# Computation of Basic Heading PPPs for Comparisons within and between Regions

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This chapter is the first in a set of three devoted to describing the aggregation methods used at various stages of the International Comparison Program (ICP) in compiling the purchasing power parities (PPPs) of currencies of countries within regions and for comparisons between regions.[1] A schematic diagram showing the various stages in the computation of PPPs appears in figure 1.3 in chapter 1. The main objectives of this chapter are to provide a detailed description of the methods used in the computation of PPPs at the basic heading (BH) level at the regional level[2] and to describe how BH-level PPPs are compiled for the purpose of making global comparisons of prices. Chapter 5 by W. Erwin Diewert focuses on the methods for aggregating price and quantity data in the computation of PPPs for higher-level aggregates such as consumption, investment, and, finally, the gross domestic product (GDP) at the regional level. Chapter 6, also by Diewert, examines the problem of linking regional comparisons above the basic heading level to make global comparisons.

The literature on aggregation methods for international comparisons has traditionally focused on aggregation at levels above the basic heading level—see Balk (2008) and Rao (2009) for excellent reviews of such methods—and relatively little weight is placed on the problem of constructing PPPs at the basic heading level. In a recent paper, Hill and Hill stress the importance of PPPs at the basic heading level:

> Perhaps the most pressing concern in the international comparisons literature is the problem of obtaining unbiased price indexes at the basic heading level (the lowest level of aggregation at which expenditure weights are available). The basic heading price indexes provide building blocks from which the overall comparison is constructed. If these building blocks are biased or otherwise flawed, then everything that builds on them will be likewise tainted. (Hill and Hill 2009, 192–93)

The importance of reliable PPPs at the basic heading level was reiterated after the release of the 2005 ICP results (World Bank 2008), which showed substantial revisions to the global GDP and

to the real incomes of major economies such as China and India. Deaton and Heston (2008) examined possible reasons for such revisions and concluded that one of the underlying factors is the price data used for compilation of PPPs at the basic heading level.

Reflecting concerns about improving the quality and reliability of PPPs at the basic heading level, this chapter is designed to provide an appreciation of the core issues surrounding the compilation of PPPs at the basic heading level, including a description of the characteristics of the price data, as well as the characteristics of the products priced in different countries. The chapter seeks to illustrate how the aggregation methods used in the ICP are designed to take full account of the information on such characteristics. Reflecting the forward-looking nature of this ICP volume, this chapter not only reviews the methods used in the 2005 ICP, but also describes the new approaches and methods being considered for the 2011 ICP. A significant new approach in the 2011 ICP is the use of a *list of core products* for the purpose of linking regions. Although there is an ongoing discussion on how to calibrate the aggregation methods to best utilize prices on core products, a consensus has emerged on the method to be used for linking PPPs at the basic heading level.[3]

This chapter draws on material from a number of sources, including (1) the extensive discussion of the issues surrounding aggregation at the basic heading level in the *ICP 2005 Methodological Handbook* (World Bank 2007); (2) the final report of the 2005 ICP issued by the Global Office (World Bank 2008); and (3) various published and unpublished papers and reports on the subject.

The chapter is organized as follows. Section 4.1 focuses on features and aspects specific to comparisons at the basic heading level. It discusses in detail the nature of price data, product characteristics, and the features of price quotations that have a direct bearing on the aggregation methods used. Section 4.2 discusses briefly the analytical requirements for the aggregation methods used at the basic heading level. Sections 4.3 and 4.4 are devoted to a description of the aggregation methods used in the compilation of PPPs at the regional level. The Jevons binary price indexes and the Gini-Éltetö-Köves-Szulc (GEKS) method of constructing transitive parities are the subject of section 4.3. Variants of the Jevons-GEKS indexes designed to use information on the *representativity* and *importance* of products[4] are also discussed in this section. Section 4.4 is devoted to a description of the Country Product Dummy (CPD) method and its variants used in the computation of basic heading PPPs.[5] Section 4.5 focuses on the problem of linking the regional basic heading PPPs and describes the methodology used in the 2005 ICP based on *Ring countries* and a *Ring product list,* as well as the new approach based on the use of *core products* proposed for the 2011 ICP. Section 4.6 presents some concluding remarks.

# 4.1 Features of Data for Computing PPPs at the Basic Heading Level

As a starting point, it is useful to note the main features of the BH-level data and what distinguishes the basic heading level from higher levels of aggregation. The features discussed in this section have a direct bearing on the methodology used in aggregating data. The number of products priced in the basic heading and the relative overlap across countries can lead to differences in results from different methods. If all products are priced in all countries and if all products are treated as equally important, then the two major approaches used within the ICP—the Jevons index with GEKS and the CPD methods—will lead to identical PPPs, thereby eliminating the need to choose between methods. In addition, it is important to understand the concept of the basic heading and the concepts of representativity and importance before identifying an appropriate index number method.

## Basic Headings

Within the pyramid approach to price comparisons, as illustrated in figure 1.1 in chapter 1, price comparisons begin with the identification of products in the regional list of items for which price data are collected through price surveys.[6] These products are grouped into different basic headings. As a working definition, the *basic heading* is the lowest level of aggregation for which expenditure share weight information is available. An immediate implication is that in the ICP no quantity data are available for the products priced in the surveys.[7] Thus price comparisons at the basic heading level are derived as aggregates of item-level price ratios, and the basic heading PPPs are similar to the *elementary indexes* gathered as part of the compilation of the consumer price index (CPI).[8] An additional consideration is that basic headings consist of groups of similar products defined within a general product classification. Similarity of products within the basic heading is supposed to ensure the similarity in the magnitudes of price relatives for products included in the basic heading.[9] However, in the ICP the composition of basic headings is necessarily dictated by the detailed classification used in the national accounts and the availability of national account expenditure share weights for a desired grouping of products. Therefore, it is possible that basic headings used in ICP price comparisons may not always include products that exhibit similar price movements. For example, the clothing and footwear aggregate consists of the following five basic headings: (1) clothing materials; (2) garments (men's, women's, and children's); (3) cleaning, repair, and hire of clothing; (4) shoes and other footwear; and (5) repair and hire of footwear. In this example, it is clear that similar products are grouped together. However, there is considerable heterogeneity within groups. The garments group includes men's, women's, and children's clothing, and, moreover, there would be heterogeneity within men's garments, which include shirts, trousers, and suits, as well as undergarments. It is difficult to determine whether all the products within the garments group are likely to exhibit similar price relatives or price movements.

From the perspective of computing a PPP for a given basic heading, the most ideal situation is one in which item-specific weights in the form of quantities or expenditure share weights are available. In the absence of such information, several characteristics of the products included in the basic heading are taken into account in computing basic heading PPPs. It is useful to review these characteristics because they play a critical role in the choice of an appropriate formula for the computation of basic heading PPPs.

## Product Characteristics

In the 2005 ICP, a new approach based on *structured product descriptions* (SPDs) was introduced. The SPDs provide a structured, systematic way in which to describe the price-determining characteristics for products that households purchase. In that way, the prices collected in different countries refer to the same products, ensuring *comparability*. The SPDs were used to prepare the product specifications given to the price collectors. For example, in the Asia-Pacific region 20 different varieties of rice were defined by the rice SPD for pricing purposes (ADB 2007). However, it is widely recognized that a product comparable in its characteristics across countries may not be *representative* of consumption in all countries in which the product is priced. In general, there is a trade-off between comparability and representativity in identifying products for inclusion in the basic headings and price surveys.

### Representativity

The concept of *representativity* was developed as part of the Eurostat–Organisation for Economic Co-operation and Development (OECD) PPP Programme as way of accounting for the possibility

that some of the items in the product lists may not be representative of consumption in some of the participating countries.[10] It makes intuitive sense that the prices of representative products should have a greater influence on the PPPs for the basic heading and that the converse should be true for unrepresentative products. The actual implementation requires a formal process for identifying representative products.

The representativity of a product is determined on the basis of two main considerations:

1. A particular item is representative in a country for a particular basic heading if the item has a significant market share as reflected by the expenditure share or by the volume of sales associated with the item. This notion of representativity is consistent with the approach one would take if quantity or expenditure data were available at the item level. In the presence of such information, the aggregation of price ratios would be weighted proportional to the expenditure shares. However, because expenditure share data are not available at the item level, labeling a product as representative has to be left to the price statistician or local experts.

2. Representative products have lower price levels than unrepresentative products, and not accounting for such differences in aggregating price data could result in biased PPPs.[11] If products could be labeled as representative or unrepresentative in each of the countries, such information could be incorporated into the aggregation process. However, it is difficult to decide a priori whether a particular product is representative on the basis of whether the price relative of the product in two countries is typical of the products in the basic heading. A possible approach could be based on the fact that a cheap product would be purchased in large quantities and therefore would be popular.[12]

An important reason for distinguishing between representative and unrepresentative products is that the relative prices of these products would be different. Products purchased in large quantities are associated with lower prices relative to other products in the basic heading, and unrepresentative products may have higher relative prices. On the basis of this finding, a product may be considered unrepresentative if its relative price is high and therefore may not be representative of the relative prices of products that are representative and included in the basic heading. This notion of a representative commodity underpins the use of the Country Product Representative Dummy (CPRD) model and the starred-GEKS (Gini-Éltetö-Köves-Szulc) methods used in international comparisons.

