Linking Large Currency Swings to Fundamentals’ Shocks

Phornchanok Cumperayot
Chulalongkorn University

Casper G. de Vries*
Erasmus University Rotterdam and Chapman University

October 17, 2017

Abstract

We test if large swings in currency prices are driven by economic fundamentals. Theoretically, we apply extreme value theory (EVT) to a monetary exchange rate model to investigate the contribution of the heavy-tailed macroeconomic fundamentals to the tail behavior of FX returns. Empirically, using the data of 34 countries from three continents, we provide evidence that both exchange rate returns and fundamentals are heavy tailed, and the variables are asymptotically dependent. The strongest tail links are between Asian and Latin American currency depreciations and fundamentals. The main drivers are monetary and financial variables. Real income shocks appear disconnected.

*Corresponding author’s address: Erasmus University Rotterdam H9, P.O. Box 1738, Rotterdam 3000 DR, The Netherlands. E-mail address: cdevries@ese.eur.nl; E-mail address of Phornchanok Cumperayot: phornchanok.c@chula.ac.th. We like to thank seminar participants at Texas A&M, Nelson Mark and Eric van Wincoop for their perceptive comments and helpful suggestions on an earlier draft.
1 Introduction

Starting from a simple monetary exchange rate model in Engel and West (2005), we apply extreme value theory (EVT) to demonstrate the association between the heavy tails of the foreign exchange (FX) returns and economic fundamentals. It is well known that large swings in currency prices induce the distribution of FX returns with heavy tails, to the extent that the probability of a large currency movement is of a higher order of magnitude than the normal distribution would indicate.¹ However, due to the low frequency nature of the macroeconomic fundamentals, few are aware that economic fundamentals, such as money growth and inflation, also exhibit heavy tails in the low frequency domain. In this article, we examine whether the well-established heavy tail feature of exchange rate returns can be attributed to the tail behavior of the macroeconomic fundamentals’ distribution. Our research is inspired by Engel and West (2005) and Engel, Mark and West (2007) that rejuvenated the monetary approach exchange rate models, and hence the link between exchange rates and macroeconomic fundamentals.

The literature on exchange rate modeling is burdened by the low explanatory power of macroeconomic fundamentals in specifications that rely on regression analysis. Forecasts based on such models have not fared well either. Short term no-change forecasts often produce a lower mean squared error, see Meese and Rogoff (1983) and Cheung, Chinn and Pascual (2005). Evidence in favor of the linkage between the nominal exchange rate and macroeconomic fundamentals is rather weak and often not robust, see Neely and Sarno (2002) and Sarno (2005). The intriguing exchange rate papers by Engel and West (2005) and Engel, Mark and West (2007), however, demonstrate that this poor performance of the fundamentals based regressions, known as the exchange rate disconnect puzzle, is due to the non-stationarity of the drivers and the high discount

¹This fact has long been documented by using specific distributions as in Westerfield (1977), Boothe and Glassman (1987) and Akgtayar, Booth and Seifert (1988); or by zooming in on the tails as in Koedijk, Schafgans and de Vries (1990), Koedijk, Stork and de Vries (1992), Koedijk and Kool (1994) and Susmel (2001).
factor. Under these conditions, observed macroeconomic fundamentals can only weakly forecast exchange rate returns, even if the model is correct. The non-stationarity has been recognized for some time. Evidence on a high discount factor is reported in Sarno and Sojli (2009).

Engel and West (2005) indicate that a lack of power to forecast does not necessarily imply the rejection of the exchange rate models, hence the link between exchange rates and macroeconomic fundamentals. Consequently, there is a growing number of studies investigating the relationship between exchange rates and the fundamentals implied by the rational expectations present-value models of exchange rates. Using long run data, Balke, Ma and Wohar (2013) find that unobserved money demand shifts, along with observed monetary fundamentals, are important contributors to the movements in the exchange rate. Djeutem and Kasa (2013) show that the use of robust forecasts by agents, based on a monetary model of exchange rates, can explain observed exchange rate volatility. If exchange rates are driven by expected future fundamentals, as the present-value models suggest, then current exchange rates contain information regarding future fundamentals. Sarno and Schmeling (2014), indeed, show that in a large cross section of long-run data exchange rates have strong and significant predictive power for nominal fundamentals.

It is a stylized fact that exchange rates are excessively volatile compared to macroeconomic fundamentals. An interesting question is what fraction of exchange rate movements can be accounted for by economic fundamentals. Engel, Mark and West (2007), for example, compare the volatility of exchange rate returns with the volatility of a constructed measure of the discounted expected future fundamentals. They find that the ratio of the two volatilities hovers between 30% and 50%. In this article, we exploit the present-value exchange rate model of Engel and West (2005) to establish the linkage between the larger shocks that drive the observed fundamentals and the large movements of the exchange rate. To uncover the dependency between the exchange rate returns and the fundamentals in the tail areas of their distributions, we rely on multivariate extreme value techniques from statistics and estimate the asymptotic dependence between the
variables.\(^2\) It is an informative method to uncover the contribution of economic fundamentals to the likelihood of extreme exchange rate movements. The estimated limit conditional probability is, moreover, a good approximation for finite quantiles in the tail area of the joint distribution. It helps set the lower bound for the probability of joint extremes in the tail area.

Shocks that drive the fundamentals in standard theoretical exchange rate models also drive the exchange rate. In this article, we first explain why the distribution of some macroeconomic fundamentals can have heavy tails using a standard monetary macroeconomic model combined with parameter uncertainty. Subsequently, we show that the properties of the distribution of the fundamentals determine the properties of the distribution of the exchange rate returns, apart from the influence of exogenous noise. Furthermore, if the distributions of some macroeconomic fundamentals have heavy tails, given the linearity of the exchange rate model, the FX returns and the macroeconomic fundamentals should exhibit asymptotic dependence. This is similar to the volatility point made by Engel, Mark and West (2007): "Both sides should display comparable levels of volatility." We investigate whether this similarity also applies to the largest movements on both sides of the equation. In contrast, if for example, the shocks of the fundamentals follow a multivariate normal distribution, even if they are correlated with the exchange rate returns, all dependency vanishes in the tail area. In this case, the heavy tail feature of the exchange rate returns could solely derive from exogenous noise.

To our knowledge, we are the first to apply EVT to study the heavy-tailed macroeconomic fundamentals and their association with large exchange rate movements. Empirically, we use monthly observations from 34 countries over the period of February 1974 to May 2016 to test the theoretical implications, for left and right tails separately. We do indeed find evidence that economic fundamentals exhibit heavy tails, and financial and monetary variable shocks appear

\(^2\)The variables are said to be asymptotically dependent if the probability of a large realization of one variable, given a large realization of the other random variable, is non-zero, even in the limit.
strongly connected with the larger movements of the exchange rates on the depreciation side, but real income growth is not. This accords with the theoretical model as well as with the findings in Balke, Ma and Wohar (2013) and Sarno and Schmeling (2014). The intensity of the interdependency in the tail area varies across regions. The intensity is generally higher for Latin American and Asian countries. Moreover, there appears to be an important asymmetry. Even though the heavy tail feature is two sided, the asymptotic dependency is only one sided on the depreciation side. With a probability of more than 20%, large downward swings in currency prices are likely to be preceded by large movements in the monetary fundamentals, such as money supply, inflation and interest rates. We find that the asymptotic dependence is stronger during crisis episodes, but not exclusive for crisis events.

The remainder of the paper is organized as follows. Section 2 builds up our theoretical arguments. We first explain that within a standard monetary macroeconomic model with multiplicative parameter uncertainty, the implied distributions of economic variables like the interest rate, inflation rate and money stock can exhibit the heavy tail feature, even if the multiplicative noise distributions have no tails at all (such as the uniform distribution). Subsequently, based on the Engel and West (2005) model we apply Feller’s Convolution Theorem (1971, VIII.8) to show that the heavy tail property of macroeconomic fundamentals can carry over to the exchange rate returns. Moreover, if the fundamentals exhibit heavy tails, then the standard exchange rate model implies the asymptotic dependence between the exchange rate returns and economic fundamentals. Section 3 explains the measures we used to estimate and test for tail fatness and asymptotic dependence. Section 4 contains the empirical results and robustness checks. It reveals the contribution of large fundamentals’ shocks on large currency swings. Conclusions are in Section 5.

3 Several robustness checks are implemented to confirm the extreme linkages.
2 Theory

In this section, we first show that multiplicative parameter uncertainty with a bounded support induces heavy tails on the distribution of the macroeconomic aggregates, even in a standard monetary macroeconomic model where the noise itself does not have heavy tails. Then, we review the present-value exchange rate model of Engel and West (2005) to establish the tail linkage, so called limit copula, between the observed fundamentals and the exchange rate returns. Subsequently, we provide a brief review of the convolution properties of heavy-tailed distributed random variables and the strong linkage that this may imply for macroeconomic shocks and exchange rate returns, i.e. asymptotic dependence.

2.1 Tail Events and Macroeconomic Fundamentals

One may wonder why macroeconomic fundamentals have distributions with heavy tails. An early statistically-oriented explanation for inflation rates was offered by Engle (1982). Engle’s ARCH model has random variables which follow a martingale process with autoregressive behavior in the second moment, causing clusters of high and low volatility. Even if the innovations are thin-tailed normally distributed, the stationary distribution ends up having heavy tails like the Pareto distribution, see de Haan, Resnick, Rootzen and de Vries (1989). In the vein of Bollerslev (1987), Cumperayot (2002) however shows that macroeconomic variables still exhibit heavy tails after filtering out the ARMA-GARCH components.\(^4\)

Here we develop an economic explanation of how the distribution of macroeconomic variables like the money stock, interest rate or inflation rate can exhibit the heavy tail feature. We show that uncertainty about the central bank’s preferences induces heavy tails on the distribution of the macroeconomic aggregates, even in a setup where the noise itself does not have heavy tails.

\(^4\)The heavy tail feature of FX returns partly also stems from the other well-documented FX feature that, at higher frequencies, exchange rate returns exhibit volatility clustering (see Diebold, 1988).
tails. The idea is not to present a fully fledged theory, as this would be outside the scope of this article, but to present a coherent argument for the macroeconomic variables involved. The next subsections then show how the heavy tail feature is carried over to the exchange rate returns.

To this end, consider the following standard monetary macroeconomic model, as presented in Walsh (2003, p.440). The aggregate supply curve reads

$$Y_t = A(\Pi_t - E_{t-1}[\Pi_t]) + \varphi_t,$$

(1)

where $Y_t$ is the logarithmic level of output, $\Pi_t$ is the inflation rate and $E_{t-1}[\Pi_t]$ is the time $t - 1$ expected inflation for time $t$, and $\varphi_t$ is a noise term. In the short run, deviations from the long-run output level are possible due to expectational errors. The elasticity of output with respect to inflation expectations’ errors is $A$.\(^5\)

Aggregate demand depends on real interest rates, i.e. the nominal interest rate minus expected inflation $I_t - E_{t-1}[\Pi_t]$:

$$Y_t = -b(I_t - E_{t-1}[\Pi_t]) + \eta_t.$$

(2)

The money market demand equation is based on the quantity equation

$$M_t = P_{t-1} + \Pi_t + \beta Y_t - \lambda I_t + v_t,$$

(3)

where $M_t$ and $P_{t-1}$ stand for the logarithms of the quantity of money and the price level, respectively; and where $v_t$ is the velocity shock. The $\lambda$ is the semi-interest rate elasticity of

\(^5\)In Online Appendix B, we offer an explanation of why macroeconomic fundamentals have distributions with heavy tails assuming Brainard (1967) type uncertainty, i.e. uncertainty regarding the structural parameter $A$.\)
money demand. A logarithmic expansion of the money supply gives

\[ M_t = \ln m + MB_t, \] (4)

where \( \ln m \) and \( MB_t \) are the logarithms of a constant money multiplier and the monetary base, respectively. The money multiplier is well known not to be constant, but in the above setup, this is captured by the velocity shocks.

The additive demand and supply shocks \((\varphi_t, \eta_t)\) are assumed to exhibit mean zero i.i.d. noise that follows a distribution with thin (exponential declining) or bounded tails (in case of bounded support). At this point, we do not need to be specific about the shocks \( v_t \) to the money market equation. The \( v_t \) shocks are not required to be independent over time, i.e. these may follow a stochastic process.\(^6\)

The central bank’s objective function is assumed as in Calvo and Reinhart (2002). In logarithmic form, the central bank maximizes the expected welfare function

\[ \max_{\Pi_t} E_t \left[ W_t \right] = \omega_{t-1} E_{t-1} [MB_t - P_t + I_t] - \frac{1}{2} (1 - \omega_{t-1}) E_{t-1} \left[ (\Pi_t - \pi^*)^2 \right], \] (5)

where the weight \( \omega_{t-1} \) reflects the degree of trade-off between the seigniorage from the central bank’s money creation and the goal to stabilize the inflation rate around its target \( \pi^* \). The weight \( \omega_{t-1} \) ranges between 0 to 1. At \( \omega_{t-1} = 0 \), all emphasis is on containing inflation, while for \( \omega_{t-1} = 1 \) seigniorage collection is the main goal. Over time and across countries, \( \omega_{t-1} \) varies.

According to Calvo and Reinhart (2002), attempting to exploit a Phillips curve by monetary authorities is of little practical relevance for most emerging markets. Instead, in many emerging economies, inflation surprises are used to generate additional revenue from money creation and to reduce the real value of nominal government debt and public sector wages. This feature

\(^6\)Walsh (2003) solves the model under the assumption that the shocks follow an AR(1) process.
generally differentiates the emerging market economies from, e.g., the eurozone countries with the European Central Bank’s single inflation stability objective in which case $\omega_{t-1} = 0$.

Based on the information available at the time $t-1$, the central bank determines the policy interest rate $I_t$ in order to maximize expected welfare. To find the optimal policy interest rate, we first eliminate $Y_t$ from the first two equations (1) and (2) to get

$$\Pi_t = -\frac{b}{A} I_t + \left(1 + \frac{b}{A}\right) E_{t-1}[\Pi_t] + \frac{\eta_t - \varphi_t}{A}.$$

By taking expectations conditional on time $t-1$ information,

$$I_t = E_{t-1}[\Pi_t].$$

From equation (2), it implies that

$$Y_t = \eta_t. \quad (6)$$

The domestic inflation rate then follows as

$$\Pi_t = I_t + \frac{\eta_t - \varphi_t}{A}.$$

To derive the seigniorage term in (5), we use equations (2), (3) and (4)

$$MB_t - P_t + I_t = (1 - \lambda) I_t + v_t - mm + \beta \eta_t.$$

Using the expression for $\Pi_t$ in terms of $I_t$, the central bank’s expected welfare function can
be written as

$$\max_{I_t} E_{t-1} [W_t] = \omega_{t-1} E_{t-1}[(1 - \lambda) I_t - mm] - \frac{1}{2} (1 - \omega_{t-1}) E_{t-1} \left[ (I_t - \pi^*)^2 \right]$$

$$- \frac{1}{2} (1 - \omega_{t-1}) E_{t-1} \left[ \left( \frac{\eta_t - \varphi_t}{A} \right)^2 \right].$$

The optimal interest rate policy follows as

$$I_t = \pi^* + (1 - \lambda) \frac{\omega_{t-1}}{1 - \omega_{t-1}}. \quad (7)$$

For the single price stability objective, i.e. when $\omega_{t-1} = 0$, $I_t = \pi^*$. However, for countries which prefer some seigniorage from money creation, a higher interest rate, induced by higher expected inflation, implies higher expected welfare.

