Optimal Composition of Sovereign Debt: An Adaptation from Modern Portfolio Theory

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Outline

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Introduction and Prerequisites

- Public debt managers meet public financing needs by raising funds in both domestic and external markets
- These funding consists of selling fixed income securities, at various maturities, currencies and interest payment structures
- Less costly borrowing options usually bring along more risk
- This presentation and corresponding paper addresses the question "What is the optimal debt composition?" and proposes a practical solution
- To apply the method presented in this presentation, the risk managers only should have following three modules at their disposal:
  - ***a simulation module***
  - ***a financing program***
  - ***a debt stock database***

Debt Simulation Model

- Turkish Treasury Debt Simulation Model (TDSM) is a Matlab-based stochastic model to assist the borrowing strategy decision making process in Turkish Treasury
- It takes initial debt stock, non-borrowing financing sources (i.e. primary balance, change in cash balance, privatization revenues etc.) , historic macroeconomic data and borrowing strategies to be tested as inputs
- It first simulates macroeconomic variables, then calculates borrowing need and gives related statistics of different borrowing strategies at the end of 5-year horizon as output
- TDSM uses the Conditional Cost-at-Risk(CC@R) metric which is an extended version of cost-at-risk metric. It focuses on the average of expected cost values in the worst-case scenarios which occur beyond the specified confidence level, rather than a single value on a percentile
- As cost indicator we used the value of "Accrued Inflation-Adjusted Debt Stock" at the end of 5-year analysis horizon. This metric captures accrued interest of CPI-linked bonds and effects of currency depreciations on stock. Extreme values of these stock figures at the end of analysis period are regarded as the risk indicators

Finding the Efficient Frontier

- To get round computational expenses and complexity of optimization models, we propose another way to find optimal debt composition. First, we decided to test borrowing strategies, with some simplifying adjustments
- 5% borrowing granularity and 5 representative borrowing instruments for in total 15 instruments

Representative Instruments

- 24-months TL zero-coupon
  - 3-months TL zero-coupon
  - 6-months TL zero-coupon
  - 12-months TL zero-coupon
  - 24-months TL zero-coupon
- 5-years TL fixed-rate
  - 2-years TL fixed-rate
  - 5-years TL fixed-rate
  - 10-years TL fixed-rate
- 10-years TL CPI-indexed
  - 10-years TL CPI-indexed
- 7-years TL floating-rate
  - 7-years TL floating-rate
- 10-years USD fixed-rate
  - 10-years USD fixed-rate
  - 30-years USD fixed-rate
  - 5-years EUR fixed-rate
  - 10-years EUR fixed-rate …

Finding the Efficient Frontier

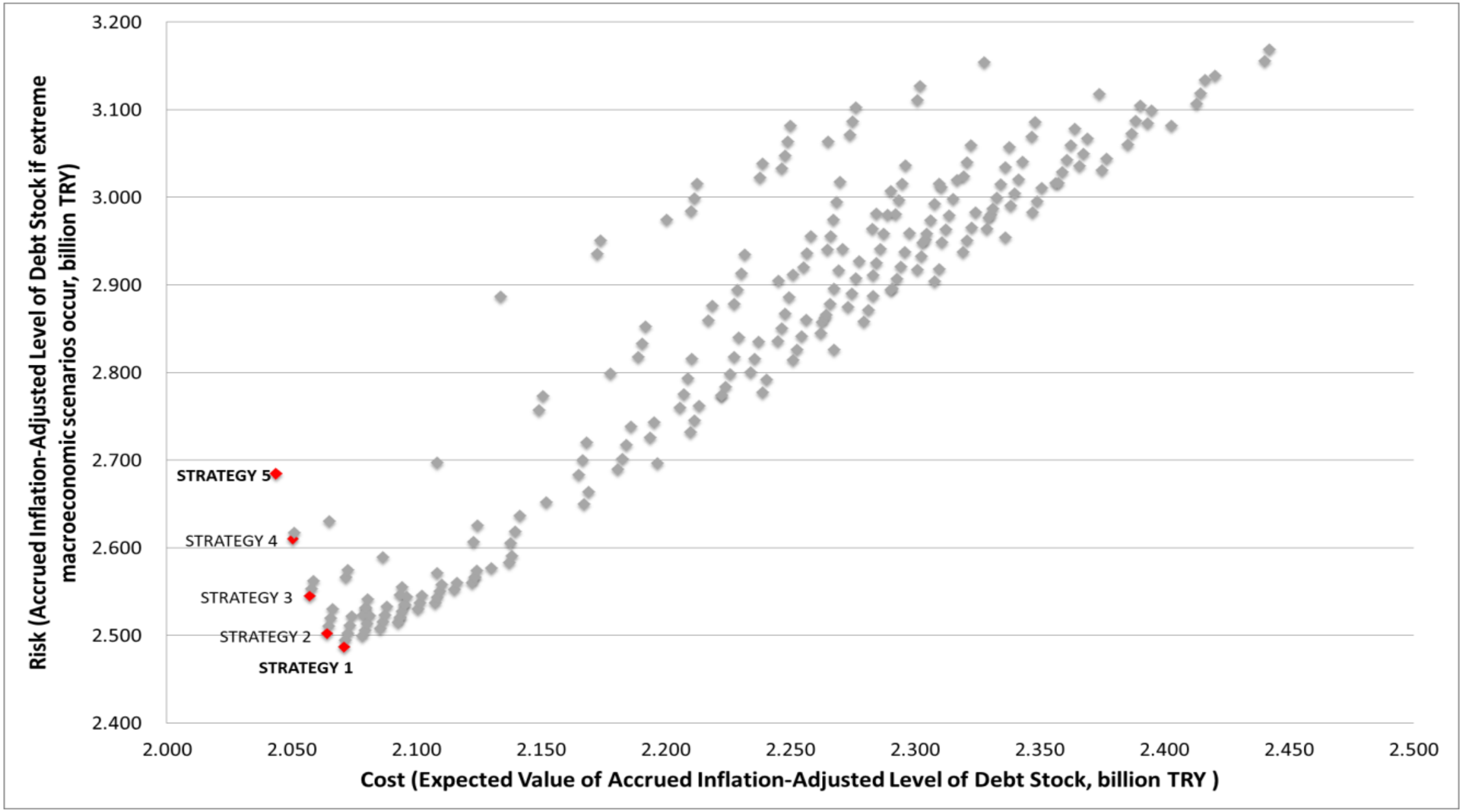
- Considering risk characteristics of specific types of securities and using judgment about borrowing markets, we set lower and upper bounds to issuances of securities