Although the conceptual basis for identifying representative products within a basic heading is sound and appealing, in practice the national and regional ICP coordinators have no real objective measures to use in deciding whether a particular product is representative. In particular, the notion of whether it is representative on the basis of relative prices is a difficult concept to implement. These issues underscored the failure of the national and regional coordinators to meaningfully identify representative products in the 2005 ICP. In the Asia and Africa regions, much confusion surrounded the identification of representative products, to the extent that including such information induced biases in the estimated PPPs at the basic heading level. Thus the information collected on the representativeness of products within basic headings was discarded at the stage of computing PPPs for basic headings. The Eurostat-OECD and Commonwealth of Independent States (CIS) regions are the only ICP regions that used the information on representativeness.

## Importance of Products

In view of the difficulties encountered during the 2005 ICP, it was decided that the concept of representativeness would be replaced by an indication of whether a product is important among the

list of products within a basic heading. Aggregation methods leading to basic heading PPPs should explicitly account for the importance of a particular item in a particular country, with unimportant products accorded a smaller weight. The notion of importance is supposed to reflect whether either the sales or the expenditure share is significant enough to warrant the label that the product is important. National statisticians have a reasonable understanding of importance as reflected by sales or shares. The notion of importance of the product is not related in any way to the relative price of the product, and therefore it is a weak alternative to the use of actual quantity data along with price data in the computation of PPPs at the basic heading level. It was generally agreed that in the 2011 ICP an important product will be accorded a weight of 3, or three times the weight attached to the remaining products, which are considered unimportant.[13]

## Price Data: National Average Prices

The price concept relevant for the ICP is the national annual average price of the items priced in each of the countries. In the 2005 ICP, national annual average prices were computed as the arithmetic averages of a large number of price quotations for each of the items in the product lists.[14] The average price should ideally cover the whole country, representing the rural and urban regions, as well as different geographic regions for larger countries. The countries supply the regional coordinator of the ICP with the following data:

- National annual average price
- Number of price quotations used in computation of the average price
- Standard deviation of the price quotations used in computing the national annual average price.

The reliability of a given national annual average price can be computed as the standard error associated with that price.[15]

The PPPs computed at the basic heading level must make use of all the information available associated with the price data (national annual average prices) from the countries participating in the comparisons. In particular, it is statistically desirable to accord lower weights to prices that have higher standard errors. Though this is a desirable approach, the standard errors of the national annual average prices have not so far been incorporated into the PPP computations.

## Basic Data for Computing Basic Heading PPPs

This section establishes the notation used throughout this chapter and describes the nature of price data used in the compilation of PPPs at the basic heading level. For the purpose of this exposition, without loss of generality, the focus is on a single region and a selected basic heading. Let $N$ represent the number of commodities included in a given basic heading and $C$ represent the number of countries included in a given region.[16] Let $p_i^c$ denote the price of the $i$-th commodity in country $c$ ($i = 1, 2, \ldots, N; c = 1, 2, \ldots, C$) where the price is assumed to be strictly positive. The problem of determining the number of items and the actual items for inclusion in the basic heading is dealt with in chapter 7 in this volume on the survey framework for the ICP. In practice, the following three scenarios are possible:

1. All the items are priced in all countries, leading to a complete tableau or matrix of prices.[17]
2. Not all items are priced in all the countries, leading to an incomplete tableau of prices.
3. Some items may be priced in only one of the countries in the region.

Table 4.1 presents two examples of incomplete tableau.

**TABLE 4.1**  Two Examples of an Incomplete Tableau for the Basic Heading Rice

*national currency units*

| | Prices | | | | | | | |
| | Tableau I | | | | Tableau II | | | |
| | Country | | | | Country | | | |
| Items in basic heading | A | B | C | D | A | B | C | D |
|---|---|---|---|---|---|---|---|---|
| Long grain | 10 | 40 | 50 | 100 | 10 | 40 | — | — |
| Medium grain | 12 | 16 | — | — | — | — | 25 | 55 |
| Short grain | — | 15 | 30 | — | — | — | 15 | 40 |
| Low quality | 4 | — | — | — | 4 | — | — | — |
| Imported | 25 | — | — | 100 | 25 | 80 | — | — |

In tableau I, price data are missing in some cells, but it would be possible to compute PPPs for the four countries in the comparison because the data exhibit overlaps between countries. Note that the fourth item, low-quality rice, is priced only in country A and therefore cannot be used in the PPP computation. Thus at the basic heading level, in order to influence the PPP computations, an item must be priced in at least two countries. Tableau II is an interesting case in which the long grain and imported rice items are priced only in countries A and B. By contrast, the medium grain and short grain items are priced only in countries C and D. Although it is possible to compare prices across A and B and between C and D, it is not possible make price comparisons between countries A and C or A and D. Similarly, B cannot be compared with either C or D. In this case, the price tableau can be reduced to two blocks, one for countries A and B and another for countries C and D, with nothing in common between the blocks.

Three important conclusions can be drawn from the examples in table 4.1:

1. When the price tableau is incomplete, it is important that the price data collected be connected and price comparisons between all the countries involved be made if and only if the price tableau is connected or irreducible. The price tableau is said to be *connected* if the price data are such that it is not possible to place the countries in two groups in which no item priced by any country in one group is priced by any other country in the second group. In tableau II, it is possible to place countries into two groups (A and B, C and D) in which no rice item priced in the first group is priced by any country in the second group and vice versa. In such cases, the process of price comparisons breaks down.[18]
2. If a product is priced in only one country, its price will have no influence on the PPPs computed.
3. When a price tableau is incomplete, the quality of price comparisons depends on the strength of interconnections and overlaps in the priced items across different countries. If the overlaps are strong, then one can make reliable price comparisons across different countries. A corollary of this observation is that countries within a particular region should strive to price as many items in the basic heading as possible.[19] (Chapter 7 on the survey framework for the ICP provides guidelines on the minimum numbers of items to be priced within each basic heading.)

# 4.2 Aggregation Methods for Computing PPPs at the Basic Heading Level: Analytical Requirements

To identify the properties expected of aggregation methods for computing PPPs at the basic heading level, it is useful to recall a working definition of the PPP and then apply it to the special case of a single product. The purchasing power parity between the currencies of countries A and B may be defined as the number of currency units of country A that have the same purchasing power as one unit of the currency of country B, defined with respect to a designated product or set of products such as a basic heading.

## PPPs for Individual Products

Suppose $p_i^j$ and $p_i^k$ are, respectively, the prices of product $i$ in countries $j$ and $k$. The PPP for country $k$ with respect to country $j$ is then given by

$$(4.1) \qquad PPP_{jk} = \frac{p_i^k}{p_i^j}.$$

Obviously, the PPP defined in (4.1) depends on the particular product selected. For a selected commodity $i$, the following transitivity property can be observed. For any three countries $j$, $k$, and $m$, it is easy to see that

$$(4.2) \qquad PPP_{jk} = \frac{p_i^k}{p_i^j} = \frac{p_i^k}{p_i^m} \cdot \frac{p_i^m}{p_i^j} = PPP_{jm} \cdot PPP_{mk}.$$

Equation (4.2) shows that the PPP between countries $j$ and $k$ is equal to the indirect PPP comparison derived through a third country, $m$. The equation guarantees the level of internal consistency required for international comparisons. This property is known as transitivity, and equation (4.2) shows that when PPPs are based on the prices of a single product, transitivity is automatically satisfied.

## Transitivity for PPPs in Multilateral and Multiproduct Comparisons

Multilateral PPPs, represented by the matrix of PPP comparisons between all pairs of countries based on price data on more than one item (i.e., in the presence of multiple products), is said to be transitive if for any three countries in the group, such as $j$, $k$, and $m$, *the direct* PPP for country $k$ with respect to country $j$ is equal to the *indirect* PPP derived through the use of the third country, $m$:

$$(4.3) \qquad PPP_{jk} = PPP_{jm} \cdot PPP_{mk} = \frac{PPP_{mk}}{PPP_{mj}}.$$

The last part of equation (4.3) requires the assumption that $PPP_{jm}$ is the reciprocal of $PPP_{mj}$.

Because the PPPs based on a single product defined in (4.1) are automatically transitive and because the PPPs based on price data for multiple items in a basic heading would require some sort of averaging of item-level PPPs, it is necessary to consider only those methods that retain the

property of transitivity, as displayed in (4.3). Unless stated otherwise, all the procedures considered in the reminder of this section satisfy transitivity.