Solving for the inflation rate, we find

$$\Pi_t = \pi^* + (1 - \lambda) \frac{\omega_{t-1}}{1 - \omega_{t-1}} + \frac{\eta_t - \varphi_t}{A}. \quad (8)$$

The money market equation (3) implies that

$$M_t = P_{t-1} + (1 - \lambda) \pi^* + (1 - \lambda)^2 \frac{\omega_{t-1}}{1 - \omega_{t-1}} + \frac{\eta_t - \varphi_t}{A} + \beta \eta_t + v_t. \quad (9)$$

Levin and Williams (2003) show how weights in the central bank’s objective function are affected by changes in the economic structure. For empirical evidence, Surico (2007) studies the asymmetric preferences of the Fed. He shows that the central bank may put different weights on positive and negative deviations of inflation, output and the interest rate from their reference values. The weight changes through time. In addition, Assenmacher-Wesche (2006) indicates
that the central bank policy in the US, UK and Germany differs across low and high inflation regimes. Weights in the central banks’ objective functions thus may alter over time and certainly vary across countries. In this article, we then show that changes in the relative weight $\omega_{t-1}$ that the central bank assigns to its objectives can be the cause for heavy tails.

To analyze the implication of parameter uncertainty on the tails of macroeconomic variables, we assume that the weight component $(1-\omega_{t-1})$ assigned to the central bank’s inflation objective follows a beta distribution of the form

$$\Pr\{1 - \omega \leq x\} = x^{\alpha_\omega}, \quad \alpha_\omega > 2. \quad (10)$$

The support of this distribution is $[0, 1]$. The beta distribution is clearly not fat tailed. However, the presence of the term $\omega_{t-1} / (1 - \omega_{t-1})$ results in heavy-tailed distributions of the interest rate $I$, inflation rate $\Pi$ and money supply $M$. The tail feature of real income $Y$, however, exclusively depends on the thin-tailed distribution assumed for $\eta$.

Given the beta distribution (10), the distribution of the relative weight $\omega_{t-1} / (1 - \omega_{t-1})$ follows as

$$\Pr\left\{ \frac{\omega}{1 - \omega} \leq x \right\} = \Pr\left\{ 1 + \frac{\omega}{1 - \omega} \leq 1 + x \right\} = \Pr\left\{ \frac{1}{1 - \omega} \leq 1 + x \right\}
= 1 - \Pr\{1 - \omega \leq \frac{1}{1 + x}\} = 1 - \left(\frac{1}{1 + x}\right)^{\alpha_\omega}
= 1 - x^{-\alpha_\omega} \left(\frac{1}{\frac{1}{1 + x} + 1}\right)^{\alpha_\omega}.$$

The relative weight has a Burr distribution on $[0, \infty)$ with a tail in which the Pareto term $x^{-\alpha_\omega}$

---

7The results below may appear specifically due to the assumption of the beta distribution in (10). What is crucial is that zero is in the support of the distribution. The result, therefore, also follows if we had assumed an exponential distribution, for example. The fact that zero is in the support indicates the case in which a country ignores the price stability objective.

8By means of Feller’s Convolution Theorem (1971, VIII.8), it is easy to show that if the fundamentals such as money supply or interest rates have heavy tails, their time differentials are also heavy tailed.
dominates as $x \to \infty$ (since $1/x + 1$ tends to 1). The expressions (7), (8) and (9) for the macroeconomic variables $I$, $II$ and $M$ contain $\omega_t - 1 / (1 - \omega_t - 1)$. Thus the policy weights provide a possible source for heavy-tailed distributed macroeconomic variables. Nevertheless, the real income $Y$ is not heavy tailed, unless the distribution of $\eta$ is heavy tailed itself.

### 2.2 The Canonical Exchange Rate Model

To establish the link between the large exchange rate movements and the observed macroeconomic fundamentals, we rely on the present-value exchange rate model of Engel and West (2005). Below, we briefly recap their model and subsequently discuss limit copula, a concept we use to assess the tail dependency of multivariate probability distributions.

From equation (3), we have

$$M_t = P_t + \beta Y_t - \lambda I_t + \nu_t.$$

Assume that a similar relation holds abroad. Then, taking the difference of the two expressions yields the quantity equation of money at home relative to a base country

$$m_t = p_t + \beta y_t - \lambda i_t + \nu_t,$$

where lower case letters denote relative country variables and $\nu$ stands for the difference between the country specific velocity shocks.

Let $s$ be the logarithm of a nominal exchange rate, quoted per one unit of the base country’s currency in terms of the domestic currency. The real exchange rate $z$ equals the nominal exchange rate $s$ minus the relative prices, i.e. $z = s - p$. Substituting this into the above equation yields

$$s_t = m_t + z_t - \beta y_t + \lambda i_t - \nu_t.$$
Furthermore, let $\rho$ be the deviation from uncovered interest parity (UIP), so that

$$i_t = E_t [s_{t+1}] - s_t + \rho_t,$$

where $i$ is the interest rate differential and $E_t[s_{t+1}]$ is the time $t$ expected exchange rate for time $t + 1$. Using the UIP relation in the expression for the exchange rate gives the monetary approach-based Cagan style exchange rate expression

$$s_t = \frac{1}{1 + \lambda} \left[ m_t + z_t \beta y_t - \nu_t \right] + \frac{\lambda}{1 + \lambda} \rho_t + \frac{\lambda}{1 + \lambda} E_t [s_{t+1}].$$

(11)

Use the following shorthand notation

$$x_t = m_t + z_t - \beta y_t - \nu_t$$

(12)

for the fundamentals, including the country relative velocity shock. Forward iteration yields the standard no-bubbles solution to equation (11)

$$s_t = \frac{1}{1 + \lambda} \sum_{j=0}^{\infty} \left( \frac{\lambda}{1 + \lambda} \right)^j E_t [x_{t+j}]$$

$$+ \frac{\lambda}{1 + \lambda} \sum_{j=0}^{\infty} \left( \frac{\lambda}{1 + \lambda} \right)^j E_t [\rho_{t+j}].$$

(13)

To fix ideas, it helps to consider specific stochastic processes for the fundamentals $x$ and the deviation from UIP $\rho$. Suppose as in Engel, Mark and West (2007), that the fundamentals have a unit root and that the changes in the fundamentals follow a stationary AR(1) process:

$$\Delta x_t = \phi \Delta x_{t-1} + \varepsilon_t, \quad E_t [\varepsilon_{t+1}] = 0, \quad \phi \epsilon(0,1).$$

(14)
Here $\Delta$ is the difference operator and $\varepsilon$ represents i.i.d. shocks that drive the composite fundamentals.

Regarding the deviations from UIP, we assume an $AR(1)$ process driven by risk factors $\mu$. The risk factors may comprise some of the fundamentals from $x$ and other risk drivers, see Burnside, Eichenbaum, Kleshchelski and Rebelo (2011) and Sarno, Schneider and Wagner (2012). Here we follow Engel and West (2005) and do not explicitly state what these factors are. However, we do allow that $\varepsilon$ and $\mu$ are possibly correlated. Furthermore, we do assume that the deviations from UIP are stationary. This is also the preferred option by Engel and West (2005, p. 496), but given the presumed large $AR(1)$ coefficient, they use the $I(1)$ specification as a shortcut. Our specification for the deviations from UIP thus reads

$$\rho_t = a\rho_{t-1} + \mu_t, \ E_t [\mu_{t+1}] = 0, \ \alpha(0,1) \tag{15}$$

where the shocks $\mu$ have zero mean and possibly the covariance $\sigma_{\varepsilon,\mu} \neq 0$. While the two may be interdependent at any instant, over time the two shocks are assumed to be i.i.d.

Given the stochastic process assumptions for the fundamentals (14), it follows that

$$\(1 - b\sum_{j=0}^\infty b^j E_t [x_{t+j}] = x_t + \frac{b\phi}{1-b\phi} \Delta x_t,$$

where the discount factor $b$ is shorthand for

$$b = \frac{\lambda}{1+\lambda}.$$

The UIP expression (15) implies

$$b \sum_{j=0}^\infty b^j E_t [\rho_{t+j}] = \frac{b}{1-ab} \rho_t.$$
We combine these expressions in equation (13) and lag the growth rate of fundamentals to obtain the following expression for $s_t$ in terms of observable fundamentals, UIP deviation and current shocks

$$s_t = x_t + \frac{b\phi^2}{1 - b\phi} \Delta x_{t-1} + \frac{b}{1 - ab} \rho_{t-1} + \frac{b\phi}{1 - b\phi} \epsilon_t. \quad (16)$$

Taking first differences of (16) yields the expression for the exchange rate returns

$$\Delta s_t = (1 - b) \frac{\phi}{1 - b\phi} \Delta x_{t-1} - \frac{(1 - a) b}{1 - ab} \rho_{t-1} + \frac{b}{1 - ab} \mu_t + \frac{1}{1 - b\phi} \epsilon_t. \quad (17)$$

The expression (17) gives the coefficients one would find in a regression of the exchange rate returns on the growth rates of the lagged fundamentals. Engel and West (2005) and Engel, Mark and West (2007) argue and demonstrate that typical values of the discount factor $b$ are close to 1, while Sarno and Sojli (2009) provide empirical evidence. By equation (17) this implies that the changes in the fundamentals become unimportant relative to their shocks $\epsilon_t$, the lagged deviation from UIP $\rho_{t-1}$ and the exogenous UIP shocks $\mu_t$. Furthermore, the exchange rate can have the appearance of a random walk if $\rho$ is small or highly persistent. As a result, beating a random walk in forecasting exchange rates is too strong a criterion for judging an exchange rate model (Engel, Mark and West, 2007).

In this article, we propose the limit copula (or tail analysis) as an alternative for evaluating the association between the exchange rate returns and the lagged fundamentals, as predicted by the model in equation (17). To elicit the dependency in the tails, we use probabilities in a particular quadrant, which are the corresponding zero moments:

$$\frac{\int_q \int_q x^0 y^0 f(x, y)dx dy}{\int_q x^0 f(x)dx} = \frac{\Pr \{X > q, Y > q\}}{\Pr \{X > q\}}.$$

If the large macroeconomic fundamental shocks are the primary drivers of the largest exchange
rate changes, we can zoom in on the tail areas. This gives the so called limit copula
\[
\lim_{q \to \infty} \frac{\Pr\{X > q, Y > q\}}{\Pr\{X > q\}} = \lim_{q \to \infty} \frac{\int_{q}^{\infty} \int_{q}^{\infty} f(x,y) \, dx \, dy}{\int_{q}^{\infty} f(x) \, dx}, \tag{18}
\]
see McNeil, Frey and Embrechts (2005, Ch. 5.2). The limit copula (18) describes the tail dependence between random variables \(X\) and \(Y\). It indicates the probability that the random variable \(Y\) is above the threshold \(q\), conditional on the random variable \(X\) being above \(q\), as the threshold \(q\) approaches infinity.

As regression analysis captures the average relationship between variables, here we assess the extreme linkages between the variables per quadrant. Our objective is to identify whether large movements of macroeconomic fundamentals \(\Delta x_{t-1}\) are associated with extreme movements of exchange rates \(\Delta s_{t}\) while treating positive and negative shocks separately. By changing the signs of the random variables in the limit conditional probability (18), the tail areas in the other three quadrants can be investigated analogously.

### 2.3 Extreme Linkages

In this section, by using statistical extreme value analysis, we explain in further detail the extreme linkages implied by the monetary model of exchange rates. In Subsection 2.3.1, we briefly discuss Feller’s Convolution Theorem (1971, VIII.8) to show the additive property of heavy-tailed distributions and to argue how the heavy tail feature of macroeconomic fundamentals can be transmitted to the distribution of the exchange rate returns. In Subsection 2.3.2, we explicitly specify the asymptotic linkage between the exchange rate returns and the macroeconomic fundamentals implied by the theoretical exchange rate model (17).
2.3.1 Convolution Theorem

We adopt the following general definition of heavy tails: A distribution function $F(q)$ is said to have heavy tails if its tails vary regularly at infinity. The upper tail varies regularly at infinity with tail index $\alpha$ if \(^9\)

$$\lim_{t \to \infty} \frac{1 - F(tq)}{1 - F(t)} = q^{-\alpha}, \quad q > 0 \text{ and } \alpha > 0. \quad (19)$$

Regular variation implies that the tail of the distribution changes at a power rate, while the shape parameter $\alpha$ determines how heavy the tail is. A lower $\alpha$ implies a fatter tail. The number of bounded moments of $F(.)$ is finite and equals the integer value of $\alpha$, i.e. the $\alpha$-moment.\(^{10}\)

This contrasts with, e.g., the normal distribution that has tail probabilities that decline at an exponential rate and all moments are bounded.

Suppose that the distribution of individual macroeconomic variables exhibits a power law, i.e.

$$\Pr\{X > q\} = 1 - F(q) \sim Aq^{-\alpha}, \text{ as } q \to \infty. \quad (20)$$

How does this translate to the composite fundamental $x$? According to Feller’s Convolution Theorem (1971, VIII.8), if $X_1$ and $X_2$ are independent with common c.d.f. $F(q)$ from (20), then

$$\Pr\{X_1 + X_2 > q\} \sim 2Aq^{-\alpha}, \text{ as } q \to \infty. \quad (21)$$

Thus, to a first order at large thresholds $q$, the probability of the sum equals the sum of the marginal probabilities. The main takeaway is that the sum of the heavy-tailed macroeconomic variables exhibits a power law with tail index $\alpha$. Furthermore, if the tail indices $\alpha$ differ, the

---

\(^9\)For the lower tail, $\lim_{t \to \infty} F(-tq)/F(-t) = q^{-\alpha}, \quad q > 0 \text{ and } \alpha > 0.$

\(^{10}\)For instance, the Pareto distribution satisfies the power law and has a number of bounded moments equal to an integer of $\alpha$. The Student-$t$ distribution has moments equal to its degree of freedom.

\(^{11}\)The expression $A \sim B$ means that $\lim (A/B) = 1.$
random variable with the thickest tail (smallest $\alpha$) dominates the sum that defines the composite fundamental.

The monetary model is the sum of the macroeconomic fundamentals and hence the convolution theorem applies if (some of) the fundamental variables have a distribution with a heavy tail. If individual macroeconomic variables exhibit power law behavior, so does the distribution of the sum of these random variables. Additionally, by extending the Feller Theorem one can show that if exchange rate returns and macroeconomic fundamentals have heavy tails, through the linearity of the exchange rate model, the variables are asymptotically dependent, i.e. the limit copula (18) is positive. On the contrary, if the variables are normally distributed, for joint distributions such as the multivariate normal distribution, this limit is zero and there is no asymptotic dependence.$^{12}$

### 2.3.2 Asymptotic Dependence and Exchange Rate Model

In this subsection, we explicitly specify the asymptotic linkage between the exchange rate returns and the macroeconomic fundamentals as implied by the theoretical exchange rate model (17). To do so, we rely on stochastic processes for the fundamentals $x$ and the deviation from UIP $\rho$ from the literature.

