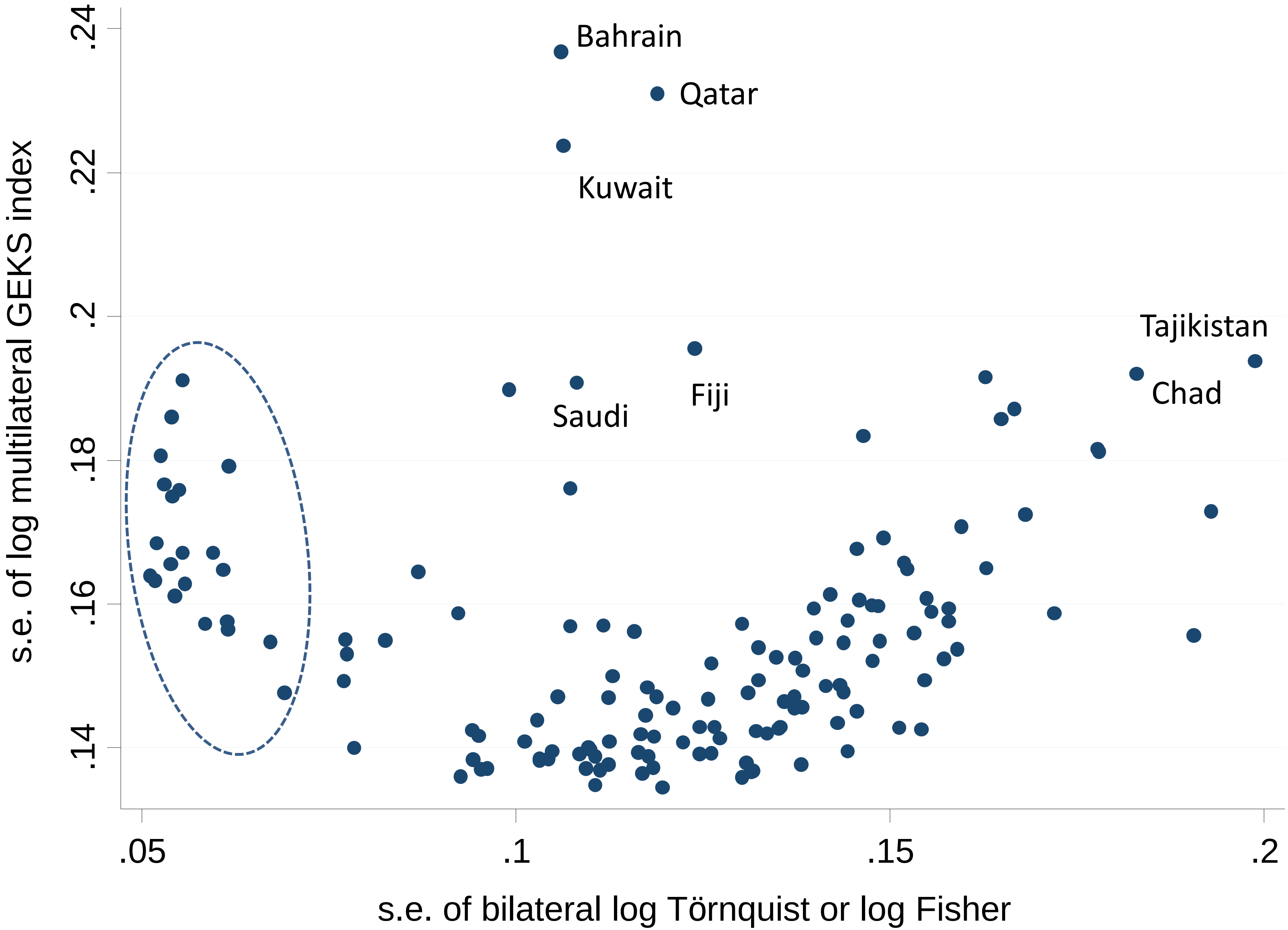
$$b = (X'SX)^{-1}X'Sy$$

$$V(b) = (X'SX)^{-1}X'S\Sigma SX(X'SX)^{-1}$$

- The V for the multilateral Fisher or EKS more work

$$4M^2V(\ln p^i) = (1-2\delta_{1i})\sum_{j=1}^{M}\sum_{k=1}^{M}(s^i+s^j)\Omega^i(s^i+s^k)+\sum_{j=1}^{M}\sum_{k=1}^{M}(s^1+s^j)\Omega^1(s^1+s^k)+$$

$$\sum_{j=1}^{M}(s^1-s^j)\Omega^j(s^1-s^j)+2\sum_{j=1}^{M}(s^i+s^j)\Omega^i(s^1-s^i)-2\sum_{j=1}^{M}(s^1-s^i)\Omega^1(s^1+s^j)$$


Bahrain
Qatar
Kuwait
Tajikistan
Chad
Saudi
Fiji
.24
.22
.2
.18
.16
.14
s.e. of log multilateral GEKS index
.05
.1
.15
.2
s.e. of bilateral log Törnquist or log Fisher

Notes on Figure 3

- Multilateral standard errors are typically larger
  - Average 15 percent instead of 12 percent
- Dispersion of ML standard errors smaller
  - Transitivity is sharing the errors
  - Poor bilateral is buttressed by ML comparisons
- Close countries have *much* larger s.e.
  - ML is a bad idea for them
  - Bringing Tajikistan into the Canada US comparison is not necessarily a good idea
  - Defense of regionalization/fixity in ICP
- Middle group of countries where costs of transitivity are balanced by the gains
- Still substantial uncertainty, big standard errors

Conclusions

- Is this a sensible way of thinking about standard errors of PPPs?
- Not completely sure
- Key idea that there exists a PPP rate between countries, and that parities for each BH are distributed around it
- And that the dispersion is a measure of uncertainty, which also matches LP ratio
- Rest is detail!