### Base Invariance of PPPs for Multilateral Comparisons

In addition to the transitivity requirement stated in (4.3), it is important that all countries participating in the price comparisons are treated symmetrically and that no country is accorded a special status. This condition is particularly relevant because it is possible to generate transitive PPPs using a "star" method, in which a single country such as the United States is at the center and all countries are compared only through the star country (see figure 1.2 in chapter 1 for an illustration). Even though comparisons based on a selected star are transitive, the comparisons are sensitive to the star country selected. For example, the relative comparisons and PPPs between two countries, say $j$ and $k$, would be different when two different countries, say the United States and Germany, are used as the star countries. Again, unless stated otherwise, all the methods discussed in the ensuing sections produce PPPs that are base-invariant.

Traditionally, there have been two main approaches to aggregation at the basic heading level. The first approach is based on the Jevons index, which is used in the computation of elementary price index numbers, and the GEKS method (Gini 1924, 1931; Éltetö-Köves 1964; Szulc 1964). An alternative approach, developed originally by Summers (1973), makes use of a regression model known as the Country Product Dummy as a way of filling or imputing missing price data. However, it was also used as a method of aggregation below the basic heading level in the earlier rounds of the ICP conducted by Kravis and his associates (Kravis, Heston, and Summers 1982). In recent years, the model has received attention through the work of Rao (1990, 2004, 2005, 2009), Sergeev (2002, 2003), Diewert (2004b, 2005, 2010b), Rao and Timmer (2003), and Hill and Timmer (2006). Although no specific references are given, the following material draws heavily on chapter 11 in the *ICP 2005 Methodological Handbook* (World Bank 2007) and on Diewert (2010b). The next two sections of this chapter describe the two major approaches to the compilation of PPPs at the basic heading level.

## 4.3  Jevons Index and GEKS Method for PPPs at the Basic Heading Level

This section describes the methodology used by Eurostat since the 1980s, as well as the current Eurostat-OECD methodology adopted for aggregating item-level price data to compute BH-level PPPs (see Roberts 2009 for a detailed description of the Eurostat-OECD PPP Programme). The basic element of the Eurostat approach is the Jevons index, which is the main index number formula used in the computation of elementary price indexes in the compilation of the consumer price index.[20] The Jevons index by itself does not yield transitive comparisons except in the special case in which all countries price all products in the basic heading. This index is suitably transformed using the GEKS approach and so is employed in the computation of basic heading PPPs. As the Eurostat-OECD program collects reliable information on the representativity of different items in the basic heading in different countries, the Jevons-GEKS method is further modified to account for the additional information on representativity. A variation of the Eurostat-OECD approach has been proposed by Sergeev (2003). Thus the following scenarios are considered here:

- All items are priced in all the countries (a complete tableau) with no weights attached to the items reflecting representativity or importance. Under this scenario, the standard Jevons index is used.

- Scenario refers to an incomplete price tableau in which not all items are priced in all countries, but all items are treated with equal weight. Here the Jevons-GEKS index is used in deriving transitive comparisons.
- Scenario refers to the most general case in which the price tableau is incomplete. At the same time, a distinction is made between representative and unrepresentative commodities. Because representative products are marked with an asterisk (*), the method used is denoted as the Jevons-GEKS* index. A variation of this method proposed by Sergeev (2003) is referred to as the Jevons-GEKS*(S) index.

## Jevons Index: Complete Price Tableau without Weights

In the simplest case in which all $N$ items are priced in all countries and are treated as equally important in the absence of any implicit weights, the PPPs at a given basic heading level can be computed using

$$(4.4) \qquad PPP_{jk}^{Jevons} = \prod_{i=1}^{N} \left[ \frac{p_i^k}{p_i^j} \right]^{\frac{1}{N}} \text{ for all } j, k = 1, 2, \ldots, C.$$

The index in (4.4) is a simple geometric mean of all the price relatives for countries $j$ and $k$ for all the commodities in the basic heading. This formula is referred to as the Jevons index in the index number literature (see Diewert 2004a for more details).

It is easy to check that the Jevons index in (4.4) results in PPPs that are transitive and base-invariant. It is also useful to note here that in this case in which all $N$ items are priced in all countries, the CPD method discussed in the next section produced PPPs identical to those based on the Jevons index.

## Jevons-GEKS Index: Incomplete Tableau without Weights

Consider the case in which not all commodities are priced in all countries. Let $N_j$ be the number of commodities, out of $N$, priced in country $j$. In addition, suppose that all the price data are connected so that price comparisons are feasible. Note that any binary comparison between countries $j$ and $k$ can be made on the basis of overlapping price data consisting of the items that are commonly priced. If a commodity is not priced in one of the two countries, that commodity cannot be included in the PPP computation. Let $N_{jk}$ represent the set and number of commodities in the basic heading that are commonly priced in countries $j$ and $k$. Then the PPP for a binary comparison between $j$ and $k$ is given by

$$(4.5) \qquad PPP_{jk}^{Jevons} = \prod_{i \in N_{jk}} \left[ \frac{p_i^k}{p_i^j} \right]^{\frac{1}{N_{jk}}}.$$

The binary PPP for countries $j$ and $k$ based on the commonly priced items given in (4.5) does not satisfy the transitivity property. The GEKS procedure is a technique that generates transitive multilateral indexes (PPPs), which may be denoted by $PPP_{jk}^{EKS}$. Details of the GEKS procedure can be found in Balk (2009) and Rao (2009). The GEKS-based PPPs are given by

$$(4.6) \qquad PPP_{jk}^{Jevons-GEKS} = \prod_{\ell=1}^{C} \left[ PPP_{j\ell}^{Jevons} \cdot PPP_{\ell k}^{Jevons} \right]^{\frac{1}{C}} = \prod_{\ell=1}^{C} \left[ \prod_{i \in N_{j\ell}} \left( \frac{p_i^\ell}{p_i^j} \right)^{\frac{1}{N_{j\ell}}} \cdot \prod_{i \in N_{\ell k}} \left( \frac{p_i^k}{p_i^\ell} \right)^{\frac{1}{N_{\ell k}}} \right]^{\frac{1}{C}}.$$

It is easy to check that the Jevons-GEKS PPPs in (4.6) satisfy transitivity. Rao, Maddison, and Lee (2002) provide an intuitive interpretation of the PPPs based on the GEKS method, which also establishes its base–invariant property. If the price tableau is complete, then it is possible to show that the use of Jevons-GEKS indexes in equation (4.6) will lead to PPPs identical to those in equation (4.4).

## Jevons-GEKS* Index: Incomplete Tableau with Asterisks for Representative Items

Consider the case in which products are labeled as "representative" or "unrepresentative" in different countries. Representative commodities are marked with an asterisk (*). In this case, the Eurostat-OECD comparisons are based on a modified Jevons-GEKS method, which is also known as the Jevons-GEKS* method. The modification is driven by the fact that for any given pair of countries $j$ and $k$ there may be (1) a set of products that is priced in both countries and is considered representative; (2) a set of products priced that is representative in country $j$ but not in country $k$; (3) a set of products priced that is representative in country $k$ but not in country $j$; or (4) a set of products priced that is not representative in either country. This approach can also be used when "representativeness' is replaced by the "importance" of commodities. The following notation is used in the equations that follow:

> Let $N_{jk}$ represent the number of products that are representative in either country $j$ or country $k$ and for which price data are reported in both countries $j$ and $k$. $N_{jk}$ will generally be smaller than the total number of commodities $N$ in the basic heading.
> Let $N_{jk}^R$ represent the set and number of products that are representative in country $j$ and that are also priced in country $k$. They may not all be representative in country $k$.
> Let $N_{kj}^R$ represent the set and number of products that are representative in country $k$ and that are also priced in country $j$. They may not all be representative in country $j$.[21]

The PPP for a binary comparison between $j$ and $k$ based only on representative or starred commodities in country $j$, denoted by $PPP_{jk}^{Jevons(j-*)}$, is then given by

$$(4.7) \qquad PPP_{jk}^{Jevons(j-*)} = \prod_{i \in N_{jk}^R} \left[ \frac{p_i^k}{p_i^j} \right]^{\frac{1}{N_{jk}^R}}.$$

However, an equally meaningful PPP measure can be defined using commodities that are representative in country $k$ and are also priced in $j$, which is denoted by $PPP_{jk}^{Jevons(k-*)}$ and given by

$$(4.8) \qquad PPP_{jk}^{Jevons(k-*)} = \prod_{i \in N_{kj}^R} \left[ \frac{p_i^k}{p_i^j} \right]^{\frac{1}{N_{kj}^R}}.$$

From a statistical or analytical perspective, the two PPP measures given in equations (4.7) and (4.8) are equally desirable because each makes use of the representative products priced in the country that are also priced in the other country. Therefore, an asterisk (*)-based Jevons index of PPP between $j$ and $k$ could be defined using a geometric average of the two PPPs in (4.7) and (4.8). The "representative" or (*)-based Jevons index is denoted by $PPP_{jk}^{Jevons(*)}$, which is given by

$$(4.9) \quad PPP_{jk}^{Jevons(*)} = \left[ PPP_{jk}^{Jevons(j-*)} \cdot PPP_{jk}^{Jevons(k-*)} \right]^{\frac{1}{2}} = \left[ \prod_{i \in N_{jk}^R} \left[ \frac{p_i^k}{p_i^j} \right]^{\frac{1}{N_{jk}^R}} \cdot \prod_{i \in N_{kj}^R} \left[ \frac{p_i^k}{p_i^j} \right]^{\frac{1}{N_{kj}^R}} \right]^{\frac{1}{2}}.$$

The new index in (4.9) is the geometric mean of the indexes in (4.7) and (4.8).[22]

Because $PPP_{jk}^{Jevons(*)}$ makes use of only information on countries $j$ and $k$ from the price tableau, the resulting indexes are not transitive even when the price tableau is complete. Therefore, it is necessary to use the GEKS procedure, which results in a transitive PPPs incorporating "representativity" information at the same time. This is the Jevons-GEKS* index. Basically, the $PPP_{jk}^{Jevons(*)}$'s are used along with the GEKS approach, leading to $PPP_{jk}^{Jevons-GEKS(*)}$ for all $j$ and $k$. These are given by

$$(4.10) \qquad PPP_{jk}^{Jevons-GEKS(*)} = \prod_{\ell=1}^{C} \left[ PPP_{j\ell}^{Jevons(*)} \cdot PPP_{\ell k}^{Jevons(*)} \right]^{\frac{1}{C}}.$$

The $PPP_{jk}^{GEKS(*)}$'s given in (4.10) are transitive and base-invariant.