Recall the fundamentals $x_t$ from (12)

$$x_t = m_t + z_t - \beta y_t - \nu_t.$$ 

We split the fundamentals into two groups, in line with the macroeconomic model in Section

$^{12}$In Appendix, we use a simple example, resembling the linear exchange rate model (17), to illustrate the asymptotic dependence between the heavy-tailed variables, and show the case of asymptotic independence when the variables are normally distributed. Also, we demonstrate that in this particular example a regression analysis would suggest no links between the variables as the regression analysis is hampered by endogeneity. In Online Appendix C, we show that the results still follow when the macroeconomic variables are not i.i.d., but cross sectionally dependent or stationary time series.
2.1 which predicts different distributions for real income innovations and the monetary related innovations. According to the stochastic process (14) $\phi < 1$, but consider the two separate stochastic processes

$$\Delta (m_t + z_t - \nu_t) = \phi \Delta (m_{t-1} + z_{t-1} - \nu_{t-1}) + \vartheta_t,$$

and

$$\Delta y_t = \phi \Delta y_{t-1} + \xi_t$$

for the two sets of fundamentals. Thus the innovations $\varepsilon$ in equation (14) are split into $\vartheta$ and $\xi$ and are attributed to different parts of the fundamentals vector. Otherwise the process for the fundamentals is as in equation (14).

The deviations from UIP follow the process in equation (15), i.e.

$$\rho_t = a \rho_{t-1} + \mu_t; \ a < 1.$$  

Given a relationship between risk premiums $\rho$ and traditional exchange rate fundamentals $\Delta x$, as in e.g. Sarno, Schneider and Wagner (2012) and Lustig, Roussanov and Verdelhan (2014), we assume that the innovations $\mu$ are a composite of the innovations $\vartheta$, $\xi$ and $\zeta$, i.e.

$$\mu_t \equiv \tau \vartheta_t + \kappa \xi_t + \zeta_t,$$

where $\vartheta$, $\xi$ and $\zeta$ are i.i.d., independent from each other, and have zero mean. The first two shocks are related to the monetary and real fundamentals, while the third innovation is unrelated to the fundamentals of the monetary model.

To derive the extreme linkage in the first quadrant, assume that all the innovations follow
Student-t distributions with the same degrees of freedom $\alpha > 2$ and unit scale $c$. From equation (17), the joint probability of the exchange rate returns and lagged fundamentals conditional on time $t-2$ information becomes

$$
\Pr \left\{ \Delta s_t > q, \Delta x_{t-1} > q \mid \Delta x_{t-2}, \rho_{t-2} \right\} = \Pr \left\{ (1 - b) \frac{\phi}{1 - b\phi} \Delta x_{t-1} - \frac{(1 - a) b}{1 - ab} \rho_{t-1} + \frac{b}{1 - ab} \mu_t + \frac{1}{1 - b\phi} (\theta_t - \beta \xi_t) > q, \Delta x_{t-1} > q \mid \Delta x_{t-2}, \rho_{t-2} \right\} .
$$

Engel and West (2005) discuss the case in which the discount factor is close to unity. For simplicity, take this to the extreme and suppose that $b = 1$.\footnote{In the literature, typical values of the discount factor $b$ are close to 1, not exactly equal to 1. However, by applying the Feller theorem it is trivial to show that the size of the high discount factor only has a marginal effect on the scale factor of the joint distribution, but has no impact on the significance of asymptotic dependence.} Hence, we have

$$
\Pr \left\{ \Delta s_t > q, \Delta x_{t-1} > q \mid \Delta x_{t-2}, \rho_{t-2} \right\} = \Pr \left\{ -\rho_{t-1} + \frac{1}{1 - a} \mu_t + \frac{\theta_t - \beta \xi_t}{1 - \phi} > q, \Delta x_{t-1} > q \mid \Delta x_{t-2}, \rho_{t-2} \right\} 
\sim \Pr \left\{ -\rho_{t-1} > q, \Delta x_{t-1} > q \mid \Delta x_{t-2}, \rho_{t-2} \right\} .
$$

Iterating one period back gives

$$
\Pr \left\{ \Delta s_t > q, \Delta x_{t-1} > q \mid \Delta x_{t-2}, \rho_{t-2} \right\} 
\sim \Pr \left\{ -\rho_{t-2} + \tau \theta_{t-1} + \kappa \xi_{t-1} + \zeta_{t-1} > q, \phi \Delta x_{t-2} + \theta_{t-1} - \beta \xi_{t-1} > q \mid \Delta x_{t-2}, \rho_{t-2} \right\} 
\sim \Pr \left\{ -\tau \theta_{t-1} - \kappa \xi_{t-1} + \zeta_{t-1} > q, \theta_{t-1} - \beta \xi_{t-1} > q \right\} 
\sim \Pr \left\{ -\tau \theta_{t-1} - \kappa \xi_{t-1} > q, \theta_{t-1} - \beta \xi_{t-1} > q \right\} .
$$

Conditional on the information at the time $t-2$, $\rho_{t-2}$ and $\Delta x_{t-2}$ do not contribute to the shocks at $t-1$. Moreover, since the risk premium shock $\zeta_{t-1}$ is unrelated to the fundamentals,
it does not contribute to $\Delta x_{t-1}$ at large levels of $q$ (under the assumption that all innovations
are Student-t distributed).

We consider several configurations for the parameters $\tau$, $\kappa$ and $\beta$. First, suppose that both
$\tau$ and $\kappa$ are negative, i.e. the case in which positive shocks to the relative fundamentals reduce $\rho$ the risk premium of holding domestic currency assets relative to the base currency assets. In
this case the relation further simplifies to

$$\Pr \{ \Delta s_t > q, \Delta x_{t-1} > q | \Delta x_{t-2}, \rho_{t-2} \}$$

$$\sim \Pr \{ -\tau \vartheta_{t-1} - \kappa \xi_{t-1} > q, \vartheta_{t-1} - \beta \xi_{t-1} > q \}$$

$$\sim \Pr \{ -\tau \vartheta_{t-1} > q, \vartheta_{t-1} > q \}. \quad (23)$$

Since both $\tau$ and $\kappa$ are assumed to be negative and $\beta$ is positive, the shocks to income $\xi$ run
in opposite directions for $\Delta s_t$ and $\Delta x_{t-1}$. Only monetary shocks $\vartheta$ contribute to large positive
levels of $\Delta s_t$ and $\Delta x_{t-1}$. The joint probability is asymptotic to

$$\Pr \{ \Delta s_t > q, \Delta x_{t-1} > q | \Delta x_{t-2}, \rho_{t-2} \} \sim \left\{ \begin{array}{ll}
c q^{-\alpha} i f \ - \tau \geq 1 \\
c (-\tau)^{\alpha} q^{-\alpha} i f \ - \tau \leq 1 \end{array} \right.$$  

where $c$ is a scale factor determined by the specifics of the Student-t distribution. The tail analysis
indicates the extreme linkage between the exchange rate returns and lagged macroeconomic
fundamentals, while in this case, a regression analysis might be impaired by the high discount
factor $b$.

Next consider the case in which $\tau$ and $\kappa$ have opposite signs. This is the case considered in
Sarno, et al. (2012) and Lustig, et al. (2014). Their empirical evidence reveals that the risk
premium on US exchange rates is countercyclical to the US economy. This means that in our
setup, $\rho$ reacts negatively to relative monetary impulses and positively to relative output growth.
In this case, $\tau$ is negative and $\kappa$ is positive. Then the joint probability is somewhat more complex

$$
\Pr \{ \Delta s_t > q, \Delta x_{t-1} > q | \Delta x_{t-2}, \rho_{t-2} \} \\
\sim \Pr \{ -\tau \vartheta_{t-1} - \kappa \xi_{t-1} > q, \vartheta_{t-1} - \beta \xi_{t-1} > q \} \\
\sim \{ \min[1, (-\tau)^{\alpha}] + \min[\beta^{\alpha}, \kappa^{\alpha}] \} cq^{-\alpha}.
$$

The intuition for this result is that the asymptotic dependency is determined by the amount of univariate probability mass located along both the $\vartheta_{t-1}$ and $\xi_{t-1}$ axes and satisfies the two inequalities $-\tau \vartheta_{t-1} - \kappa \xi_{t-1} > q$ and $\vartheta_{t-1} - \beta \xi_{t-1} > q$. Also in this setup, the probability of joint extreme movements does not necessarily approach zero as the discount factor $b$ is close to 1.

To conclude, we can expect heavy tails in the exchange rate return distribution if the fundamentals have distributions that are heavy tailed. Moreover, this induces asymptotic dependence between the right-hand-side and left-hand-side variables of equation (17). That is, as the shocks become more extreme, in the limit the probability of large currency swings condition on large fundamental shocks is positive. In general, it is not the case that if the marginal distributions have heavy tails, the random variables are necessarily asymptotically dependent. For example, Student-t distributed random variables combined with a Gaussian copula are correlated but asymptotically independent. The linearity of the model (17) combined with the marginal heavy tail feature, however, does induce the asymptotic dependency between $\Delta s$ on the one hand and the $\Delta x_{t-1}$ components and their contemporaneous shocks on the other hand. The dependency is preserved in the tail area, and even if the correlation coefficient is equal to 0, there can still be asymptotic dependence.

If one finds no support for asymptotic dependence between the exchange rate returns and economic fundamentals, this can be due to one of the following two explanations. One possi-
bility is that the fundamentals-based exchange rate model does not apply, so that the noise is exogenous and is unrelated to the macroeconomic fundamentals. Alternatively, even if two random variables are (imperfectly) correlated, but follow e.g. a multivariate normal distribution, then all dependency vanishes asymptotically. Thus if we reject asymptotic dependence, there are two possible explanations. If we find that asymptotic dependence is not rejected, this at least suggests a strong linkage between the fundamentals and the exchange rate returns via the (larger) shocks that drive both variables.

3 Estimation

In this section, we explain how we estimate and test for tail fatness and asymptotic dependence. Specifically, we apply the tail index estimator of Dekkers, Einmahl and De Haan (1989), abbreviated as DEdH, and the test of asymptotic dependence follows from de Haan and Ferreira (2007).

3.1 Tail Index Estimator

To test whether the exchange rate returns and economic fundamentals exhibit heavy tails, we estimate the inverse tail index, i.e. \( \gamma = 1/\alpha \), and asymptotic 95% confidence intervals. A higher \( \gamma \) implies a fatter tail. From Dekkers, Einmahl and De Haan (1989), let \( X(i) \) be the descending order statistic \( X(1) \geq X(2) \geq ... \geq X(n) \) from the sample of size \( n \). Considering the upper tail, define the first two conditional log-moments

\[
H = \frac{1}{M} \sum_{i}^{M} \log \frac{X(i)}{X(m)}
\]

\(^{14}\)Note that the power law in exchange rate returns does not necessarily stem from the fundamentals, but may be due to the noise terms.
and

\[ K = \frac{1}{M} \sum_{i}^{M} (\log \frac{X(i)}{X(m)})^2, \]

where \( X(m) \) is a suitable threshold and there are \( M \) observations above the threshold. Note that \( H \) is the familiar Hill (1975) tail index estimator, which is predicated on heavy tails. The Dekkers, Einmahl and De Haan (1989), or DEdH, estimator for the inverse of the tail index \( \gamma = 1/\alpha \) reads

\[ \gamma = 1 + H + \frac{1}{2} \frac{K}{H - \frac{K}{M}} \]

and

\[ \sqrt{M} (\gamma - \gamma) \]

is asymptotically normally distributed with variance \( 1 + \gamma^2 \) (as long as \( \gamma \geq 0 \), i.e. as long as the support of the distribution is unbounded). A variable is said to have a heavy tail when the 95% confidence band of the DEdH estimate is within the positive range. If the confidence interval ranges from negative to positive, the null hypothesis of an exponentially thin tail cannot be rejected.

In fact, the Hill estimator has been shown to be asymptotically unbiased and more efficient than alternative estimators, including the DEdH estimator (see, e.g., Koedijk, et al., 1992). However, it only applies if the data are heavy tailed, i.e. if \( \gamma > 0 \). The advantage of the DEdH estimator is that it applies to all types of tails. Specifically in the case where the distribution exhibits heavy tails, \( \gamma = 1/\alpha > 0 \); in the case where the tails are exponentially thin, as in the case of e.g. the normal distribution, \( \gamma = 0 \); and in the case that the support is bounded \( \gamma < 0 \). The disadvantage is that the variance of the DEdH estimator exceeds the variance of the Hill estimator in the case \( \gamma = 1/\alpha > 0 \). Therefore, while the DEdH estimator allows us to test for tail fatness, the estimator is more conservative in rejecting the null hypothesis of a thin tail.
For macroeconomic fundamentals, unlike the FX returns, their tail behavior has rarely been investigated. The DEdH estimator helps measure their tail fatness without any prejudice.

3.2 Asymptotic Dependence Measure

To test for asymptotic dependence, we evaluate the limit conditional probability

$$\lim_{p \to 0} \frac{\Pr \{ X > q_x(p), Y > q_y(p) \}}{\Pr \{ X > q_x(p) \}} = p,$$

(25)

where

$$\Pr \{ X > q_x(p) \} = \Pr \{ Y > q_y(p) \} = p.$$

Note that the asymptotic dependence measure (25) conditions on the same probability level $p$, instead of the same quantile level $q$ previously discussed. The quantiles $q_x(p)$ and $q_y(p)$ are generally different. This popular alternative is commonly used in applied work when the scale of the random variables can differ considerably, see e.g. Poon, Rockinger and Tawn (2004).

To measure the extreme linkage, we use a count measure and consider only a subset $k$ with the more extreme observations from the tail area. Specifically, let $n$ be the sample size and $k$ be a sequence of numbers such that $k(n) \to \infty$ as $n \to \infty$, but $k(n)/n \to 0$. The probability $p$ is proxied by $k/(n+1)$. Let $X_{(i)}$ and $Y_{(i)}$ denote the descending order statistics of $X_i$ and $Y_i$. The corresponding empirical distribution functions are respectively $F_n(x)$ and $G_n(y)$. The empirical counterpart for the denominator, thus, reads

$$p \approx 1 - F_n(X_{(k)}) = 1 - G_n(Y_{(k)}) = \frac{k}{n+1}.$$

For the numerator, we count the number pairs $(X_i, Y_i)$ for which both $X_i \geq X_{(k)}$ and $Y_i \geq Y_{(k)}$
and divide by the sample size. This gives the following count estimator

\[
\hat{S}(k) = \frac{1}{n+1} \sum_{i=1}^{n} \frac{1}{k/\left(n+1\right)} \sum_{i=1}^{n} \{X_i \geq X_{(k)}, Y_i \geq Y_{(k)}\} = \frac{1}{k} \sum_{i=1}^{n} \{X_i \geq X_{(k)}, Y_i \geq Y_{(k)}\},
\]

(26)

where \(1_{\{\}}\) is the indicator function. Similar measures exist for the other quadrants by simply switching the signs of \(X\) and/or \(Y\).

