

The Economics of Kenneth J. Arrow

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 - e.g., can all markets clear simultaneously?
(supply = demand in each market)

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- how does *uncertainty* about outcome affect economic behavior

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 - alternative x chosen over y if more individuals prefer x to y than y to x

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Paradox of Voting

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- called *Impossibility Theorem*
 - dictatorship not satisfactory

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 - relatively simple argument

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- each firm described by its *technology* (relationship of inputs to outputs)
- each firm chooses *production plan*, choice of inputs and outputs (feasible for its technology) to maximize *profit* (revenue from selling outputs minus cost of inputs, given prices)

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budget = income from endowment + shares of firms' profits

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- many models in applications are special cases of Arrow-Debreu

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 - but patent holder is monopolist, so dissemination suboptimal

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- foundation of endogenous growth literature

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- state of world: complete description of all relevant eventualities
 - weather, natural disasters, new discoveries
- can index goods not only by physical qualities and by date of delivery, but by state of world
 - ensures that competitive equilibrium will be Pareto optimal even in world with uncertain states
 - but huge number of contingent goods

G = number of goods tomorrow
 S = number of states
 $G \times S$ = number of contingent goods
- can introduce *security* for each state θ (Arrow security)
 - pays off just in that state θ
 - S security markets today
 - G “spot” markets tomorrow

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A. *Securities in General Equilibrium*

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B. Measures of Risk Aversion

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- coefficient of relative risk aversion =
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