These were used by the Eurostat-OECD program until the recently proposed modification of the Jevons-GEKS* method stemming from the work of Sergeev (2003). As noted earlier, this modification is known as the Jevons-GEKS*(S) method. The Sergeev (2003) approach explicitly recognizes and provides additional weights to those items that are representative in both countries and also priced in both countries.

## Jevons-GEKS*(S) Index: Incomplete Tableau with Asterisks for Representative Items with Differential Weights

The Jevons-GEKS*(S) method is similar to the GEKS* method and so a similar notation may be used:

Let $N_{jk}^{R}$ represent the set and number of products that are representative in country $j$ and that are also priced in country $k$ but are not representative in $k$.

Let $N_{kj}^{R}$ represent the set and number of products that are representative in country $k$ and that are also priced in country $j$ but are not representative in country $j$.

Let $N_{jk}^{**}$ be the number of commodities that are priced in both countries and are also representative in both countries.

Then the PPP for a binary comparison between $j$ and $k$ based only on representative commodities in country $j$ that are not representative in $k$ is denoted by $PPP_{jk}^{Jevons(j-*)}$ and given by

$$(4.11) \qquad PPP_{jk}^{Jevons(j-*)} = \prod_{i \in N_{jk}^{R}} \left[ \frac{p_i^k}{p_i^j} \right]^{\frac{1}{N_{jk}^{R}}}.$$

However, an equally meaningful PPP measure can be defined using commodities that are representative in country $k$ and that are also priced in $j$ but are not representative in $j$, which is denoted by $PPP_{jk}^{Jevons(j-*)}$ and given by

$$(4.12) \qquad PPP_{jk}^{Jevons(k-*)} = \prod_{i \in N_{kj}^{R}} \left[ \frac{p_i^k}{p_i^j} \right]^{\frac{1}{N_{kj}^{R}}}.$$

The Sergeev (2003) method uses a third index that is based purely on those commodities priced in both countries and representative in both countries. The third index is given by

$$(4.13) \qquad PPP_{jk}^{Jevons(**)} = \prod_{i \in N_{kj}^{**}} \left[ \frac{p_i^k}{p_i^j} \right]^{\frac{1}{N_{kj}^{**}}}.$$

From a statistical or analytical perspective, there is no way of choosing between the three PPP measures given in equations (4.11), (4.12), and (4.13) because each, respectively, makes use of the representative products priced in the country that are also priced in the other country. Therefore, an asterisk (*)–based index of PPP between $j$ and $k$ may be defined using a weighted geometric average of the three PPPs in (4.11), (4.12), and (4.13), in which the weights are proportional to the number of products in different groups. In particular, the Sergeev (2003) method gives double the weight for the index based on representative products in both countries. The resulting modified "representative" product–based PPP, denoted by $PPP_{jk}^{Jevons*(S)}$, is given by

$$(4.14) \qquad PPP_{jk}^{Jevons*(S)} = \left[ PPP_{jk}^{Jevons(**)} \right]^{w_1} \left[ PPP_{jk}^{Jevons(j-*)} \right]^{w_2} \left[ PPP_{jk}^{Jevons*(k-*)} \right]^{w_3}$$

where

$$w_1 = \frac{2 \cdot N_{jk}^{**}}{2 \cdot N_{jk}^{**} + N_{jk}^{R} + N_{kj}^{*}} \text{ and } w_2 = w_3 = 0.5 \cdot \frac{N_{jk}^{R} + N_{kj}^{*}}{2 \cdot N_{jk}^{**} + N_{jk}^{R} + N_{kj}^{*}}.$$

A simple example may be useful in understanding the weighting scheme used here. Suppose that 12 items are commonly priced in countries $j$ and $k$. Let items 1–7 be the products that are representative in $j$ but not in $k$; let products 8–10 be representative in both countries; and let products 11 and 12 be representative in $k$ but not in $j$. Then $N_{jk}^{R} = 7$; $N_{jk}^{**} = 3$; $N_{kj}^{R} = 2$; and, in this case, $w_1 = \frac{2 \cdot 3}{2 \cdot 3 + 7 + 2} = \frac{6}{15} = 0.4$; $w_2 = w_3 = 0.3$. Now to complete the procedure by generating transitive indexes, it is necessary to use the GEKS procedure on all bilateral comparisons of the form, leading to

$$(4.15) \qquad PPP_{jk}^{Jevons-GEKS*(S)} = \prod_{\ell=1}^{C} \left[ PPP_{j\ell}^{Jevons*(S)} \cdot PPP_{\ell k}^{Jevons*(S)} \right]^{\frac{1}{C}}.$$

The resulting indexes are transitive and based on the binary indexes that take into account the representativity of the items priced in different countries.

The Eurostat-OECD method represents a viable approach that makes use of the representativity status of products priced in different countries. A few points of interest are noted here. First, in a given binary comparison the procedures described in section 4.4 use data corresponding to those items that are representative in one country and also priced in the other country. In this process, information can be lost. For example, data on representative price items in country $j$ that are not priced in country $k$ do not enter the computation. Similarly, the prices of products that are considered unrepresentative products in both countries do not influence the binary comparison. Second, a more important point is that the Jevons-GEKS* and Jevons-GEKS*(S) indexes do rely heavily on price comparisons for commodities that are representative in one country but not in the other. Intuitively, such comparisons tend to be distorted because the commodity is representative in one country but not representative in the other. There is no guarantee that these distortions cancel each other.[23] In that case, it is likely that these distortions are in fact accumulating, and that the resulting basic heading parities may be highly distorted. This may not be a major problem when comparisons are made in a region in which all the countries are fairly similar and there is a significant overlap of products. The Eurostat-OECD approach addresses this issue through the use of equi-representativity, which endeavors to equalize the number of representative products in each country. Third, it is difficult to generalize the Jevons-GEKS* index to attach different weights to the representative and unrepresentative items in the basic heading.[24] And, fourth, the Eurostat-OECD approach, which uses the Jevons index,

requires one single price observation for each item from each country. This is not a major restriction because it is common practice to use the national annual average price of the item as input into the basic heading PPP computations. However, when each country provides all the price quotations, the Eurostat-OECD approach cannot make direct use of the price quotations, and these detailed data need to be aggregated into an average before they can be used. The regression-based approach that underpins the Country Product Dummy method solves this problem to a certain degree.

# 4.4 The Country Product Dummy Method for the Computation of Basic Heading PPPs

The Country Product Dummy method was first introduced by Summers (1973). He proposed a simple regression-based method to fill missing price data in an incomplete tableau of prices at the basic heading level. The method was later used in various phases of the ICP conducted by the research team at the University of Pennsylvania. The report by Kravis, Heston, and Summers (1982) on the third phase of the ICP is a detailed account of how the CPD method provided an aggregation method at the basic heading level. However, the Eurostat-OECD program has continued to use the GEKS methods and its variants over the last three decades. And yet it has shown renewed interest in the CPD method because of the recent work of Rao (2004, 2005, 2009) and Diewert (2004b, 2005) and also because of the recent use of the method in PPP computations for poverty (Deaton and Dupriez 2009).

This section describes the CPD method and shows how it can be used in the computation of basic heading PPPs, especially when information is available on the representativity status of items in different countries.

## The Basic CPD Model

Following the notation used in section 4.1, let $p_i^j$ represent the price of item $i$ in country $j$ ($i = 1, 2, \ldots, N; j = 1, 2, \ldots, C$). It is useful to state the CPD model in a form that is directly relevant for international comparisons. Thus the basic statistical model underlying the CPD method can be stated as

$$(4.16) \qquad p_i^j = PPP_j\, P_i\, u_{ij}; \quad j = 1, 2, \ldots, C; i = 1, 2, \ldots, N$$

where $PPP_j$ is the purchasing power parity of the currency of the $j$-th country; $P_i$ is the international average price of the $i$-th commodity; and $u_{ij}$'s are independently and identically distributed random variables. In this chapter, these disturbances are assumed to be lognormally distributed, or that ln $u_{ij}$'s are normally distributed with a zero mean and a constant variance $\sigma^2$. Several features of the CPD model are noteworthy.