The asymptotic confidence band follows from de Haan and Ferreira (2007, ch.7). Specifically, define

\[
\hat{L}(k) = \frac{1}{k} \sum_{i=1}^{n} \{X_i \geq X_{(k)}, Y_i \geq Y_{(k)}\}.
\]

Note that

\[
\hat{S}(k) = 2 - \hat{L}(k).
\]

So that the asymptotic variance of \(\hat{S}\) equals the asymptotic variance of \(\hat{L}\). The latter variance is estimated by means of

\[
\hat{\sigma}_L^2 = \hat{L}(k) \left( L_1^2 - 2L_1 \right) + \left( L_2^2 - 2L_2 \right) + 4L_1L_2 \left( 2 - \hat{L} \right),
\]

where

\[
L_1 = \frac{1}{k^{3/4}} \sum_{i=1}^{n} \{X_i \geq X_{(k+k^{3/4})}, Y_i \geq Y_{(k)}\} - k^{3/4}\hat{L}(k)
\]

and

\[
L_2 = \frac{1}{k^{3/4}} \sum_{i=1}^{n} \{X_i \geq X_{(k)}, Y_i \geq Y_{(k+k^{3/4})}\} - k^{3/4}\hat{L}(k).
\]

Since \(k + k^{3/4}\) can be non-integer, this is rounded to the nearest integer. De Haan and Ferreira (2007, ch.7) prove that

\[
\sqrt{k} \left( \hat{L}(k) - L \right)
\]
is asymptotically normally distributed with mean zero and variance $\hat{\sigma}^2_L$. From this, confidence bands can be easily constructed. If the 95% confidence band ranges from negative to positive, the null hypothesis of asymptotic independence cannot be rejected.

We will now illustrate how the count estimator $\hat{S}(k)$ behaves when pairs of variables are asymptotically dependent and independent. In Figure 1, we show the plots of $\hat{S}(k)$ and its 95% asymptotic confidence band for different thresholds $k$ using simulated data with correlation of 0.7. The plot in the left panel shows the estimates for Student-t random variables with 3 degrees of freedom, which are asymptotically dependent. The right panel displays normally distributed random variables with no asymptotic dependence. From the figure, we observe that the Student-t based plot immediately jumps up to a stable plateau, but the normal based plot only gradually rises from a lower level. For asymptotic independence, the plot often stays in the neighborhood of 0 before slowly rising towards an end point of 1.

4 Empirical Results

Our empirical aim is twofold. First, we examine whether or not some of the macroeconomic fundamental shocks are heavy-tailed distributed. Second, if this is the case, we investigate the asymptotic dependency between the variables on both sides of the exchange rate model. The data are monthly observations from 34 countries over the period of February 1974 to May 2016. To deal with the small number of observations resulting from the low frequency of macroeconomic variables, we pool the data by region: combining European, Asian and Latin American countries in three groups. The number of observations used to estimate and test for heavy tails and the extreme linkage between exchange rate returns and economic fundamentals then ranges from

\[15\] To be specific, the two series of simulated uniformly distributed random variables are transformed to Student-t random variables with 3 degrees of freedom and normally distributed random variables, both with correlation coefficient of 0.7.

\[16\] The data set and descriptive statistics are shown in Online Appendix A.
3843 to 6096. The panel analysis of the joint tail events is partly justified by the similarity of tail indices across countries within the regions.\textsuperscript{17}

Furthermore, we study the extreme linkage between exchange rate returns and lagged individual fundamentals, similar to the regression analysis in Engel and West (2005). A problem with using the composite fundamental $\Delta x_{t-1}$ in practice is that the composite relies on knowing the coefficients of the individual fundamentals. Measuring these coefficients is, however, difficult. Moreover, different theories tend to suggest different sets of fundamentals. In this article, we identify the relevance of each fundamental separately. We examine whether large movements of macroeconomic fundamentals are associated with large currency swings, and which fundamentals matter most for the extreme movements of exchange rates.

The extreme value theory discussed earlier offers an advantage over a regression based analysis when part of the model under the null hypothesis is not well specified. It helps circumvent the need to first measure the contribution of the specific fundamental to the composite. Nevertheless, extreme value analysis itself can be rather sensitive to the number of observations included in the tail area. Too few observations can enlarge the variance of the estimate, while too many observations reduce the variance at the expense of biasedness due to including observations from the central range. Below, we report the estimates using a typical tail size of 2.5\% of the overall sample.\textsuperscript{18}

### 4.1 Tail Indices

Table 1 reports the estimated tail index $\alpha$, i.e. the inverse of the DEdH estimate $\gamma$ described in Section 3.1, using 2.5\% the tail observations from the overall sample. A lower $\alpha$ implies a fatter tail, while the integer value of the tail index $\alpha$ indicates the number of bounded moments.

\textsuperscript{17}To save space, estimated tail indices for individual countries are available upon request.

\textsuperscript{18}The main conclusions of our study are still valid when using 5\% or 1\% of the overall sample as the tail data. Figure 1 shows the stability of $S(k)$ in the tail area.
In Table 1, we only show the estimates of the tail index $\alpha$ which are significantly positive at the 5% level as well as their lower and upper bounds constructed from the asymptotic 95% confidence intervals of the estimated $\gamma$ (in parentheses).\textsuperscript{19} In the table, the right tail indicates large depreciations of the domestic currency relative to the base currency and dramatic increases in the domestic fundamentals relative to the fundamentals of the base country, while the left tail shows the opposite pattern. In the case of Asia and Latin America, the domestic variables are relative to US variables, and the US dollar (USD) is the base currency. For the European countries, apart from the US, we also consider the domestic variables relative to Germany. After the introduction of the euro in 1999, for the countries which do not adopt the euro, we extend their sample relative to the eurozone. The Deutsche mark (DM), and thereafter the euro (EUR), are then the base currency.

For Asian and Latin American countries, all exchange rate returns and changes in macroeconomic fundamentals exhibit heavy tails. In almost all cases, the estimated $\alpha$ is below 4 which indicates that the fourth moment is unbounded. Half of the Asian and Latin American variables have a tail index $\alpha$ below 2. In these cases, the variance of the variables is not bounded. The fundamentals are clearly heavy tailed, compared to the thin-tailed normal distribution which has all moments bounded. For the Asian and Latin American countries, we also find evidence that real income exhibits heavy tails. In our theoretical model, real income can exhibit heavy tails if aggregate demand shocks are heavy tailed.

For the European countries, there are only a number of cases in which the variables significantly exhibit heavy tails. When considered relative to the US, only the negative growth rates of money supply and price level, and both positive and negative changes in the interest rates have heavy tails. The exchange rate returns do not have heavy tails. When the German Mark or

\textsuperscript{19}Thus, Table 1 only shows the $\alpha$ estimates for heavy-tailed variables. Table A2 in Online Appendix A gives the estimates of the inverse tail index, i.e. $\gamma = 1/\alpha$, and asymptotic 95% confidence intervals for all the variables.
the euro is used as the base currency, the heavy tail cases are the depreciation of the European currencies, the positive growth rate of money supply and both tails of the interest rate changes.

In sum, Table 1 shows that not only the distribution of exchange rate returns has heavy tails, but this also holds for macroeconomic fundamentals. Furthermore, both nominal as well as real variables appear to be heavy tailed. The tail fatness, however, varies from one region to another. We have demonstrated in Section 2 that, given the linearity of the exchange rate model, the presence of large shocks (or heavy tails) on both sides of equation (17) implies a specific kind of dependency, i.e. asymptotic dependence. Next, we empirically estimate and test for the asymptotic dependence.

### 4.2 Extreme Linkages

To test the hypothesis that large swings in exchange rates are linked to large changes in the heavy-tailed macroeconomic fundamentals, we apply the linkage estimator (26) which conditions on the same probability level, see equation (25). Using 2.5% of the data, Table 2 and 3 give the estimates of the linkage measure and asymptotic 95% confidence intervals (in parentheses) for large depreciations and appreciations of the domestic currency, respectively. The depreciation side is on the right tail of the exchange rate returns’ distribution, and opposite for the appreciation side. The first column shows the pair of variables under investigation. Positive and negative signs indicate positive and negative relations between the two variables.

For instance, $\Delta s, \Delta m_{-1}, +$ represents the positive relation between exchange rate returns and lagged relative money supply growth. In Table 2, the results show the linkage between the right tails of the distributions of exchange rate returns and money supply growth, i.e. between large depreciations of the domestic currency and large increases in domestic money supply relative to the foreign money supply. The linkage between the left tails, showing large appreciations and declines in relative money supply, is in Table 3. For the interest rate $\Delta i_{-1}$, theory is ambiguous
regarding the relation between the exchange rate and the interest rate differential. We therefore examine both positive (+) and negative (−) relations.

For the European fundamentals and exchange rate returns, both tables show that the limit conditional probability (25) is close to zero and insignificant in all cases regardless of the base currency. This may be due to the fact that we do not find much evidence of heavy tails for the European variables. For Asian and Latin American currencies, we however find strong evidence for asymptotic dependence between depreciations of the domestic currency and increases in money supply $\Delta m_{t-1}$, prices $\Delta p_{t-1}$ and interest rates $\Delta i_{t-1}$. Nonetheless, for depreciations of the domestic currency and declines in real output $\Delta y_{t-1}$, the link is not significant in any of the regions; the estimates are close to zero and the null hypothesis of asymptotic independence cannot be rejected. Large depreciations of the domestic currency are mainly a monetary phenomenon, and strong links are found for Asian and Latin American currencies.

Interestingly, even though Table 1 suggests that income may be heavy tailed for Latin American and Asian countries, we do not find evidence of asymptotic dependency between real income changes and exchange rate returns. The absence of tail dependence for real income is in line with the earlier example in equation (23), if we assume a negative $\kappa$. In this case, only monetary shocks contribute to large positive levels of $\Delta s_t$ and $\Delta x_{t-1}$. If $\kappa$ is negative, it would imply that for Asian and Latin American countries, the risk premium of holding their domestic currencies is countercyclical to their home economies, not the US economy as documented in Sarno, et al. (2012) and Lustig, et al. (2014).20

Alternatively, suppose that the risk premium of holding the domestic currencies is countercyclical to the US economy as reported in Sarno, et al. (2012) and Lustig, et al. (2014), i.e. $\kappa$ is positive. Our empirical results then match the theoretical prediction in equation (24) if $\beta = 0$.

---

20Evidence in Sarno, et al. (2012) and Lustig, et al. (2014) is from the developed market economies. It is, therefore, interesting to further investigate whether this also holds for emerging market economies.
That is when the real income does not play a part in (12), but does figure in the risk premium $\rho$. In this case (24) reduces to

\[
\Pr \left\{ \Delta s_t > q, \Delta x_{t-1} > q \mid \Delta x_{t-2}, \rho_{t-2} \right\} \\
\sim \Pr \left\{ -\tau \vartheta_{t-1} - \kappa \xi_{t-1} > q, \vartheta_{t-1} > q \right\} \\
\sim \min \left[ 1, (-\tau)^\alpha \right] cq^{-\alpha}.
\]

The extreme link between $\Delta s_t$ and $\Delta x_{t-1}$ is then dominated by monetary shocks. To conclude, large increases in money supply, prices and interest rates are followed by dramatic depreciations of the domestic currency, with probabilities of more than 20% approximately.

Estimates for the appreciation side are given in Table 3. The limit conditional probability (25) is close to zero for all cases, even for the monetary variables, indicating asymptotic independence. Extreme contractionary monetary policy or high deflation do not trigger large appreciations of the domestic currency. Thus, even though Table 1 reveals that the exchange rate returns and macroeconomic fundamentals are heavy tailed on both sides for Asia and Latin America, we do not find evidence of asymptotic dependency with lagged macroeconomic fundamentals. According to equation (22), this could mean that on the appreciation side, the risk premium does not respond much to shifts in the fundamentals, such as a sudden restraint in the money supply. Such asymmetric responses of the exchange rate returns to macroeconomic fundamentals are in line with the copula-based evidence of Patton (2006) for the DM-USD and Yen-USD exchange rates, and Ehrmann and Fratzscher’s (2005) analysis of how macroeconomic news impacts exchange rates.

To show how the count estimator $S(k)$ differs for the cases of asymptotic dependence and independence, we end with two plots of the linkage estimator (26), using the actual data from the Latin American countries. In Figure 2, the left panel shows the case of asymptotic dependence...
between the Latin American currency depreciations and the positive growth of lagged relative money supply. For the asymptotic independence, the right panel illustrates the case of the Latin American currency depreciations against the negative growth of lagged relative real income. There is a marked difference between the two plots. When the variables are asymptotically dependent, the conditional probability almost immediately jumps upwards, as in the left panels of Figure 1. The graph for the case of asymptotic independent variables has more resemblance with the right-hand-side panels of Figure 1. The plot often stays in the neighborhood of 0 before slowly rising.

In summary, we detect significant connections between large swings in currencies and fundamentals. The extreme linkages occur exclusively for Asian and Latin American currencies, not for Europe. Furthermore, we uncover asymmetric responses of the exchange rate returns to large changes in macroeconomic fundamentals, as the asymptotic dependence is only significant for the depreciation of the domestic currency. The monetary variable displays asymptotic dependence with the exchange rate depreciation. Therefore, the results lend further support to traditional exchange rate models in the vein of research initiated by Engel and West (2005) and Engel, Mark and West (2007).

4.3 Robustness Checks

We examine the robustness of the asymptotic links between exchange rate returns and macroeconomic fundamentals, considering several dimensions.\textsuperscript{21} To summarize, we first lagged the macroeconomic fundamentals up to 12 lags and find that the estimates of the linkage measure do not change substantially, see Table A3. This is indicative of the persistence of tail (in)dependence. Our findings are not lag dependent. For pairs of asymptotically dependent variables, once we observe the extreme movements of macroeconomic fundamentals, there exist possibilities of large

\textsuperscript{21}The complete results are in Online Appendix D. In this subsection, we only summarize the main results.
currency swings in the future.

Second, in Table A4 we show the estimates of the tail linkage between the time $t$ macro-economic fundamentals and lagged exchange rate returns. This follows on from the research of Sarno and Schmeling (2014), who argue that if exchange rates are driven by expected future fundamentals, then those exchange rates contain information regarding future fundamentals. We find that those macroeconomic fundamentals which are asymptotically dependent with exchange rate returns in the previous subsection are also significantly associated with the lagged exchange rate return (see Table A4). For asymptotically independent variables, the results also remain the same. This confirms the extreme linkages between exchange rate returns and macroeconomic fundamentals, as suggested by the present-value model of exchange rates.

Third, we investigate whether the asymptotic links between exchange rate returns and macroeconomic fundamentals disappear when using a lower data frequency. When using quarterly data, as shown in Table A5, our conclusions remain the same but with slightly higher point estimates.

Fourth, in Table A6, we show that the associations between large swings in currency prices and lagged macroeconomic fundamental shocks, captured by the linkage measure (25), are extreme events in non-crisis circumstances. To do so, we use the exchange rate regime classification advanced by Ilzetzki, Reinhart and Rogoff (2008). There are five coarse regimes: the pegged, limited flexibility, managed floating, freely floating and freely falling regimes. After excluding the freely falling regime, the asymptotic dependence between currency depreciations and increases in money supply, price and interest rate declines from 20-30% to a single digit number, but the estimates are still significant. The results thus indicate that the degrees of asymptotic

---

22 We use the monthly coarse classification of exchange rate regimes from Ilzetzki, Reinhart and Rogoff (2008). For details on the de facto exchange rate regime classification, the reader is referred to Reinhart and Rogoff (2004) and Ilzetzki, Reinhart and Rogoff (2008).

23 Note that observations are classified as freely falling when one of the following conditions applies. First, the annual inflation rate is above 40%, but excluding months during which the exchange rate still follows an official pre-announced arrangement (crawl or band). Second, the six months immediately follow a currency crisis, but only for the cases where the crisis marks a transition from a fixed (or quasi-fixed) regime to a managed or freely floating regime.
dependence between exchange rate returns and monetary fundamentals are much stronger during crisis episodes. Yet, the asymptotic dependence is not exclusive for crisis events. Regardless of crisis or non-crisis episodes, large movements in monetary variables can potentially be followed by dramatic depreciations of the domestic currency. Real income, on the other hand, remains disconnected.