Limits on Representative Instruments

	Lower Bound	Upper Bound
24-months TL zero-coupon	5%	60%
5-years TL fixed-rate	5%	60%
10-years CPI-indexed TL	5%	60%
7-years floating-rate TL	5%	60%
10-years fixed-rate USD	0%	30%

- Using a 5-years simulation horizon, we tested 244 strategies under 5000 randomly generated scenarios
- To capture optimal debt stock composition through the optimal borrowing strategy, we started with a state where there is zero debt stock. However, we kept borrowing need during simulation period same as what real debt stock and mean macroeconomic projections imply. Thus, risk and cost levels of different strategies were not affected by initial zero-stock state

Finding the Efficient Frontier



Finding the Efficient Frontier

Composition of Efficient Strategies:

Strategies (lowest to highest risk)	Zero coupon	Fixed-rate coupon	CPI-indexed	FRN	FX-denominated
1	60%	25%	5%	5%	5%
2	60%	5%	15%	20%	0%
3	60%	15%	10%	15%	0%
4	60%	15%	10%	5%	10%
5	60%	5%	5%	5%	25%

Correlation Matrix

	Zero2Y	Fixed5Y	CPI	FRN	USD10Y
Zero2Y	1.00	0.98	0.82	0.88	0.75
Fixed5Y	0.98	1.00	0.82	0.87	0.75
FRN	0.88	0.87	0.80	1.00	0.55
CPI	0.82	0.82	1.00	0.80	0.57
USD10Y	0.75	0.75	0.57	0.55	1.00

- High correlation of costs between instruments issued by a single issuer has a strong impact on diversification gains. Therefore an instrument which has slightly better cost and risk characteristics dominates those efficient strategies
- The strongest cost and risk trade-off occurs between FX and TL instruments. This is understandable because USD 10-year bonds has the lowest correlation with other instruments
- In that regard, new instruments such as a GDP-linked bond here may enhance risk and cost metrics of portfolios because of its lower correlation with other instruments

Selecting the Optimal Strategy

- Modern Portfolio Theory proposes an approach to reach optimal portfolio composition. But, this approach is to determine optimal asset composition for investors holding different assets. We modified this approach considering borrower position of sovereigns
- First, for each point on the efficient frontier, we calculate the delta of risk value (marginal conditional cost-at-risk (MCCaR)) and the additional cost occurred by the issuance of one additional unit of that instrument for each instrument type (marginal cost)
- Then, we calculated the “marginal risk/marginal cost” ratios for each instrument type. After that, among the points on the efficient frontier, the one in which marginal risk/marginal cost ratio is the closest to each other is regarded as the optimal portfolio

$$\frac{\text{MCCaR}_{p1,i1}}{\text{Marginal Cost}_{p1,i1}} = \frac{\text{MCCaR}_{p1,i2}}{\text{Marginal Cost}_{p1,i2}} = \cdots = \frac{\text{MCCaR}_{p1,i5}}{\text{Marginal Cost}_{p1,i5}}$$