First, prices used in the CPD model may be considered a single price observation for each item in each country in which it is priced. The CPD model is general enough to accommodate the case in which several price quotations are available for each commodity in each country—a case considered in Diewert (2004b). When individual price quotations are used, it would be possible to extend the CPD model to incorporate additional characteristics associated with each quotation, including information on the type of outlet and on the rural/urban location for the transaction.

Second, in the ICP only single price observations representing the annual average prices of items in the basic heading are used. If information is available on the standard error associated with the average price, then this information can be incorporated into the model using different variances for different products.

Third, the CPD model in (4.16) is usually referred to as the *law of one price*, reflected by a single average price for a commodity across all the countries and a single measure of price level for each country represented by $PPP_j$.

Finally, the CPD model can be best described as a hedonic regression model in which the characteristics used are the country and the commodity specifications. The CPD model can be written as a standard hedonic model by using logarithmic prices. Taking natural logs on both sides, the model can be written as

$$(4.17) \qquad \ln p_{ij} = \ln PPP_j + \ln p_i + \ln u_{ij}$$
$$= \alpha_j + \gamma_i + v_{ij}$$

where $v_{ij}$ are random disturbance terms that are independently and identically (normally) distributed with a zero mean and variance $\sigma^2$.[25] The CPD model can be seen as a simple fixed-effects model in which country effects provide estimates of purchasing power parities and commodity-specific effects provide estimates of international prices.

The parameter $\alpha_j$ is interpreted as the general price level in country $j$ relative to prices in other countries included in the comparison. It is possible to express $\alpha_j$ relative to a reference country (say country 1). Then $\alpha_j$ represents the purchasing power parity of country $j$, showing the number of country $j$ currency units that have the same purchasing power as one unit of currency of country 1 or the reference country.

Then the PPP for country $j$ is given by

$$(4.18) \qquad PPP_j = \exp(\hat{\alpha}_j).$$

Because the estimated PPP depends on the estimated parameter values, it is possible to derive the standard errors associated with $PPP_j$, which is not possible when the Jevons method discussed in section 4.3 is used.

## The CPD Regression Model

The simple model in (4.17) is called the Country Product Dummy method because it can be expressed as a regression equation in which all the explanatory/regressor variables are essentially dummy variables (one for each country and one for each commodity). The basic model $\ln p_{ij} = \alpha_j + \gamma_i + v_{ij}$ can be written as

$$(4.19) \qquad y_{ij} = \ln p_{ij} = \alpha_1 D_1 + \alpha_2 D_2 + \cdots + \alpha_C D_C + \eta_1 D_1^* + \eta_2 D_2^* + \cdots + \eta_N D_N^* + v_{ij}$$

where $D_j$ ($j = 1, 2, \ldots, C$) and $D_i^*$ ($i = 1, 2, \ldots, N$) are, respectively, country and commodity dummy variables. Equation (4.19) can be written as

$$y_{ij} = x_{ij} \beta + v_{ij}$$

where $x_{ij} = [D_1 \, D_2 \ldots D_C \, D_1^* \, D_2^* \ldots D_N^*]$ and $\beta = [\alpha_1 \, \alpha_2 \, \alpha_C \, \gamma_1 \, \gamma_2 \, \gamma_N]'$ and where the values of the dummy variables are determined at the *ij*-th observation.

The main advantage of the CPD model in (4.19) is that it is possible to use very sophisticated econometric tools to derive interesting results—see Rao (2004) for more details on how the CPD model can be used in dealing with some of the data-related issues to be discussed shortly.[26] In

their recent work, Hajargasht and Rao (2010) derive the Iklé-Dikhanov-Balk (IDB), Rao, and Geary-Khamis methods by using different distributional assumptions and thus are able to compute standard errors or measures of reliability associated with PPPs from these methods.

## CPD Method: Complete Price Tableau and Item-Specific Weights

Now consider the case in which all items in the basic heading are priced in all countries. Here the Jevons index is used—when all countries price the same set of products.

In this case, for the aggregation at the basic heading level where there no weights, the parameters $\alpha_j$ and $\eta_i$ can be estimated using simple unweighted or ordinary least squares by minimizing

$$(4.20) \qquad \sum_{i=1}^{N} \sum_{j=1}^{C} (\ln p_{ij} - \alpha_j - \gamma_i)^2.$$

The first-order conditions for optimization with respect to $\alpha_j$ and $\eta_i$ lead to the following system of $C + N$ equations in as many unknowns:

$$\alpha_j = \frac{1}{N} \sum_{i=1}^{N} \ln p_{nc} - \sum_{i=1}^{N} \gamma_i \ for \ j = 1, 2, \ldots, C \ and$$

$$\gamma_i = \frac{1}{C} \sum_{j=1}^{C} \ln p_{nc} - \sum_{j=1}^{C} \alpha_c \ for \ n = 1, 2, \ldots, N.$$

This system can be solved by imposing a linear restriction on the unknown parameters. For example, if $\alpha_1 = 0$ is the restriction imposed, it can be easily shown that, for each $j = 2, \ldots, C$,

$$(4.21) \qquad \hat{\alpha}_j = \frac{1}{N} \sum_{i=1}^{N} \left[ \ln p_{nj} - \ln p_{n1} \right] \quad or \quad PPP_j = \exp(\hat{\alpha}_j) = \prod_{i=1}^{N} \left[ \frac{p_{ij}}{p_{i1}} \right]^{\frac{1}{N}}.$$

Using the solution in (4.20), comparisons of price levels between countries $j$ and $k$, represented by $PPP_{jk}$, can be derived as

$$(4.22) \qquad PPP_{jk} = \frac{\exp(\hat{\alpha}_k)}{\exp(\hat{\alpha}_j)} = \prod_{i=1}^{N} \left[ \frac{p_{ik}}{p_{ij}} \right]^{\frac{1}{N}}.$$

The $PPP_{jk}$ obtained using the CPD model in (4.22) is identical to the Jevons index presented in equation (4.4) in section 4.3. As in the case of the Jevons index, the index in (4.22) is obviously transitive and base-invariant. The only difference is that because the CPD method uses a regression model, it is possible to derive the standard error associated with each $PPP_{jk}$. It was shown in Rao (2004) that the estimated variance of $PPP_{jk}$ is given by

$$EstVar\,(\hat{\alpha}_j) = \frac{2}{N}\,\hat{\sigma}^2$$

where $\hat{\sigma}^2$ is an unbiased estimator of $\sigma^2$, which is given by

$$(4.23) \qquad \hat{\sigma}^2 = \frac{\displaystyle\sum_{j=1}^{C}\sum_{i=1}^{N} e_{ij}^{2}}{CN - (C + N - 1)}$$

where $e_{ij} = \ln p_{ij} - \hat{\alpha}_j - \hat{\gamma}_j$ is the least squares residual. Using (4.23), the estimated variance of $PPP_j$ with a numeraire country—say country 1—is given by

$$(4.24) \qquad EstVar\,(P\hat{P}P_j) \approx EstVar\,(\hat{\alpha}_j) \cdot (\hat{\alpha}_j)^2\,.$$

Equation (4.24) can then be used in deriving the estimated variance for PPPs with any other countries as the reference country.

## CPD Method: Incomplete Price Tableau

In practice, rarely are all items priced in all countries. In fact, the general experience in international comparison exercises is that only a few items are priced in each of the participating countries, resulting in a rather sparse price tableau. This section examines the nature and role of the CPD method in this context, and it is contrasted with the alternative aggregation method based on variants of the Jevons method used by the Eurostat-OECD program.

The CPD model described in equations (4.17) and (4.19) can be used in conjunction with incomplete data if the price data set is connected as illustrated in table 4.2, which appears later in this chapter.[27] The CPD model and the least squares estimation shown in equations (4.20) and (4.21) can be used with appropriate modifications. Rao (2004) provides algebraic expressions and the necessary proofs to support the following properties of the CPD method relative to the Jevons-GEKS approach described in section 4.3:

- The CPD and the GEKS methods provide identical estimates of PPPs when the price tableau is complete, or equivalently when all countries price the same set of products. There is no real problem of choice. However, the GEKS method, which expresses the $PPP_{jk}$ as a geometric mean of the price relatives for all the items, provides no measure of reliability as in the case of the CPD approach.
- When the price tableau is incomplete, the CPD and the Jevons-GEKS methods provide different numerical values. The CPD method makes use of all the price information in a single step, whereas the Jevons-GEKS method uses the information in two stages. At the first stage, binary comparisons are made using only the prices of items that are priced in a given pair of countries. Obviously, data on the prices of items that are priced in one country but not the other are ignored. Indirect use is made of the price data for the other items through the GEKS extension of the binary Jevons indexes. Once again, no standard errors are available for the PPPs derived using the Jevons-GEKS method.
- When the price tableau is incomplete, one can estimate the CPD model and fill in the missing price data to complete the tableau of prices. The CPD-based PPPs remain unchanged if the CPD model is applied a second time after filling in the missing prices. This is an indication that the price data were fully used under the CPD method. However, the Jevons-GEKS method applied to the incomplete tableau differs from the Jevons method applied to the tableau after the missing price data are imputed using the CPD model. This means that the Jevons-GEKS method can be improved through the use of

CPD fillers, which in turn implies that the GEKS method does not make full use of the price data in the incomplete tableau.