Finally, we adopt a non-parametric approach to illustrate the asymptotic dependence and independence between pairs of variables. In Figure A1, we plot the conditional probability of a large currency depreciation given a large macroeconomic shock on the Y-axis. The X-axis shows $1 - p$, as both variables are conditioned on the same probability level $p$ as in equation (25). The contrast between the cases of asymptotic dependence and independence, reported in Table 2, are clear in Figure A1. For the variables, which are asymptotically independent, e.g., the cases between $\Delta s$ and $\Delta y_{-1}$, the conditional probability moves along the diagonal line and eventually approaches null as shocks to economic growth $\Delta y_{-1}$ become very large. For the cases of asymptotic dependence, e.g., between $\Delta s$ and $\Delta m_{-1}$ of Asia and Latin America, the probability of joint extremes lingers at a positive number for large fundamental shocks. Therefore, in those cases, the conditional probability is non-zero even in the limit. Noteworthily, while Table 2 contains the estimated limit conditional probability, Figure A1 empirically shows the conditional probability for finite quantiles which is typically even larger. In other words, the estimated limit conditional probability sets the lower bound for the probability of joint extremes in the tail area.

5 Conclusion

Exchange rate returns are well known to have distributions with heavy tails. In this article, we ask whether this heavy tail feature can be explained by the observed macroeconomic fundamentals. Standard models of the exchange rate imply that the exchange rate returns are driven by the
growth rates of the macroeconomic fundamentals. Using statistical extreme value theory, we then argue that if macroeconomic fundamentals have distributions with heavy tails the exchange rate returns can be heavy-tailed distributed by implication. In addition, we demonstrate that the heavy tails on both sides of the linear exchange rate models imply a specific kind of dependency, i.e. asymptotic dependence, between the exchange rate returns and the macroeconomic shocks.

To determine whether there in fact exists such a connection, or if the larger movements in the exchange rate returns are exclusively due to exogenous noise, we use a data set consisting of monthly observations from 34 countries from February 1974 to May 2016. To deal with the small number of observations resulting from the low frequency of macroeconomic variables, we pool the data by region: combining European, Asian and Latin American countries into three groups. Based on the estimates of the tail shape parameter, the Asian and Latin American FX returns and macroeconomic fundamentals clearly have heavy tails. Moreover, the currency depreciations of the Asian and Latin American countries and the lagged monetary variables are significantly asymptotically dependent. With a probability of roughly more than 20%, large downward swings in currency prices are likely to be preceded by large movements in the monetary fundamentals, such as money supply, price and interest rate. Several robustness checks confirm the asymptotic dependency between these variables.

No asymptotic dependency was detected for European currencies, whether we use the US dollar, the German mark or the euro as an anchor. This may be due to the fact that we do not find much evidence of heavy tails for the European variables. In addition, real income does not show tail dependency with nominal exchange rates for any of the three regions. This is also in line with the theoretical analysis, which shows that the monetary fundamentals may be tail dependent with exchange rate returns, but this does not apply to real income. Therefore, we may conclude that the heavy tail feature of the FX returns is, at least partially, attributable to the tail behavior of the macroeconomic fundamentals. The responses of the exchange rate returns
to large changes in the fundamentals, however, are asymmetric. The asymptotic dependence is only significant for the depreciation of the domestic currency. This article thereby lends support to traditional exchange rate models connecting the largest depreciations to extreme movements in the fundamentals.

6 Appendix: Feller’s Convolution Theorem

In this appendix, we apply the Feller theorem to demonstrate that if exchange rate returns and macroeconomic fundamentals have heavy tails, through the linearity of the exchange rate model, the variables are asymptotically dependent. On the contrary, if the variables are normally distributed, there is no asymptotic dependence. To do so, consider the simple linear specification as an illustration. Suppose

\[ Y = X + (\vartheta - 2\xi) \]  

(27)

and where the explanatory variable \( X \) is driven by two random shocks

\[ X = \zeta + \xi. \]  

(28)

The error term in equation (27) is also driven by two shocks \( \vartheta \) and \( \xi \). Shocks in \( \zeta \) do affect both \( X \) and \( Y \) in the same positive way. A shock in \( \vartheta \) does affect \( Y \), but does not show up in \( X \); a shock in \( \xi \) does affect both \( X \) and \( Y \), but in opposite directions. A shock in \( \xi \) increases \( X \), but in equation (27) this effect of \( X \) on \( Y \) is overwhelmed by the negative impact of \( \xi \) in the noise term of \( Y \). In equation (17), this can occur if the monetary policy shocks, e.g., a higher growth rate of money supply, increase both \( \Delta x_{t-1} \) and the risk premium \( \rho_{t-1} \), as these appear with opposite signs in the solution for the exchange rate returns.

For illustration purpose, suppose that all three shocks are independently Student-t distributed.
with the same degrees of freedom $\alpha > 2$. As a result $\sigma^2 = \sigma^2 = \sigma^2$, while the covariances are zero. Then for large $q$

$$\Pr(|\zeta| > q) = \Pr(|\vartheta| > q) = \Pr(|\xi| > q) \sim 2c q^{-\alpha},$$

where

$$c = \Gamma ((\alpha + 1)/2) \alpha^{(\alpha-1)/2} / \Gamma (\alpha/2) \sqrt{\alpha \pi}.$$ 

The absolute value and symmetry of the tails explains the factor 2 in the expression. Thus, for the upper tail for example we have $\Pr\{\xi > q\} \sim cq^{-\alpha}$.

By Feller’s convolution theorem

$$\Pr \{\zeta + \xi > q\} \sim 2cq^{-\alpha} \quad (29)$$ 

and

$$\Pr \{\zeta - \xi + \vartheta > q\} \sim 3cq^{-\alpha}.$$ 

For the joint distribution $\Pr \{\zeta + \xi > q, \zeta - \xi + \vartheta > q\}$, notice that the shocks $\xi$ appear with opposite signs. Therefore a large positive shock in $\xi$ likely contributes to an extreme realization of the sum $\zeta + \xi$, but has the opposite effect on $\zeta - \xi + \vartheta$. A slight extension of the Feller theorem then shows that for large $q$

$$\Pr \{\zeta + \xi > q, \zeta - \xi + \vartheta > q\} \sim \Pr \{\zeta > q, \zeta > q\} \sim cq^{-\alpha}.$$
This gives the following limit

\[
\frac{\Pr\{X > q, Y > q\}}{\Pr\{X > q\}} = \frac{\Pr\{\zeta + \xi > q, \zeta - \xi + \vartheta > q\}}{\Pr\{\zeta + \xi > q\}} \approx \frac{\Pr\{\zeta > q, \zeta + q > q\}}{\Pr\{\zeta + q > q\}} \to \frac{1}{2}
\]  

(30)

The variables \(X\) and \(Y\) are, then, said to be asymptotically dependent since the limit conditional probability of a large realization of \(Y\), given a large realization of \(X\), is non-zero.24

Alternatively, if the \((\zeta, \vartheta, \xi)\) are independent mean zero standard normal shocks, then the sum in equation (29) is also normally distributed. Laplace’s classical expansion for the tail probabilities is the density to the (large) quantile, which immediately shows that the tail probabilities are of exponential nature. Briefly, if say both \(\zeta\) and \(\xi\) are independent standard normally distributed, then for large \(q\)

\[
\Pr\{\zeta > q\} = \Pr\{\xi > q\} \sim \frac{1}{q \sqrt{2\pi}} e^{-\frac{q^2}{2}},
\]

so that

\[
\Pr\{\zeta + \xi > q\} \sim \frac{1}{q \sqrt{\pi}} e^{-\frac{q^2}{4}}, \text{ as } q \to \infty.
\]

The important difference between summing the Student-t random variables and the normal case is that, in the former case, the power \(\alpha\) remains as it is, but in the normal case the power changes from \(-1/2\) in the exponent to \(-1/4\). In the above example, since the two random variables \(\zeta + \xi\) and \(\zeta - \xi + \vartheta\) are uncorrelated, the two normally distributed random variables are in this case

\[\text{24} \text{ An OLS regression gives}
\]

\[
\hat{\beta} = \frac{\text{Cov}\{\zeta - \vartheta; \zeta + \xi\}}{\text{Var}\{\zeta + \xi\}} \approx \frac{\sigma_\zeta^2 - \sigma_\xi^2}{\sigma_\zeta^2 + \sigma_\xi^2} \to 0
\]

in large samples. The regression analysis is hampered by averaging and endogeneity as the shocks \(\vartheta - 2\xi\) in equation (27) are correlated with the explanatory variable \(X\). As we consider each tail separately, shocks in \(X\) and \(Y\) are not averaged out. Thus the tail analysis shows that \(Y\) does depend on \(X\), while a regression analysis would suggest this is not the case.
also independent and hence

\[
\text{Pr}\{\zeta + \xi > q, \zeta - \xi + \vartheta > q\} = \text{Pr}\{\zeta + \xi > q\} \text{Pr}\{\zeta - \xi + \vartheta > q\}
\]

\[
\approx \frac{1}{q} \frac{1}{\sqrt{\pi}} e^{-\frac{1}{2}q^2} \cdot \frac{\sqrt{3}}{q} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}q^2}
\]

\[
= \frac{\sqrt{3/2}}{q^2} \frac{1}{\pi} e^{-\frac{3}{2}q^2}
\]

which tends to zero faster than \(\text{Pr}\{\zeta + \xi > q\}\) so that there is no asymptotic dependence. For more details, see e.g., De Vries (2005).
<table>
<thead>
<tr>
<th>Base Currency</th>
<th>Europe</th>
<th>Europe</th>
<th>Asia</th>
<th>Latin America</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DM/EUR</td>
<td>USD</td>
<td>USD</td>
<td>USD</td>
</tr>
<tr>
<td>(\Delta s)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>left</td>
<td>-</td>
<td>-</td>
<td>3.36</td>
<td>0.81</td>
</tr>
<tr>
<td>right</td>
<td>-</td>
<td>3.20</td>
<td>(2.16, 7.57)</td>
<td>(0.66, 1.05)</td>
</tr>
<tr>
<td>(1.93, 9.45)</td>
<td></td>
<td></td>
<td>(1.53, 3.30)</td>
<td>(0.94, 1.64)</td>
</tr>
<tr>
<td>(\Delta m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>left</td>
<td>-</td>
<td>4.74</td>
<td>1.70</td>
<td>1.73</td>
</tr>
<tr>
<td>right</td>
<td>2.52</td>
<td>-</td>
<td>(1.28, 2.51)</td>
<td>(1.28, 2.67)</td>
</tr>
<tr>
<td>(2.45, 8.97)</td>
<td></td>
<td></td>
<td>(1.75, 4.58)</td>
<td>(2.71, 10.89)</td>
</tr>
<tr>
<td>(\Delta y)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>left</td>
<td>-</td>
<td>-</td>
<td>1.70</td>
<td>2.13</td>
</tr>
<tr>
<td>right</td>
<td>-</td>
<td>-</td>
<td>(1.75, 4.58)</td>
<td>(2.71, 10.89)</td>
</tr>
<tr>
<td>(1.63, 5.49)</td>
<td></td>
<td></td>
<td>(1.75, 4.58)</td>
<td>(2.71, 10.89)</td>
</tr>
<tr>
<td>(\Delta p)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>left</td>
<td>-</td>
<td>2.70</td>
<td>5.91</td>
<td>2.79</td>
</tr>
<tr>
<td>right</td>
<td>-</td>
<td>-</td>
<td>(3.00, 185.19)</td>
<td>(1.83, 5.82)</td>
</tr>
<tr>
<td>(1.76, 5.83)</td>
<td></td>
<td></td>
<td>(1.79, 7.24)</td>
<td>(1.83, 5.82)</td>
</tr>
<tr>
<td>(\Delta i)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>left</td>
<td>1.82</td>
<td>2.13</td>
<td>1.06</td>
<td>0.58</td>
</tr>
<tr>
<td>right</td>
<td>2.26</td>
<td>3.42</td>
<td>1.03</td>
<td>0.59</td>
</tr>
<tr>
<td>(1.29, 3.10)</td>
<td></td>
<td>(1.48, 3.80)</td>
<td>(0.86, 1.39)</td>
<td>(0.48, 0.74)</td>
</tr>
<tr>
<td>(1.52, 4.46)</td>
<td></td>
<td>(2.05, 10.29)</td>
<td>(0.83, 1.34)</td>
<td>(0.48, 0.74)</td>
</tr>
</tbody>
</table>

Table 1 shows the significantly positive DEdH estimates of the tail index and asymptotic 95% confidence intervals (in parentheses), using 2.5% tail observations from the overall sample. The variables are the exchange rate returns \(\Delta s\), the rate of change in relative money supply \(\Delta m\), the rate of change in relative real income \(\Delta y\), the rate of change in relative price \(\Delta p\) and the change in the interest rate differential \(\Delta i\). The right tail indicates large depreciations of the domestic currency and large increases in the domestic fundamentals relative to the base country, while the left tail shows the opposite pattern. For all three groups, i.e. Europe, Asia and Latin America, the US dollar (USD) is the base currency and the domestic fundamentals are relative to the US. However, for the European countries, we also consider the variables relative to Germany, and to the eurozone (after the introduction of the euro in 1999) for the countries which do not adopt the euro. The Deutsche mark (DM), and subsequently the euro (EUR), are then the base currency.
Table 2: Estimates of Extreme Linkage

<table>
<thead>
<tr>
<th>Base Currency</th>
<th>Europe</th>
<th>Europe</th>
<th>Asia</th>
<th>Latin America</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DM/EUR</td>
<td>USD</td>
<td>USD</td>
<td>USD</td>
</tr>
<tr>
<td>$\Delta s, \Delta m_{-1,+}$</td>
<td>0.0104</td>
<td>0.0194</td>
<td>0.2676</td>
<td>0.2276</td>
</tr>
<tr>
<td></td>
<td>(-0.0250, 0.0458)</td>
<td>(-0.0263, 0.0652)</td>
<td>(0.1437, 0.3915)</td>
<td>(0.1121, 0.3432)</td>
</tr>
<tr>
<td>$\Delta s, \Delta y_{-1,-}$</td>
<td>0.0510</td>
<td>0.0364</td>
<td>0.0229</td>
<td>0.0417</td>
</tr>
<tr>
<td></td>
<td>(-0.0217, 0.1237)</td>
<td>(-0.0254, 0.0982)</td>
<td>(-0.0215, 0.0673)</td>
<td>(-0.0287, 0.1120)</td>
</tr>
<tr>
<td>$\Delta s, \Delta p_{-1,+}$</td>
<td>0.0404</td>
<td>0.0636</td>
<td>0.2466</td>
<td>0.2358</td>
</tr>
<tr>
<td></td>
<td>(-0.0269, 0.1077)</td>
<td>(-0.0181, 0.1454)</td>
<td>(0.1337, 0.3594)</td>
<td>(0.1213, 0.3502)</td>
</tr>
<tr>
<td>$\Delta s, \Delta i_{-1,+}$</td>
<td>0.0928</td>
<td>0.0833</td>
<td>0.1862</td>
<td>0.2373</td>
</tr>
<tr>
<td></td>
<td>(-0.0126, 0.1982)</td>
<td>(-0.0095, 0.1761)</td>
<td>(0.0675, 0.3049)</td>
<td>(0.1156, 0.3509)</td>
</tr>
<tr>
<td>$\Delta s, \Delta i_{-1,-}$</td>
<td>0.0309</td>
<td>0.0648</td>
<td>0.1034</td>
<td>0.1017</td>
</tr>
<tr>
<td></td>
<td>(-0.0280, 0.0808)</td>
<td>(-0.0184, 0.1481)</td>
<td>(0.0174, 0.1895)</td>
<td>(0.0049, 0.1985)</td>
</tr>
</tbody>
</table>

Table 2 shows the estimates of the extreme linkage measure and asymptotic 95% confidence intervals (in parentheses) using 2.5% of the data for large depreciations of the domestic currency relative to the base currency. The first column shows the pair of variables under investigation. The variables are the exchange rate returns $\Delta s$, the rate of change in relative money supply $\Delta m$, the rate of change in relative real income $\Delta y$, the rate of change in relative price $\Delta p$ and the change in the interest rate differential $\Delta i$. Positive and negative signs indicate positive and negative relations between the two variables.