$$\frac{\text{MCCaR}_{p2,i1}}{\text{Marginal Cost}_{p2,i1}} = \frac{\text{MCCaR}_{p2,i2}}{\text{Marginal Cost}_{p2,i2}} = \cdots = \frac{\text{MCCaR}_{p2,i5}}{\text{Marginal Cost}_{p2,i5}}$$

$$\ldots \quad = \quad \ldots \quad = \quad \ldots$$

$$\frac{\text{MCCaR}_{p5,i1}}{\text{Marginal Cost}_{p5,i1}} = \frac{\text{MCCaR}_{p5,i2}}{\text{Marginal Cost}_{p5,i2}} = \ldots = \frac{\text{MCCaR}_{p5,i5}}{\text{Marginal Cost}_{p5,i5}}$$

Key Takeaways

- Simple approach to find the optimal composition of debt and the relevant borrowing strategies
- The sole requirements are a simulation module, a financing program and a debt stock database.
- User needs to decide the granularity and the upper-lower bounds for the strategies
- Macroeconomic assumptions play a key role
- Although, it is not always the case that the debt managers have the possibility to use the borrowing instruments in desired size, maturity, level, interest and currency type; results assist decision makers by showing the direction
- It is always advised that, public debt managers to continue supporting risk management activities with scenario and sensitivity analyses

Appendix: Method Used in Finding Optimal Debt Portfolio

- According to approaches based on Modern Portfolio Theory, optimal portfolio is defined as the composition, in which the ratio of excess return divided by the per unit change in the risk value of the portfolio when an additional investment of that position is added to the portfolio, is equal to the same constant for each position. In other words, optimal composition is obtained when this ratio for a position is equal to the ratios of other positions.
- To recall, Sharpe ratio is a ratio shows the return of an instrument above risk-free return (excess return) per unit of standard deviation of the portfolio.

$$\text{Sharpe ratio} = \frac{Portfolio\ Return - Riskfree\ Return}{Portfolio\ Standard\ Deviation} \quad (1)$$

- Value-at-Risk is defined as the value that can be expected to be lost during severe adverse market fluctuations. (Marisson, 2002) If we take Value-at-Risk instead of standard deviation as risk indicator, the ratio turns into as equation (2).

$$\frac{Portfolio\ Return - Riskfree\ Return}{Portfolio\ Value - at - Risk\ (VaR)} \quad (2)$$

Appendix: Method Used in Finding Optimal Debt Portfolio

- This ratio maximizes when the weights of each position makes the equality below hold for each position or instrument type

$$\frac{\text{Return}_{\,instrument\ i} - Riskfree\ Return}{(\text{Marginal Value}-\text{at}-\text{Risk}_i)} = \frac{\text{Return}_{instrument\ j} - Riskfree\ Return}{(\text{Marginal Value}-\text{at}-\text{Risk}_j)} \quad (3)$$

- Since there is no "return" and "risk-free return" notions for sovereign Treasuries, we looked at how an additional issuance of an instrument changes the risk taken and the cost borne relative to other instruments. Therefore, considering sovereign borrower position, the general equation in question is modified and applied as follows:

$$\frac{\text{MCCaR}_{1,1}}{\text{Marginal Cost}_{1,i1}} = \frac{\text{MCCaR}_{1,2}}{\text{Marginal Cost}_{1,2}} = \cdots = \frac{\text{MCCaR}_{1,I}}{\text{Marginal Cost}_{1,I}}$$

$$\frac{\text{MCCaR}_{2,1}}{\text{Marginal Cost}_{2,1}} = \frac{\text{MCCaR}_{2,2}}{\text{Marginal Cost}_{2,2}} = \cdots = \frac{\text{MCCaR}_{2,I}}{\text{Marginal Cost}_{2,I}}$$

$$\ldots \quad = \quad \ldots \quad = \quad \ldots$$

$$\frac{\text{MCCaR}_{P,1}}{\text{Marginal Cost}_{P,1}} = \frac{\text{MCCaR}_{\boldsymbol{P},2}}{\text{Marginal Cost}_{\boldsymbol{P},2}} = \ldots = \frac{\text{MCCaR}_{\boldsymbol{P},\boldsymbol{I}}}{\text{Marginal Cost}_{\boldsymbol{P},\boldsymbol{I}}}$$

Appendix: Method Used in Finding Optimal Debt Portfolio

- In Marginal Cost and Marginal Risk calculations, one percent of total borrowing requirement is set to be one unit of additional issuance.
- However, to isolate the effect of this issuance to risk and cost measures, this additional issuance is not meet by decreasing the issuance of other debt instruments but by increasing the borrowing 101 percent of borrowing requirement.
- Since Marginal Risk/Marginal Cost ratio is examined only within a strategy for each type of instruments in that strategy, 101 percent borrowing has no effect on comparing between different borrowing strategies.