## CPD Method: Information on Representativity and Importance

This section discusses two possible modifications of the simple CPD model depicted in (4.17) and (4.19). The first modification refers to the case in which additional information on the representativity of each item priced in each of the countries is available. In this case, it is possible to extend the CPD regression model to directly account for the possible upward (or downward) bias caused by the prices of *unrepresentative items*. The representativeness concept was used in the 2005 ICP. The second modification concerns the notion of *importance*, whereby each item is classified as important or unimportant in each of the countries. The notion of importance has been adopted for the 2011 round of the ICP. A discussion of extensions of the CPD model in these two cases follows.

### Country Product Representative Dummy Model

Recalling the discussion of *representativity* in section 4.2 of this chapter as well as in chapter 1, the basic idea is that representative products tend to be cheaper than unrepresentative products within a basic heading.[28] This means that in addition to the country and product dimensions used in the CPD model, another dimension of representativity is considered critical and is therefore included.

Accommodating representativity information is quite straightforward through the introduction of another dummy variable representing the additional dimension. In this case, for each price observation a representativeness dummy variable, $R$, is defined so that the value of $R$ equals 0 if the price observation corresponds to a representative item and equals 1 if the particular item is not representative. The basic CPD model in equation (4.19) may be extended to include the representativeness dummy as follows:[29]

$$(4.25) \quad \begin{aligned} y_{ij} = \ln p_{ij} &= \alpha_1 D_1 + \alpha_2 D_2 + \cdots + \alpha_C D_C + \eta_1 D_1^* + \eta_2 D_2^* + \cdots + \eta_N D_N^* + \delta R + v_{ij} \\ &= \sum_{j=1}^{C} \alpha_j D_j + \sum_{i=1}^{N} \eta_i D_i^* + \delta R + v_{ij}. \end{aligned}$$

The parameters in the model can be estimated using the standard least squares method after imposing the numeraire restriction setting one of the $\alpha_j$'s equal to unity. The resulting estimates of PPPs are essentially adjusted for the upward bias caused by those price observations that are not representative. It is expected that in the general case in which unrepresentative items are more expensive the estimate of $\delta$ will be positive.[30]

Using the CPRD model, it is much easier to handle the bias induced by the prices of unrepresentative items through the magnitude of $\delta$. Such an adjustment is not possible in the case of the Jevons-GEKS procedure. In addition, it is clear that considerable price information is lost in the general architecture of the Jevons-GEKS* and Jevons-GEKS*(S) methods described in section 4.3.

Based on the advantages of the CPRD model and because it uses all the information contained in the data set, the CPRD model was recommended for use in the 2005 ICP round for aggregation at the basic heading level. However, the CPRD model could not be used in the Africa and Asia-Pacific regions because of the problems associated with the determination of representative and unrepresentative products. The CPRD method was used in the South America region comparisons. The Eurostat-OECD program used the Jevons-GEKS*(S) method described in section 4.3.

### CPD Model with Importance Weights Attached to Price Observations

Because of the practical problems associated with identifying the representativeness of items priced in different countries, the 2011 ICP will use the notion of *importance* of the product priced. In particular, each item priced will be identified as either *important or unimportant*. The *importance* information will be used by means of attaching weights to price observations. In its meeting in April 2011, the Technical Advisory Group (TAG) of the ICP recommended that a weight of 3 be attached to products identified as important and a weight of 1 be attached to items deemed unimportant.[31]

It is fairly simple to attach weights in the estimation of the parameters of the CPD model. It is equivalent to running *weighted least squares* in the place of simple unweighted least squares. Suppose $w_{ij}$ is the weight attached to the price quotation for the *i*-th commodity in the *j*-th country. Then the weighted least squares simply minimizes

$$(4.26) \qquad \sum_{i=1}^{N} \sum_{j=1}^{C} w_{ij} (\ln p_{ij} - \alpha_j - \gamma_i)^2 =$$

$$\sum_{i=1}^{N} \sum_{j=1}^{C} \left[ w_{ij} (\ln p_{ij} - \alpha_1 D_1 - \alpha_2 D_2 - \cdots - \alpha_C D_C - \eta_1 D_1^* - \eta_2 D_2^* - \cdots - \eta_N D_N^*)^2 \right]$$

with respect to the unknown parameters, which in turn results in estimates of PPPs. The TAG recommendation is to use $w_{ij} = 3$ if the commodity is important or representative and $w_{ij} = 1$ if it is unimportant.

As an extension of this procedure, it may be possible to include information on approximate expenditure or sales shares in the place of arbitrarily stated weights. The main feature of the model in (4.26) is that it is exactly the model one would use if expenditure share weights were available. Rao (2009) discusses the expenditure share–weighted CPD model.

The extensions and variations of the CPD model discussed in this section are limited to the cases applicable to the estimation of basic heading PPPs. It is clear that the CPD method offers major advantages over the alternative based on the Jevons index and variations of the Jevons-GEKS index. Because of the noise in the price data arising from the fact that the data are collected through price surveys, methods such as the CPD based on statistical models are best suited to account for noise and also to provide estimates of reliability in the form of standard errors associated with the PPPs at the basic heading level.

## A Numerical Example

A numerical example designed to illustrate the various methods of aggregation used in deriving PPPs at the basic heading level is presented in table 4.2. In particular, the properties of the Jevons-GEKS, Jevons-GEKS*, Jevons-GEKS*(S), and CPD, CPRD, and weighted CPD methods are illustrated using a simple example.[32]

The example refers to the basic heading materials for maintenance and repair of the dwelling. Four items are listed under this basic heading, and four countries are used in this example. All the items are priced in all the countries, which is useful in illustrating the equivalence of the CPD and Jevons-GEKS methods in special cases. The representativity of different products is indicated by an asterisk (*), shown in the adjacent column, attached to the price quotation.

In this example, all four commodities are priced in all four countries. Therefore, the GEKS parities should be identical to those derived using the CPD model. Only one item is considered representative in country 1, whereas in countries 2 and 3 three products are representative. By contrast, only one item is representative in country 4.

**TABLE 4.2**  Price Data for Aggregation at the Basic Heading Level (Materials for Maintenance and Repair of the Dwelling)

| Description | Qty. | Unit | Country 1 | Rep. | Country 2 | Rep. | Country 3 | Rep. | Country 4 | Rep. |
|---|---|---|---|---|---|---|---|---|---|---|
| Paint, indoor use | 10 | l | 33.88 | | 34.90 | * | 753.36 | * | 89.45 | * |
| Paint, outdoor use | 10 | l | 49.19 | | 71.34 | | 1317.93 | * | 149.05 | |
| Silicone | 300 | g | 4.54 | * | 5.29 | * | 84.74 | | 7.54 | |
| Cement | 25 | kg | 4.57 | | 6.30 | * | 60.07 | * | 5.55 | |

The following aggregation methods are considered in the example:

- Simple unweighted CPD model
- CPRD model with a representativeness dummy
- CPD model with weights for representative items equal to 3 and equal to 1 for unrepresentative items similar to the TAG recommendation
- Jevons-GEKS(*) method in which representativeness is taken into account
- Jevons-GEKS*(S) method, which gives additional weight to commodities representative in both countries.

The computed PPPs using different methods are presented in table 4.3.

Several features of table 4.3 are worth noting. First, the PPPs derived by the GEKS method without (*) are identical to the values obtained using the unweighted CPD. This result is consistent with the analytical result that shows the equivalence of these two methods when the price tableau is complete. However, when the representativity information is incorporated, the methods lead to different results. Otherwise, there are no obvious trends. The weighted CPD method with the 3:1 weighting scheme seems to perform quite well. The CPRD method appears to produce the lowest PPP values, followed by the GEKS* method.

Because the CPD method makes it possible to employ a range of econometric techniques on the price data used and also produces standard errors for the estimated PPPs, the use of the weighted CPD appears to be a particularly good choice.

# 4.5  Linking Regions at the Basic Heading Level in ICP 2005 and Looking Forward to ICP 2011

The main objective of this section to outline the methodology used in linking PPPs at the basic heading level in the 2005 ICP and provide a brief overview of the methodology being considered for the 2011 ICP. The problem of linking PPPs above the basic heading level is considered in depth in chapter 6 by Diewert. In essence, that chapter uses the linked PPPs at the basic heading level as inputs into the aggregation process.