The depreciation side is on the right tail of the exchange rate returns' distribution. Hence, $\Delta s, \Delta m_{-1,+}$ represents the linkage between the right tails of the distributions of exchange rate returns and lagged money supply growth, i.e. between large depreciations of the domestic currency and large increases in domestic money supply (relative to foreign money supply).

For all three groups, i.e. Europe, Asia and Latin America, the US dollar (USD) is the base currency and the domestic fundamentals are relative to the US. However, for the European countries, we also consider the variables relative to Germany, and to the eurozone (after the introduction of the euro in 1999) for the countries which do not adopt the euro. The Deutsche mark (DM), and subsequently the euro (EUR), are then the base currency.

Table 3: Estimates of Extreme Linkage

<table>
<thead>
<tr>
<th>Base Currency</th>
<th>Europe</th>
<th>Europe</th>
<th>Asia</th>
<th>Latin America</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DM/EUR</td>
<td>USD</td>
<td>USD</td>
<td>USD</td>
</tr>
<tr>
<td>$\Delta s, \Delta m_{-1,+}$</td>
<td>0.0104</td>
<td>0.0485</td>
<td>0.0423</td>
<td>0.0407</td>
</tr>
<tr>
<td></td>
<td>(-0.0245, 0.0453)</td>
<td>(-0.0214, 0.1185)</td>
<td>(-0.0153, 0.0998)</td>
<td>(-0.0185, 0.0998)</td>
</tr>
<tr>
<td>$\Delta s, \Delta y_{-1,-}$</td>
<td>0.0102</td>
<td>0.0273</td>
<td>0.0153</td>
<td>0.0104</td>
</tr>
<tr>
<td></td>
<td>(-0.0243, 0.0447)</td>
<td>(-0.0255, 0.0800)</td>
<td>(-0.0213, 0.0518)</td>
<td>(-0.0256, 0.0465)</td>
</tr>
<tr>
<td>$\Delta s, \Delta p_{-1,+}$</td>
<td>0.0202</td>
<td>0.0364</td>
<td>0.0342</td>
<td>0.0732</td>
</tr>
<tr>
<td></td>
<td>(-0.0270, 0.0674)</td>
<td>(-0.0239, 0.0967)</td>
<td>(-0.0167, 0.0852)</td>
<td>(-0.0060, 0.1523)</td>
</tr>
<tr>
<td>$\Delta s, \Delta i_{-1,+}$</td>
<td>0.0309</td>
<td>0.0463</td>
<td>0.0276</td>
<td>0.0763</td>
</tr>
<tr>
<td></td>
<td>(-0.0289, 0.0908)</td>
<td>(-0.0204, 0.1130)</td>
<td>(-0.0183, 0.0734)</td>
<td>(-0.0092, 0.1617)</td>
</tr>
<tr>
<td>$\Delta s, \Delta i_{-1,-}$</td>
<td>0.0515</td>
<td>0.0093</td>
<td>0.0276</td>
<td>0.0593</td>
</tr>
<tr>
<td></td>
<td>(-0.0255, 0.1286)</td>
<td>(-0.0230, 0.0416)</td>
<td>(-0.0186, 0.0737)</td>
<td>(-0.0147, 0.1334)</td>
</tr>
</tbody>
</table>

Table 3 shows the estimates of the extreme linkage measure and asymptotic 95% confidence intervals (in parentheses) using 2.5% of the data for large appreciations of the domestic currency relative to the base currency. The first column shows the pair of variables under investigation. The variables are the exchange rate returns $\Delta s$, the rate of change in relative money supply $\Delta m$, the rate of change in relative real income $\Delta y$, the rate of change in relative price $\Delta p$ and the change in the interest rate differential $\Delta i$. Positive and negative signs indicate positive and negative relations between the two variables.

The appreciation side is on the left tail of the exchange rate returns' distribution. Hence, $\Delta s, \Delta m_{-1,+}$ represents the linkage between the left tails of the distributions of exchange rate returns and lagged money supply growth, i.e. between large appreciations of the domestic currency and large increases in domestic money supply (relative to foreign money supply).

For all three groups, i.e. Europe, Asia and Latin America, the US dollar (USD) is the base currency and the domestic fundamentals are relative to the US. However, for the European countries, we also consider the variables relative to Germany, and to the eurozone (after the introduction of the euro in 1999) for the countries which do not adopt the euro. The Deutsche mark (DM), and subsequently the euro (EUR), are then the base currency.
Figure 1: Extreme Linkage Measure and 95% Confidence Interval for Simulated Student-t and Normal Data with Correlation 0.7

Figure 2: Extreme Linkage Measure and 95% Confidence Interval: Asymptotic Dependency and Asymptotic Independency
References


Engel, C., N.C. Mark, K.D. West, K. Rogoff and B. Rossi, "Exchange Rate Models Are Not as Bad as You Think [with Comments and Discussion]," *NBER Macroeconomics Annual* 22 (2007), 381-443.


Sarno, L. and E. Sjöli, "The Feeble Link between Exchange Rates and Fundamentals: Can We Blame the Discount Factor?," *Journal of Money, Credit and Banking* 41 (2009), 437-442.


Appendix A: Data, Descriptive Statistics and Tail Estimates

The data are monthly observations on the exchange rate, money supply (M2), production index, interest rate and price index from the IMF International Financial Statistics (IFS). The data range from February 1974 to May 2016. The list of 34 countries used in our study consists of Argentina, Austria, Bolivia, Brazil, Chile, China, Colombia, Denmark, Ecuador, Finland, France, Germany, India, Indonesia, Ireland, Israel, Italy, Japan, Jordan, Malaysia, Mexico, Netherlands, Norway, Pakistan, Peru, Philippines, Singapore, South Korea, Spain, Sweden, Turkey, UK, Uruguay and Venezuela. To deal with the small number of observations resulting from the low frequency of macroeconomic variables, we pool the data by region: combining European, Asian and Latin American countries in three groups.

Table A1 provides descriptive statistics for the exchange rate returns $\Delta s$, relative money supply growth $\Delta m$, relative real income growth $\Delta y$, relative inflation $\Delta p$ and changes in the interest rate differential $\Delta i$, for Europe, Asia and Latin America, respectively. In the case of Asia and Latin America, the domestic variables are relative to the US variables, and the US dollar (USD) is the base currency. For the European countries, apart from the US, we also consider variables relative to Germany and the Euro zone. For the countries which

---

*Corresponding author’s address: Erasmus University Rotterdam H9, P.O. Box 1738, Rotterdam 3000 DR, The Netherlands. E-mail address: cdevries@ese.eur.nl; E-mail address of Phornchanok Cumperayot: phornchanok.c@chula.ac.th.
adopt the euro, the data relative to Germany end in December 1998. For the countries which do not adopt the euro, following the introduction of the euro in 1999, we extend their sample until May 2016 relative to the data from the Euro zone. The Deutsche mark (DM), and thereafter the euro (EUR), are then the base currency.

In Table A1, the Jarque-Bera (J-B) normality test rejects the null hypothesis of a normal population distribution at the 1% significance level ($p$-values equal zero) in all cases, while the skewness and kurtosis values indicate non-normal distributions of the variables. The kurtosis is particularly high for the exchange rate returns and most of the fundamentals from Asia and Latin America. A high kurtosis can be indicative of heavy tails. In Table A2, we then test for heavy tails by considering the left and right tails of the distribution separately.

Table A2 reports the estimates of the inverse tail index, i.e. $\gamma = 1/\alpha$, and asymptotic 95% confidence intervals (in parentheses) using the DEdH estimator. The right tail, denoted "right" in the table, indicates large depreciations of the domestic currency and increases in the domestic fundamentals relative to the base country, while the left tail, denoted "left", shows the opposite pattern. A higher $\gamma$ implies a fatter tail. The estimator is used as a pre-test for the tail type. If the confidence interval ranges from negative to positive, the null hypothesis of an exponentially thin tail cannot be rejected.

For Asian and Latin American countries, all exchange rate returns and changes in macroeconomic fundamentals can reject the null hypothesis of a thin tail at the 5% significance level. For the European countries, when considered relative to the US, only the negative growth rates of money supply and price level, and both positive and negative changes in the interest rates have heavy tails. When the German Mark or the euro is used as the base currency, the heavy tail cases are the depreciation of the European currencies, the positive growth rate of money supply and both tails of the interest rate changes.

Appendix B: Brainard (1967) Type Uncertainty and Heavy Tails

To explain the heavy tail feature of macroeconomic fundamentals, we use a monetary macroeconomic model in Walsh (2003, p.440), as presented in the main article, and assume Brainard (1967) type uncertainty. The
aggregate supply curve then reads

\[ Y_t = A_t (\Pi_t - E_{t-1}[\Pi_t]) + \varphi_t, \tag{1} \]

where the elasticity of output with respect to inflation expectations’ errors \( A_t \) is variable. In addition to uncertainty about the central bank’s preferences, here we assume uncertainty regarding the structural parameter. Since the seminal work of Brainard (1967), the topic of policy effectiveness has received considerable attention. Model estimates and new data frequently lead to parameter revisions, see Sack (2000). We capture the model uncertainty by assuming that the coefficient for the short-run Phillips effect \( A_t \) is an i.i.d. random variable with a positive mean. Swamy and Tavlas (2007) offer theoretical arguments in favor of a random coefficient specification, while Vavra (2014) offers empirical support for the time-varying slope of the Phillips curve. The variability of the slope \( A_t \) in equation (1) captures changes in wage indexation to inflation. Over time and across countries, indexation has varied considerably. The level of indexation determines the responsiveness of output to inflation. Full indexation implies \( A = 0 \). This corresponds to the classical dichotomy. In case of partial indexation in the short run, output responds to changes in inflation.

Similar to the main article, the inflation rate and the money supply become

\[ \Pi_t = \pi^* + (1 - \lambda) \frac{\omega_{t-1}}{1 - \omega_{t-1}} + \frac{\eta_t - \varphi_t}{A_t} \tag{2} \]

and

\[ M_t = P_{t-1} + (1 - \lambda) \pi^* + (1 - \lambda)^2 \frac{\omega_{t-1}}{1 - \omega_{t-1}} + \frac{\eta_t - \varphi_t}{A_t} \beta \eta_t + v_t, \tag{3} \]

respectively.

Consider the term \((\eta_t - \varphi_t)/A_t\). Suppose that \( A_t \) has a beta distribution

\[
\Pr\{A \leq x\} = x^{\alpha_A}, \quad \alpha_A > 2
\]
and the support of this distribution is $[0, 1]$. The distribution of the inverse of $A$ is

$$
\Pr\left\{ \frac{1}{A} \leq x \right\} = 1 - \Pr\{A \leq \frac{1}{x}\} = 1 - \frac{1}{x^{\alpha_A}},
$$

with support $x \in (1, \infty)$. Thus the inverse of $A$ has a heavy-tailed Pareto distribution and has moments only up to $\alpha_A$, while the beta distribution is clearly not fat tailed. The presence of the term $1/A$ results in heavy-tailed distributions of the interest rate $I$, inflation rate $\Pi$ and money supply $M$.

As a result, the unconditional distribution of $(\eta_t - \varphi_t)/A_t$ is also heavy tailed. To see this, let $Q = \eta - \varphi$ and consider the distribution of $Q/A$. Suppose the distribution of $Q$ is such that at least the $\alpha$-th moment is bounded; thus $E_Q[Q^\alpha] < \infty$. Using the conditioning argument of Breiman and the distribution in (4) then shows that

$$
\Pr\left\{ \frac{Q}{A} > x \right\} = E_Q[\Pr\left\{ \frac{q}{A} > x|Q = q \right\}]
= E_Q[\Pr\left\{ \frac{1}{A} > \frac{x}{q}|Q = q \right\}]
= E_Q\left[ \left( \frac{Q}{x} \right)^{\alpha_A} \right] = E_Q[Q^{\alpha_A}x^{-\alpha_A}].
$$

Note that the numerator only plays a minor role, since all that is required is that the expectation $E_Q[Q^{\alpha_A}]$ exists. Therefore, due to the random Phillips curve coefficient $A$ in the denominators of equations (2) and (3), the unconditional distributions of $\Pi$ and $M$ have heavy tails.

**Appendix C: Tail Additivity**

The monetary-approach exchange rate model is linear in the macroeconomic fundamentals. In the main article, we show that according to Feller’s Convolution Theorem (1971, VIII.8) if the i.i.d. macroeconomic variables are heavy tailed, then the exchange rate also has a distribution with heavy tails. Furthermore, the macroeconomic variable with the thickest tail (smallest $\alpha$) dominates the sum that defines the composite fundamental. In this Appendix, we argue that this result still follows if the macroeconomic variables are not

---

1The fact that zero is in the support of $A$ reflects the possibility that the short-run supply curve may be vertical, i.e. it coincides with the long-run supply curve.
i.i.d., but cross sectionally dependent or stationary time series.

From an economic point of view, the macroeconomic fundamentals can be driven by a common component that is heavy-tailed distributed.\(^2\) Consider, therefore, the multivariate extension of regular variation. Suppose that the vector \(x\) of fundamental variables is multivariate regularly varying in the sense that

\[
\lim_{t \to \infty} \frac{1 - F(tx)}{1 - F(t1)} = W(x), \quad x > 0,
\]

where \(W(.)\) is a function such that \(W(\lambda x) = \lambda^{-\alpha}W(x), \quad \alpha > 0, \quad \lambda > 0\) and \(1\) is the unit vector. Suppose the marginal distributions of the fundamentals have heavy tails, i.e.

\[
\Pr(X > q) = 1 - F(q) \sim Aq^{-\alpha}, \quad as \quad q \to \infty. \tag{6}
\]

So that the scales are of the same order, and all the marginal distributions have the same tail index \(\alpha\). Then, for any non-zero weight vector \(w\), \(P\{w^Tx > q\} \sim Cq^{-\alpha}, \quad as \quad q \to \infty\). Here the scale constant \(C\) depends on the type of dependence, i.e. it requires specific knowledge of the copula. Nevertheless, the weighted sum of random variables that determines the distribution of the exchange rate still has a Pareto-like upper tail with the tail index \(\alpha\). Moreover, it is still the case that if the marginal distributions have different tail indices, the fundamental with the heaviest tail determines the tail index of the exchange rate returns.

In addition, it is worth noting that the a-temporal convolution result still holds when the economic variables are stationary time series. This is because the convolution is a ‘cross-section’ like aggregation at a specific point in time. The convolution theorem can be used to study the aggregation of time series over time. Suppose, for example, that \(m\) follows the following \(MA(1)\) process

\[
m_t = \varepsilon_t + \delta \varepsilon_{t-1}, \quad and \quad \delta > 0,
\]

and where the innovations \(\varepsilon\) are i.i.d. with distribution function as in equation (6). Then, by Feller’s

\(^2\)See the monetary variables in equations (2) and (3), for example.
\(^3\)The expression \(A \sim B\) means that \(\lim (A/B) = 1\).
Convolution Theorem

\[ \Pr\{m > q\} \sim A(1 + \delta^\alpha)q^{-\alpha}, \text{ as } q \to \infty. \]

Furthermore, \( P[m_t + m_{t-1} > q] \sim A[1 + (1 + \delta^\alpha) q^{-\alpha}, \text{ as } q \to \infty. \) Note that the convolution results show that the scales of the random variables change due to the moving average process, but not the tail index \( \alpha. \)

As the exchange rate and macroeconomic variables display bouts of quiescence and turbulence, changes in the economic variables are often captured by ARMA-GARCH-type models. When time series are not i.i.d. but serially dependent, the occurrence of extremes may affect the distribution of order statistics, but not the tail index \( \alpha. \) That is the exchange rate return distribution still has hyperbolic tails.

**Appendix D: Results of Robustness Checks**

We examine the robustness of the asymptotic links between exchange rate returns and macroeconomic fundamentals. Table A3 - A6 and Figure A1 provide the results. Explanations of the tests are given in the main article (Section 4.3) and in notes below the tables.

---

4 More complicated time series models can also be handled. For instance, Engle’s (1982) original contribution modeled the inflation rate by the ARCH process. De Haan, Resnick, Rootzen and de Vries (1989) show that the tail of the stationary distribution of the ARCH process is regularly varying. Basrak, Davis and Mikosch (2002) discuss the convolution of GARCH processes.
consider the domestic variables

Table A1 shows descriptive statistics for the exchange rate returns $\Delta s$, relative money supply growth $\Delta m$, relative real income growth $\Delta y$, relative inflation $\Delta p$ and changes in the interest rate differential $\Delta i$ for Europe, Asia and Latin America. The variables are relative to the US and the US dollar (USD) is the base currency. However, for the group of European countries, we also consider the domestic variables relative to Germany, and to the eurozone (after the introduction of the euro in 1999) for the countries which do not adopt the euro. The Deutsche mark (DM), and thereafter the euro (EUR), are then the base currency.

<table>
<thead>
<tr>
<th>Region</th>
<th>$\Delta s$</th>
<th>$\Delta m$</th>
<th>$\Delta y$</th>
<th>$\Delta p$</th>
<th>$\Delta i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Europe</td>
<td>0.0018</td>
<td>0.0013</td>
<td>0.0007</td>
<td>0.0019</td>
<td>1.5E-06</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0003</td>
<td>0.0003</td>
<td>-0.0005</td>
<td>0.0014</td>
<td>0.0000</td>
</tr>
<tr>
<td>Median</td>
<td>0.2208</td>
<td>0.5703</td>
<td>1.0209</td>
<td>0.0423</td>
<td>0.0581</td>
</tr>
<tr>
<td>Max</td>
<td>-0.1036</td>
<td>-0.2490</td>
<td>-1.1087</td>
<td>-0.0282</td>
<td>-0.0545</td>
</tr>
<tr>
<td>Min</td>
<td>0.0168</td>
<td>0.0252</td>
<td>0.1504</td>
<td>0.0055</td>
<td>0.0019</td>
</tr>
<tr>
<td>Std. Dev</td>
<td>1.9916</td>
<td>3.5906</td>
<td>0.0006</td>
<td>1.0502</td>
<td>2.1680</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>21.5115</td>
<td>82.5704</td>
<td>17.1546</td>
<td>8.1680</td>
<td>458.8525</td>
</tr>
<tr>
<td>J-B test</td>
<td>59099.88</td>
<td>1022081</td>
<td>32932.90</td>
<td>5129.58</td>
<td>33500286</td>
</tr>
<tr>
<td>Obs.</td>
<td>3956</td>
<td>3843</td>
<td>3945</td>
<td>3956</td>
<td>3869</td>
</tr>
<tr>
<td>Asia</td>
<td>0.0006</td>
<td>0.0017</td>
<td>-0.0003</td>
<td>0.0007</td>
<td>3.25E-07</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.0007</td>
<td>0.0002</td>
<td>0.0035</td>
<td>9.70E-05</td>
<td>0.0000</td>
</tr>
<tr>
<td>Median</td>
<td>0.1991</td>
<td>0.3202</td>
<td>0.9380</td>
<td>0.0688</td>
<td>0.0578</td>
</tr>
<tr>
<td>Max</td>
<td>-0.1314</td>
<td>-0.2468</td>
<td>-1.0013</td>
<td>-0.0535</td>
<td>-0.0547</td>
</tr>
<tr>
<td>Min</td>
<td>0.0314</td>
<td>0.0211</td>
<td>0.1582</td>
<td>0.0060</td>
<td>0.0017</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.4043</td>
<td>1.2061</td>
<td>-0.3598</td>
<td>1.3394</td>
<td>1.1433</td>
</tr>
<tr>
<td>Obs.</td>
<td>4424</td>
<td>4126</td>
<td>4392</td>
<td>4414</td>
<td>4326</td>
</tr>
<tr>
<td>Latin America</td>
<td>$\Delta s$</td>
<td>$\Delta m$</td>
<td>$\Delta y$</td>
<td>$\Delta p$</td>
<td>$\Delta i$</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0555</td>
<td>0.0093</td>
<td>0.0023</td>
<td>0.0050</td>
<td>-2.36E-07</td>
</tr>
<tr>
<td>Median</td>
<td>0.0000</td>
<td>0.0055</td>
<td>0.0023</td>
<td>0.0015</td>
<td>-8.33E-06</td>
</tr>
<tr>
<td>Max</td>
<td>0.8025</td>
<td>1.4456</td>
<td>2.1472</td>
<td>0.2413</td>
<td>0.2934</td>
</tr>
<tr>
<td>Min</td>
<td>-0.3487</td>
<td>-0.6980</td>
<td>-2.0442</td>
<td>-0.0697</td>
<td>-0.3972</td>
</tr>
<tr>
<td>Std. Dev</td>
<td>0.0350</td>
<td>0.0355</td>
<td>0.1123</td>
<td>0.0171</td>
<td>0.0107</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.9947</td>
<td>10.7632</td>
<td>0.1050</td>
<td>4.2662</td>
<td>-6.3496</td>
</tr>
<tr>
<td>J-B test</td>
<td>101.0635</td>
<td>522.6942</td>
<td>120.1407</td>
<td>38.1431</td>
<td>672.3097</td>
</tr>
<tr>
<td>Obs.</td>
<td>6969</td>
<td>5696</td>
<td>5242</td>
<td>5853</td>
<td>5801</td>
</tr>
</tbody>
</table>

Table A1 shows descriptive statistics for the exchange rate returns $\Delta s$, relative money supply growth $\Delta m$, relative real income growth $\Delta y$, relative inflation $\Delta p$ and changes in the interest rate differential $\Delta i$ for Europe, Asia and Latin America. The variables are relative to the US and the US dollar (USD) is the base currency. However, for the group of European countries, we also consider the domestic variables relative to Germany, and to the eurozone (after the introduction of the euro in 1999) for the countries which do not adopt the euro. The Deutsche mark (DM), and thereafter the euro (EUR), are then the base currency.
Table A2: DEdH Estimates of Inverse Tail Index

<table>
<thead>
<tr>
<th></th>
<th>Europe</th>
<th>Europe</th>
<th>Asia</th>
<th>Latin America</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Base Currency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δs</td>
<td>DM/EUR</td>
<td>USD</td>
<td>USD</td>
<td>USD</td>
</tr>
<tr>
<td>left</td>
<td>0.0593</td>
<td>-0.0311</td>
<td>0.2975</td>
<td>1.2338</td>
</tr>
<tr>
<td>right</td>
<td>(0.1380, 0.2567)</td>
<td>(-0.2173, -0.1550)</td>
<td>(0.1321, 0.4628)</td>
<td>(0.9554, 1.5123)</td>
</tr>
<tr>
<td>Δm</td>
<td>left 0.1396</td>
<td>0.2110</td>
<td>0.5889</td>
<td>0.5787</td>
</tr>
<tr>
<td></td>
<td>(0.1058, 0.5185)</td>
<td>(-0.4676, -0.0817)</td>
<td>(0.3032, 0.6546)</td>
<td>(0.6092, 1.0666)</td>
</tr>
<tr>
<td>Δy</td>
<td>left -0.0648</td>
<td>-0.5574</td>
<td>0.5883</td>
<td>0.4692</td>
</tr>
<tr>
<td></td>
<td>(0.1820, 0.6125)</td>
<td>(-0.0470, 0.3416)</td>
<td>(0.2183, 0.5707)</td>
<td>(0.0091, 0.3689)</td>
</tr>
<tr>
<td>Δp</td>
<td>left 0.1478</td>
<td>0.3698</td>
<td>0.1693</td>
<td>0.3588</td>
</tr>
<tr>
<td></td>
<td>(0.0513, 0.3469)</td>
<td>(0.1714, 0.5681)</td>
<td>(0.0054, 0.3333)</td>
<td>(0.1718, 0.5458)</td>
</tr>
<tr>
<td>Δi</td>
<td>left 0.5494</td>
<td>0.4704</td>
<td>0.9421</td>
<td>1.7135</td>
</tr>
<tr>
<td></td>
<td>(0.3223, 0.7765)</td>
<td>(0.2629, 0.6779)</td>
<td>(0.7185, 1.1657)</td>
<td>(1.3571, 2.0700)</td>
</tr>
<tr>
<td>Δi</td>
<td>right 0.4418</td>
<td>0.2928</td>
<td>0.9747</td>
<td>1.7089</td>
</tr>
<tr>
<td></td>
<td>(0.2242, 0.6594)</td>
<td>(0.0972, 0.4884)</td>
<td>(0.7474, 1.2019)</td>
<td>(1.3532, 2.0647)</td>
</tr>
</tbody>
</table>

Table A2 shows the DEdH estimates of the inverse tail index and asymptotic 95% confidence intervals (in parentheses) using 2.5% of the tail observations from the overall sample. The variables are the exchange rate returns Δs, the rate of change in relative money supply Δm, the rate of change in relative real income Δy, the rate of change in relative price Δp and change in the interest rate differential Δi. The right tail indicates large depreciations of the domestic currency and dramatic increases in the domestic fundamentals relative to the base country, while the left tail shows the opposite pattern. For all three groups, i.e. Europe, Asia and Latin America, the US dollar (USD) is the base currency and the domestic fundamentals are relative to the US. However, for the European countries, we also consider the variables relative to Germany, and to the eurozone (after the introduction of the euro in 1999) for the countries which do not adopt the euro. The Deutsche mark (DM), and subsequently the euro (EUR), are then the base currency.
### Table A3: Estimates of Extreme Linkage between Currency Depreciations and Lagged Macroeconomic Fundamentals

#### European Currencies (DM/EUR)

<table>
<thead>
<tr>
<th>Lag</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta s_{t}$, $\Delta m_{t}$, $+$</td>
<td>0.0104</td>
<td>0.0521</td>
<td>0.0521</td>
<td>0.0094</td>
<td>0.0000</td>
<td>0.0013</td>
<td>0.0421</td>
<td>0.0000</td>
<td>0.0032</td>
<td>0.0226</td>
<td>0.0211</td>
<td>0.0042</td>
</tr>
<tr>
<td>$\Delta s_{t}$, $\Delta \gamma_{t}$, -</td>
<td>0.0010</td>
<td>0.0204</td>
<td>0.0101</td>
<td>0.0406</td>
<td>0.0002</td>
<td>0.0010</td>
<td>0.0714</td>
<td>0.0204</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0409</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\Delta s_{t}$, $\Delta \rho_{t}$, -</td>
<td>0.0277</td>
<td>0.1257</td>
<td>0.0681</td>
<td>0.0264</td>
<td>0.1080</td>
<td>0.0425</td>
<td>0.0447</td>
<td>0.0258</td>
<td>0.0097</td>
<td>0.0928</td>
<td>0.0404</td>
<td>0.0409</td>
</tr>
<tr>
<td>$\Delta s_{t}$, $\Delta \epsilon_{t}$, -</td>
<td>0.0604</td>
<td>0.0906</td>
<td>0.0612</td>
<td>0.0612</td>
<td>0.0608</td>
<td>0.0016</td>
<td>0.0006</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0010</td>
</tr>
<tr>
<td>$\Delta s_{t}$, $\Delta \delta_{t}$, +</td>
<td>0.0010</td>
<td>0.0204</td>
<td>0.0101</td>
<td>0.0406</td>
<td>0.0002</td>
<td>0.0010</td>
<td>0.0714</td>
<td>0.0204</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0409</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\Delta s_{t}$, $\Delta \beta_{t}$, -</td>
<td>0.0277</td>
<td>0.1257</td>
<td>0.0681</td>
<td>0.0264</td>
<td>0.1080</td>
<td>0.0425</td>
<td>0.0447</td>
<td>0.0258</td>
<td>0.0097</td>
<td>0.0928</td>
<td>0.0404</td>
<td>0.0409</td>
</tr>
<tr>
<td>$\Delta s_{t}$, $\Delta \epsilon_{t}$, +</td>
<td>0.0604</td>
<td>0.0906</td>
<td>0.0612</td>
<td>0.0612</td>
<td>0.0608</td>
<td>0.0016</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0010</td>
</tr>
</tbody>
</table>

#### European Currencies (USD)

<table>
<thead>
<tr>
<th>Lag</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta s_{t}$, $\Delta m_{t}$, $+$</td>
<td>0.0104</td>
<td>0.0291</td>
<td>0.0971</td>
<td>0.0007</td>
<td>0.0194</td>
<td>0.0194</td>
<td>0.0194</td>
<td>0.0007</td>
<td>0.0291</td>
<td>0.0194</td>
<td>0.0007</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\Delta s_{t}$, $\Delta \gamma_{t}$, -</td>
<td>0.0064</td>
<td>0.0364</td>
<td>0.0650</td>
<td>0.0183</td>
<td>0.0183</td>
<td>0.0183</td>
<td>0.0183</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\Delta s_{t}$, $\Delta \rho_{t}$, -</td>
<td>0.0075</td>
<td>0.0972</td>
<td>0.0207</td>
<td>0.0254</td>
<td>0.0968</td>
<td>0.0259</td>
<td>0.0259</td>
<td>0.0259</td>
<td>0.0259</td>
<td>0.0259</td>
<td>0.0259</td>
<td>0.0259</td>
</tr>
<tr>
<td>$\Delta s_{t}$, $\Delta \epsilon_{t}$, -</td>
<td>0.0064</td>
<td>0.0364</td>
<td>0.0650</td>
<td>0.0183</td>
<td>0.0183</td>
<td>0.0183</td>
<td>0.0183</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\Delta s_{t}$, $\Delta \delta_{t}$, +</td>
<td>0.0010</td>
<td>0.0204</td>
<td>0.0101</td>
<td>0.0406</td>
<td>0.0002</td>
<td>0.0010</td>
<td>0.0714</td>
<td>0.0204</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0409</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\Delta s_{t}$, $\Delta \beta_{t}$, -</td>
<td>0.0277</td>
<td>0.1257</td>
<td>0.0681</td>
<td>0.0264</td>
<td>0.1080</td>
<td>0.0425</td>
<td>0.0447</td>
<td>0.0258</td>
<td>0.0097</td>
<td>0.0928</td>
<td>0.0404</td>
<td>0.0409</td>
</tr>
<tr>
<td>$\Delta s_{t}$, $\Delta \epsilon_{t}$, +</td>
<td>0.0604</td>
<td>0.0906</td>
<td>0.0612</td>
<td>0.0612</td>
<td>0.0608</td>
<td>0.0016</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0010</td>
</tr>
</tbody>
</table>