## Linking BH-Level PPPs in ICP 2005

The 2005 ICP embraced a fully regionalized approach to the compilation of PPPs and international comparisons of real gross domestic product and its components. As a part of the regionalized

**TABLE 4.3** Multilateral PPPs for Basic Heading (Materials for Maintenance and Repair of the Dwelling, Four Items) by Different Methods

| Method of calculation | Purchasing power parities | | | |
|---|---|---|---|---|
| | Country 1 | Country 2 | Country 3 | Country 4 |
| Jevons-GEKS* | 1.000 | 1.170 | 18.722 | 2.160 |
| Jevons-GEKS without * | 1.000 | 1.245 | 19.548 | 2.004 |
| Jevons-GEKS*(S) | 1.000 | 1.088 | 18.456 | 2.339 |
| CPRD | 1.000 | 1.164 | 18.280 | 2.004 |
| CPD unweighted (weights 1:1) | 1.000 | 1.245 | 19.548 | 2.004 |
| CPD weighted (weights 3:1) | 1.000 | 1.232 | 18.917 | 2.115 |
| Exchange rate | 1.000 | 2.150 | 8.664 | 1.000 |

*Note:* GEKS*(S) = Gini-Éltetö-Köves-Szulc (Sergeev); CPRD = Country Product Representative Dummy; CPD = Country Product Dummy.

approach, all the participating countries were classified into six regions: Africa, Asia-Pacific, CIS, Eurostat-OECD, South America, and Western Asia. In the 2005 ICP, the Arab Republic of Egypt and the Russian Federation belonged to two different regions at the same time. Egypt participated in the Africa and Western Asia regional comparisons, whereas Russia participated in the Eurostat-OECD and CIS regions. Two major steps were involved in compiling the BH-level PPPs for the 146 participating countries in the global comparison. First, price comparisons were undertaken in each of the regions, coordinated by a regional agency that worked under the guidance of the ICP Global Office.[33] At the conclusion of the activities at the regional level, PPPs at the basic heading level were compiled in each region for the 155 basic headings in the GDP comparisons. Also in each region, one country was selected as the numeraire or reference country, so that the PPPs were expressed in the currency units of the reference country. In order to use these regional sets of PPPs within a global comparison, it was necessary to express the regional basic heading PPPs in the currency units of a global numeraire. A major requirement in this process was *fixity*, which stipulates that the relativities between purchasing powers of currencies of countries within a region must remain unaltered in the process of conversion to a global numeraire. For a more complete description of the general approach used in making global comparisons, see chapter 1 of this volume.

The methodology used in the 2005 ICP to convert the regional BH-level PPPs maintaining fixity is described in this section. The essential steps involved in the process are as follows:

- PPPs were compiled for the currencies of countries within each region using a regional numeraire currency for all 155 basic headings in the GDP.
- A set of 18 Ring countries[34] was identified: six countries from Africa, four from the Asia-Pacific, and two each from the Eurostat-OECD and Western Asia regions. The CIS region was linked using Russia as the bridge country.
- All the Ring countries conducted additional surveys to collect prices of items in the *Ring product list*. The Ring list was developed by the Global Office after examining the regional product lists. This process was used only for household consumption. For all the other aggregates, regional comparisons were based on a global list of items.

- The price data collected by the 18 Ring countries were used in compiling *linking factors*, which were in turn used to convert the regional numeraire currency units into the global numeraire currency. The U.S. dollar was used as the global numeraire.

The methodology used for linking both at the basic heading level and at higher levels of aggregation was developed by Diewert and is described in detail in Diewert (2008, 2010a). The aggregation methodology used in the computation of linking factors is described in the rest of this section.

### Notation

Suppose there are $R$ regions in the comparison, with $C(r)$, for $r = 1, 2, \ldots, R$, countries in region $r$. In the 2005 ICP, $R$ equaled 6, and table 1.3 of chapter 1 in this volume shows the distribution of countries by region. Let $L(r)$ ($r = 1, 2, \ldots, R$) be the number of linking countries for a total of $L = \sum_{r=1}^{R} L(r)$. In the 2005 ICP, $L$ equaled 18.

### The Method

Let $PPP_{rcn}$ represent the PPP for the $n$-th basic heading in country $c$ belonging to region $r$. Without loss of generality, let the first country in each region be the numeraire country. The following steps are used in the computation of linking factors for the $n$-th basic heading:

> *Step 1*. Consider all the Ring countries in region $r$. Let $p_{ircn}^{L}$ represent the price of the $i$-th item priced in linking country $c$ in region $r$. These are expressed in the national currency units of country $c$.
>
> *Step 2*. Convert all the prices of Ring list items in the $n$-th basic heading in linking country $c$ in region $r$ into the numeraire or reference country currency units using

$$(4.27) \qquad\qquad p_{ircn}^{L*} = \frac{p_{ircn}^{L}}{PPP_{rcn}}.$$

> This means that the price in the linking country is converted using $PPP_{rcn}$. For example, if the PPP for the rice basic heading in Sri Lanka in the Asia-Pacific region is 5.85 to the Hong Kong dollar, which was the numeraire currency in the Asia-Pacific region, then the prices of Ring products belonging in the rice basic heading collected from Sri Lanka are all divided by 5.85, thereby expressing Ring product prices in Hong Kong dollars. At the end of this step, the prices of all the Ring list items priced by Ring countries in the Asia-Pacific region are expressed in Hong Kong dollars.
>
> *Step 3*. Use the converted Ring list prices for all the Ring countries in a CPD regression model to derive PPPs for the numeraire currencies in different regions. The CPD model used can be written as

$$(4.28) \qquad\qquad \ln p_{ircn}^{L*} = \alpha_{rn} + \beta_n + u_{ircn}$$

> where $\exp(\alpha_{rn})$ represents the purchasing power parity of the currency of region $r$ in terms of currency units of the global numeraire or reference currency (say the U.S. dollar) for the $n$-th basic heading.
>
> *Step 4*. Apply the CPD regression in (4.28) in step 3 for all 155 basic headings.

*Step 5.* Convert the basic heading PPPs within each region expressed in the reference currency of the region (e.g., Hong Kong dollars in the Asia-Pacific) into the global numeraire using the PPPs derived in step 4. At the end of this step, the linked PPPs at the basic heading level are given by

$$(4.29) \qquad\qquad PPP^{*}_{rcn} = PPP_{rcn} \cdot \exp(\alpha_{rcn}).$$

For example, if the basic heading PPP for rice in Sri Lanka in the Asia-Pacific comparison is 5.85 Sri Lankan rupees per HK$1 and if the linking factor computed using Ring prices leads to, say, HK$6 per US$1, then the PPP for the rice basic heading for Sri Lanka in the global reference currency, the U.S. dollar, is SL Rs 35.10 per U.S. dollar.[35]

Sergeev (2009) raises an interesting issue that grows out of the different number of Ring or linking countries from different regions—that is, $L(r)$ varies with $r$. In the 2005 ICP, six countries from Africa and four countries from Asia were compared with four countries from the OECD.[36] The main point is that within the CPD framework it can be shown that the international average prices are averages of prices from the linking countries. Therefore, regions with a greater number of linking countries may appear to exert a larger influence.[37] However, the rationale for including more than one country in a region in the Ring list is that for those regions that are large and those that exhibit a large variation in prices it is necessary to use more price data drawn from a diverse set of countries representing the region. There are, then, two possible solutions to the issue raised by Sergeev (2009):

1. Express the uncertainty attached to prices from a region through a larger variance for the corresponding disturbance term in the CPD model and then apply the generalized least squares method, which accords less weight to those observations with larger variance. This eliminates the problem discussed by Sergeev. However, it is difficult to know the extent of variability in prices across countries within a region.
2. Alternatively, a suggestion made by Sergeev in his paper may be implemented. Instead of using country-specific prices for each of the linking countries in a region, simply take the geometric mean of the prices after converting them into the currency of the numeraire country. In this case, one can simply use

$$(4.30) \qquad\qquad p^{L^{*}}_{irn} = \prod_{c=1}^{C} \left[ \frac{p^{L^{*}}_{ircn}}{\alpha_{r1n}} \right]^{\frac{1}{L(r)}}.$$

In the next step, use these geometric means as inputs into the CPD model to generate the regional linking factors described in equation (4.28). Use of the geometric mean ensures that each region is represented by a single vector of prices in the CPD regression model, thereby guaranteeing that no region exerts more than a proportionate influence on the BH-level parities that are used in deriving linking factors at higher levels of aggregation.

Because the different regions exhibit different levels of variability and because the use of geometric average prices from Ring countries entails a loss of information on prices, it is appropriate to continue with the current practice of using different numbers of Ring countries in the basic heading computations.

Finally, an additional comment is in order here on the possible use of representativity and importance information in the derivation of the linking factors. Because the CPD regression in equation (4.28) uses price observations from a number of countries within each region and because

each country may have a different representativity status for a product in the Ring list, it would be possible to make use of such information in deriving the regional linking factors for different basic headings. However, use of representativity information would not be possible if Sergeev's suggestion for using the regional average price in (4.30) is implemented (Sergeev 2009). It is difficult to attach any meaning for representativity for a price that is the average over a number of different countries.

## A New Approach Based on Core Products in ICP 2011

An evaluation of the Ring country approach used in the 2005 ICP raised some issues: the choice of Ring countries from each of the regions and the likely influence of a less than ideal choice of reference countries on the linking factors that have a direct impact on the global PPPs at the GDP level and the resulting real income comparisons. Construction of the Ring product lists and the pricing of these products in a limited set of Ring countries from each region were also considered to be less than satisfactory.

In an effort to improve the 2005 methodology, it was decided to link the 2011 regional comparisons through price data collected for *core products*. The core product list is supposed to represent the products used in both developed and developing countries. A list of more than 600 core products was prepared by the Global Office. The core products are expected to strengthen the link between regional and global comparisons through the following steps:

- Include the core products in the product lists of all the regions.
- Encourage the regions to price as many core products as possible in their regional price surveys.
- Have regions use the price data on core products, as well as the region-specific product lists in the computation of PPPs for different basic headings.
- Ensure that linking factors to link regional basic heading PPPs use the prices of core products collected by all the participating countries in all the regions and not just the prices collected for a selected set of Ring countries.
- Base the CPD regression in (4.28) on prices of all the core products in all the countries participating in the 2011 round of the ICP. The number of countries is expected to be about 180. Use of price data from all countries will conceivably produce a more robust and reliable set of linking factors that are likely to improve the quality of the global comparisons.

Exploratory empirical analyses using core product prices collected in the first two quarters of the 2011 ICP will be conducted to examine the nature and reliability of the linking factors resulting from the new approach. Meanwhile, research is continuing into alternative ways of using core product prices. Hill (2011) is proposing methods to use the core product prices in computing PPPs at and above the basic heading levels, imposing within-region fixity in the ICP.

## 4.6  Conclusion

The Jevons-GEKS and the CPD are the two main aggregation methods used to compile PPPs at the basic heading level. When all items are priced in all countries and if all items are considered to be equally important, then these two methods lead to the same numerical values for the estimated

PPPs. However, the CPD method has the advantage of providing standard errors that can be used as measures of reliability. In the case in which some products are representative or important, then three variants of the GEKS method and two variants of the CPD method are available. On a conceptual level, the CPD with weights reflecting the importance of the product is superior to the use of the CPRD, which relies on a systematic bias induced by unrepresentative products. As for the Jevons-GEKS and its variants, in general methods based on Jevons-GEKS appear to discard some price data, which is not the case when the CPD-based methods are used. The Jevons-GEKS*(S) method, thanks to Sergeev (2003), appears to perform well among the GEKS-based methods.

Looking forward to the 2011 ICP, the Jevons-GEKS and the CPD-based methods should be reexamined in light of the decision to use a list of "core products" that will be priced in all countries in all regions. The use of core products is designed to eliminate the reliance on a few selected Ring countries for the purpose of linking regions. A number of methods are currently being developed and discussed at various meetings of the Technical Advisory Group. The CPD method is ideally placed for the current approach of using core product prices collected from all the countries in the global comparisons. The Jevons-GEKS method would be of limited applicability in this case because the method can be used only if average prices representing each of the regions are available. However, use of average prices implies a loss of information. Although the search is continuing to identify suitable aggregation methods to link regional basic heading PPPs using the core product price data, in the interim the CPD method is currently best suited to make use of all the core product prices collected from all countries.

NOTES

1. The author wishes to acknowledge comments from W. Erwin Diewert, Robert Hill, Sergey Sergeev, and Frederic A. Vogel on the material covered in this chapter. Additional thanks are due to Sergeev for providing data and results for the numerical example included in this chapter.

2. PPPs compiled at the regional level are based exclusively on data for countries in the region. The PPPs are expected to satisfy the usual properties of transitivity and base invariance only for those countries. By contrast, global comparisons make use of data from all the countries included in the global comparison, and PPPs at that level satisfy transitivity across countries from different regions.

3. At its meeting in April 2011, the ICP's Technical Advisory Group recommended procedures for linking PPPs across regions at the basic heading level.

4. Representativeness of products is discussed briefly in chapter 1, and it is further elaborated in this chapter.

5. The CPD and the CPRD (Country Product Representative Dummy) methods were recommended for use in the 2005 ICP at the basic heading level.

6. The survey framework for the collection of price data is described in chapter 7 by Frederic A. Vogel.

7. Lack of quantity or expenditure share data makes it impossible to use standard index number formulas such as the Laspeyres, Paasche, Fisher, and Tornqvist.

8. For a description of elementary indexes and the methods used, see the manual for the compilation of the CPI issued by the International Labour Organization et al. (2004).

9. In the case of temporal movements in prices, products within a basic heading are expected to show similar price changes over time.

10. For more details on the Eurostat-OECD treatment of representativity, see the paper by Roberts (2009), which provides a comprehensive account of the methodology used by Eurostat and the OECD.

11. Consider a hypothetical example in which country A prices only representative products and country B prices only unrepresentative products. In this case, the PPP for country B relative to country A would overstate the price level in country B.

12. This may not hold for all products. For example, beef is not representative of meat consumption in India, but it is not relatively more expensive than other meats such as lamb or chicken. In fact, beef and pork are generally cheaper because they are not commonly consumed.

13. This is the recommendation made at the April 2011 meeting of the Technical Advisory Group. These weights will be used in conjunction with the CPD method. Details of the procedure are given in section 4.3.

14. For a discussion of determining the number of quotations and related issues, see chapter 7 on the survey framework for the ICP.

15. If $\sigma$ is the standard deviation associated with a given price (average) and if $n$ is the number of price quotations used, then $\sigma/\sqrt{n}$ is the standard error associated with the given average price.

16. Generally, $N$ (number of items) would vary with the basic heading, and therefore ideally a subscript needs to be added. Similarly, the number of countries varies across different regions, and therefore $C$ needs to have a regional descriptor. Because the focus is on a single region and on a single basic heading, for expositional purposes subscripts for the basic heading and the regions are dropped.

17. This case is the same as the one in which all the countries in the region price the same subset of items, which implies that the remaining items are not priced in any of the countries and can therefore be dropped from the computations.

18. In the case in which the price data are not connected, then even the idea of using a spatial chain of countries will fail because there is no way to connect countries A and B with countries C and D in the second block.

19. Because price data collection is resource-intensive, a balance should be struck between the cost of collection and the need to price as many items as possible to strengthen the price comparisons.

20. A comprehensive discussion of elementary price indexes and the properties of the Jevons index can be found in Diewert (2004a). The publication *Consumer Price Index Manual: Theory and Practice* is an excellent source of material on methods used in the compilation of the consumer price index (International Labour Organization et al. 2004). Chapter 20 of that publication is based on Diewert (2004a).

21. Hill and Hill (2009) use a more complex notation in presenting these methods.

22. Hill and Hill (2009) interpret this formula as a variant of the standard Tornqvist index, which uses the geometric mean of price relatives, with the average budget shares in two periods as the weight.

23. This is easily true in the hypothetical case in which no items are priced that are considered representative in both countries.

24. For example, it would be difficult to accord a weight of 3 to a price observation for a representative or important product and a weight of 1 to unrepresentative products. This difficulty will require a further modification of the Sergeev (2003) suggestion.

25. The model in (4.16) is not identified, and it requires normalization before the parameters of the model can be estimated.

26. Hill and Syed (2010) demonstrate how the CPD model can be used along with individual price quotations from different countries to obtain estimates of rural-urban price differentials and the outlet effects.
27. This is the case where not all items in the basic heading are priced in all countries.
28. The CPD model can equally incorporate the opposite case in which unrepresentative products are cheaper.
29. The model presented here is in a format slightly different from that used in the *ICP 2005 Methodological Handbook* (World Bank 2007).
30. In a recent study, Hill and Syed (2010) present estimates of the representativity coefficients computed using data from a selected set of countries in the 2005 ICP Asia-Pacific region. They found that the coefficients could be negative for some basic headings and positive for others and that no definite conclusions could be drawn. Their findings may reflect in part the difficulties the national price statisticians had in determining whether a particular product was representative in their countries.
31. It was noted that the unweighted use of price observations amounts to giving equal weight to products that are important and that are unimportant.
32. The author is greatly indebted to Sergey Sergeev for providing the data and the computations required for the numerical example.
33. See chapter 2 of the *ICP 2005 Methodological Handbook* for more details on the organizational structure of the ICP (World Bank 2007).
34. This group included one economy, Hong Kong SAR, China.
35. A numerical example of the computation of the linking factors using illustrative data can be found in appendix H of World Bank (2008).
36. Ring countries for the 2005 ICP round were Brazil; Cameroon; Chile; Arab Republic of Egypt; Estonia; Hong Kong SAR, China; Japan; Jordan; Kenya; Malaysia; Oman; the Philippines; Senegal; Slovenia; South Africa; Sri Lanka; the United Kingdom; and Zambia.
37. Equations (4.26)–(4.28) in Sergeev (2009) can be derived from the normal equations associated with the ordinary least squares method.

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