#### European Currencies (EUR)

<table>
<thead>
<tr>
<th>Lag</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta s_{t}$, $\Delta m_{t}$, $+$</td>
<td>0.0104</td>
<td>0.0291</td>
<td>0.0971</td>
<td>0.0007</td>
<td>0.0194</td>
<td>0.0194</td>
<td>0.0194</td>
<td>0.0007</td>
<td>0.0291</td>
<td>0.0194</td>
<td>0.0007</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\Delta s_{t}$, $\Delta \gamma_{t}$, -</td>
<td>0.0064</td>
<td>0.0364</td>
<td>0.0650</td>
<td>0.0183</td>
<td>0.0183</td>
<td>0.0183</td>
<td>0.0183</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\Delta s_{t}$, $\Delta \rho_{t}$, -</td>
<td>0.0075</td>
<td>0.0972</td>
<td>0.0207</td>
<td>0.0254</td>
<td>0.0968</td>
<td>0.0259</td>
<td>0.0259</td>
<td>0.0259</td>
<td>0.0259</td>
<td>0.0259</td>
<td>0.0259</td>
<td>0.0259</td>
</tr>
<tr>
<td>$\Delta s_{t}$, $\Delta \epsilon_{t}$, -</td>
<td>0.0064</td>
<td>0.0364</td>
<td>0.0650</td>
<td>0.0183</td>
<td>0.0183</td>
<td>0.0183</td>
<td>0.0183</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\Delta s_{t}$, $\Delta \delta_{t}$, +</td>
<td>0.0010</td>
<td>0.0204</td>
<td>0.0101</td>
<td>0.0406</td>
<td>0.0002</td>
<td>0.0010</td>
<td>0.0714</td>
<td>0.0204</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0409</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\Delta s_{t}$, $\Delta \beta_{t}$, -</td>
<td>0.0277</td>
<td>0.1257</td>
<td>0.0681</td>
<td>0.0264</td>
<td>0.1080</td>
<td>0.0425</td>
<td>0.0447</td>
<td>0.0258</td>
<td>0.0097</td>
<td>0.0928</td>
<td>0.0404</td>
<td>0.0409</td>
</tr>
<tr>
<td>$\Delta s_{t}$, $\Delta \epsilon_{t}$, +</td>
<td>0.0604</td>
<td>0.0906</td>
<td>0.0612</td>
<td>0.0612</td>
<td>0.0608</td>
<td>0.0016</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0010</td>
</tr>
</tbody>
</table>

Note: The table presents the estimated coefficients for the linkage between currency depreciations and lagged macroeconomic fundamentals. Each cell contains the coefficient estimate for the corresponding lag and lagged variable.
<table>
<thead>
<tr>
<th>Lag</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta s$</td>
<td>$\Delta m_{i\text{z}}$</td>
<td>+</td>
<td>0.2768</td>
<td>0.1884</td>
<td>0.2513</td>
<td>0.1972</td>
<td>0.1761</td>
<td>0.1972</td>
<td>0.1901</td>
<td>0.13479</td>
<td>0.1761</td>
<td>0.1158</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta s$</td>
<td>$\Delta m_{i\text{z}}$</td>
<td>-</td>
<td>0.0220</td>
<td>0.0153</td>
<td>0.0155</td>
<td>0.0229</td>
<td>0.0153</td>
<td>0.0229</td>
<td>0.0153</td>
<td>0.0060</td>
<td>0.0153</td>
<td>0.0154</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta s$</td>
<td>$\Delta y_{i\text{z}}$</td>
<td>+</td>
<td>0.2466</td>
<td>0.2329</td>
<td>0.2260</td>
<td>0.2334</td>
<td>0.2192</td>
<td>0.1986</td>
<td>0.1986</td>
<td>0.2329</td>
<td>0.2280</td>
<td>0.1996</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta s$</td>
<td>$\Delta y_{i\text{z}}$</td>
<td>-</td>
<td>0.1162</td>
<td>0.1739</td>
<td>0.1517</td>
<td>0.1795</td>
<td>0.1448</td>
<td>0.1586</td>
<td>0.1724</td>
<td>0.1724</td>
<td>0.1724</td>
<td>0.1448</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta s$</td>
<td>$\Delta p_{i\text{z}}$</td>
<td>+</td>
<td>0.0579</td>
<td>0.3040</td>
<td>0.0984</td>
<td>0.2923</td>
<td>0.0725</td>
<td>0.2881</td>
<td>0.0415</td>
<td>0.2482</td>
<td>0.01507</td>
<td>0.2685</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta s$</td>
<td>$\Delta p_{i\text{z}}$</td>
<td>-</td>
<td>0.1054</td>
<td>0.0966</td>
<td>0.0621</td>
<td>0.0800</td>
<td>0.1103</td>
<td>0.0828</td>
<td>0.0779</td>
<td>0.0759</td>
<td>0.0621</td>
<td>0.0690</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table A3: Estimates of the Extreme Linkage between Domestic Currency Depreciations and Lagged Macroeconomic Fundamentals (Continued)

#### Asian Currencies (USD)

#### Latin American Currencies (USD)

Table A3 shows the estimates of the extreme linkage between domestic currency depreciations and lagged macroeconomic variables (from lag 1 to lag 12), and asymptotic 95% confidence intervals (in parentheses) using 2.5% of the data. The variables are the exchange rate returns $\Delta s$, relative money supply growth $\Delta m_{i\text{z}}$ relative real income growth $\Delta y_{i\text{z}}$, relative inflation $\Delta p_{i\text{z}}$ and changes in the interest rate differential $\Delta r$. The first column shows the pair of variables under investigation. Positive and negative signs indicate positive and negative relations between the two variables. The depreciation side is on the right tail of the exchange rate returns distribution. Hence, $\Delta s$, $\Delta m_{i\text{z}}$ + represents the linkage between the right tails of the distributions of exchange rate returns and lagged money supply growth, i.e. between large depreciations of the domestic currency and increases in domestic money supply relative to foreign money supply. For all three groups, i.e. Europe, Asia and Latin America, the US dollar (USD) is the base currency and the domestic fundamentals are relative to the US. However, for the European countries, we also consider the variables relative to Germany, and to the eurozone (after the introduction of the euro in 1999) for the countries which do not adopt the euro. The Deutsche mark (DM), and subsequently the euro (EUR), are the base currency.
Table A4: Estimates of Extreme Linkage between Macroeconomic Fundamentals and Lagged Currency Depreciations

<table>
<thead>
<tr>
<th>Base Currency</th>
<th>Europe</th>
<th>Europe</th>
<th>Asia</th>
<th>Latin America</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \bar{m} ), ( \Delta \bar{s}_t )</td>
<td>DM/EUR</td>
<td>USD</td>
<td>USD</td>
<td>USD</td>
</tr>
<tr>
<td>(+)</td>
<td>0.0313</td>
<td>0.0388</td>
<td>0.2394</td>
<td>0.2520</td>
</tr>
<tr>
<td>(-)</td>
<td>(-0.0297, 0.0922)</td>
<td>(-0.0244, 0.1021)</td>
<td>(0.1273, 0.3516)</td>
<td>(0.1351, 0.3690)</td>
</tr>
<tr>
<td>( \Delta \bar{y} ), ( \Delta \bar{s}_t )</td>
<td>+</td>
<td>0.0408</td>
<td>0.0273</td>
<td>0.0305</td>
</tr>
<tr>
<td>(-)</td>
<td>(-0.0286, 0.197)</td>
<td>(-0.0252, 0.0798)</td>
<td>(-0.0210, 0.0821)</td>
<td>(-0.0285, 0.0910)</td>
</tr>
<tr>
<td>( \Delta \bar{p} ), ( \Delta \bar{s}_t )</td>
<td>+</td>
<td>0.0707</td>
<td>0.0818</td>
<td>0.3151</td>
</tr>
<tr>
<td>(-)</td>
<td>(-0.0134, 0.1548)</td>
<td>(-0.0062, 0.1699)</td>
<td>(0.1822, 0.4480)</td>
<td>(0.1629, 0.4062)</td>
</tr>
<tr>
<td>( \Delta \bar{i} ), ( \Delta \bar{s}_t )</td>
<td>+</td>
<td>0.0619</td>
<td>0.0556</td>
<td>0.1724</td>
</tr>
<tr>
<td>(-)</td>
<td>(-0.0242, 0.1479)</td>
<td>(-0.0231, 0.1342)</td>
<td>(0.0599, 0.2849)</td>
<td>(0.0783, 0.2946)</td>
</tr>
<tr>
<td>( \Delta \bar{y} ), ( \Delta \bar{m} )</td>
<td>+</td>
<td>0.0825</td>
<td>0.0556</td>
<td>0.1310</td>
</tr>
<tr>
<td>(-)</td>
<td>(-0.0114, 0.1764)</td>
<td>(-0.0189, 0.1300)</td>
<td>(0.0313, 0.2307)</td>
<td>(0.0140, 0.2063)</td>
</tr>
</tbody>
</table>

Table A4 shows the estimates of the extreme linkage between \( t \) macroeconomic variables and \( t-1 \) domestic currency depreciations, and asymptotic 95% confidence intervals (in parentheses) using 2.5% of the data. The variables are the exchange rate returns \( \Delta s \), relative money supply growth \( \Delta \bar{m} \), relative real income growth \( \Delta \bar{y} \), relative inflation \( \Delta \bar{p} \) and changes in the interest rate differential \( \Delta \bar{i} \). The first column shows the pair of variables under investigation. Positive and negative signs indicate positive and negative relations between the two variables. For instance, \( \Delta \bar{m} \), \( \Delta \bar{s}_t \) represents the linkage between the right tails of the distributions of \( t \) relative money supply growth and \( t-1 \) exchange rate returns. For all three groups, i.e. Europe, Asia and Latin America, the US dollar (USD) is the base currency and the domestic fundamentals are relative to the US. However, for the European countries, we also consider the variables relative to Germany, and to the eurozone (after the introduction of the euro in 1999) for the countries which do not adopt the euro. The Deutsche mark (DM), and subsequently the euro (EUR), are the base currency.

Table A5: Estimates of Extreme Linkage between Currency Depreciations and Lagged Macroeconomic Fundamentals, using quarterly data

<table>
<thead>
<tr>
<th>Base Currency</th>
<th>Europe</th>
<th>Europe</th>
<th>Asia</th>
<th>Latin America</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \bar{s} ), ( \Delta \bar{m}_t )</td>
<td>DM/EUR</td>
<td>USD</td>
<td>USD</td>
<td>USD</td>
</tr>
<tr>
<td>(+)</td>
<td>0.0625</td>
<td>0.0294</td>
<td>0.3191</td>
<td>0.2927</td>
</tr>
<tr>
<td>(-)</td>
<td>(-0.0061, 0.2112)</td>
<td>(-0.0082, 0.1408)</td>
<td>(0.1065, 0.5318)</td>
<td>(0.0853, 0.5001)</td>
</tr>
<tr>
<td>( \Delta \bar{y} ), ( \Delta \bar{m}_t )</td>
<td>+</td>
<td>0.0313</td>
<td>0.0000</td>
<td>0.0233</td>
</tr>
<tr>
<td>(-)</td>
<td>(-0.0071, 0.1339)</td>
<td>(-0.0095, 0.0095)</td>
<td>(-0.0552, 0.1017)</td>
<td>(-0.0825, 0.2700)</td>
</tr>
<tr>
<td>( \Delta \bar{p} ), ( \Delta \bar{m}_t )</td>
<td>+</td>
<td>0.0909</td>
<td>0.0556</td>
<td>0.3750</td>
</tr>
<tr>
<td>(-)</td>
<td>(-0.0836, 0.2654)</td>
<td>(-0.0818, 0.1929)</td>
<td>(0.1622, 0.5878)</td>
<td>(0.1492, 0.5825)</td>
</tr>
<tr>
<td>( \Delta \bar{i} ), ( \Delta \bar{m}_t )</td>
<td>+</td>
<td>0.0313</td>
<td>0.1944</td>
<td>0.2292</td>
</tr>
<tr>
<td>(-)</td>
<td>(-0.0723, 0.1348)</td>
<td>(-0.0238, 0.4127)</td>
<td>(0.0252, 0.4331)</td>
<td>(0.1213, 0.5967)</td>
</tr>
<tr>
<td>( \Delta \bar{y} ), ( \Delta \bar{m} )</td>
<td>+</td>
<td>0.0938</td>
<td>0.0556</td>
<td>0.1250</td>
</tr>
<tr>
<td>(-)</td>
<td>(-0.0825, 0.2700)</td>
<td>(-0.0746, 0.1857)</td>
<td>(-0.0399, 0.2899)</td>
<td>(-0.0651, 0.3215)</td>
</tr>
</tbody>
</table>

Table A5 shows the estimates of the extreme linkage between domestic currency depreciations and lagged macroeconomic variables, and asymptotic 95% confidence intervals (in parentheses) using 2.5% of the data. The variables are \textit{quarterly}. They are the exchange rate returns \( \Delta s \), relative money supply growth \( \Delta \bar{m} \), relative real income growth \( \Delta \bar{y} \), relative inflation \( \Delta \bar{p} \) and changes in the interest rate differential \( \Delta \bar{i} \). The first column shows the pair of variables under investigation. Positive and negative signs indicate positive and negative relations between the two variables. The depreciation side is on the right tail of the exchange rate returns distribution. Hence, \( \Delta \bar{s} \), \( \Delta \bar{m}_t \) represents the linkage between the right tails of the distributions of exchange rate returns and lagged money supply growth, i.e. between large depreciations of the domestic currency and increases in domestic money supply relative to foreign money supply. For all three groups, i.e. Europe, Asia and Latin America, the US dollar (USD) is the base currency and the domestic fundamentals are relative to the US. However, for the European countries, we also consider the variables relative to Germany, and to the eurozone (after the introduction of the euro in 1999) for the countries which do not adopt the euro. The Deutsche mark (DM), and subsequently the euro (EUR), are the base currency.
Table A6: Estimates of Extreme Linkage between Currency Depreciations and Lagged Macroeconomic Fundamentals, excluding crisis episodes

<table>
<thead>
<tr>
<th></th>
<th>Whole sample</th>
<th>Whole sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) Incl. crises obs.</td>
<td>(2) Excl. crises obs.</td>
</tr>
<tr>
<td>( \Delta s ), ( \Delta m_{-i} ) +</td>
<td>0.3913</td>
<td>0.0647</td>
</tr>
<tr>
<td></td>
<td>(0.3009, 0.4817)</td>
<td>(0.0158, 0.1137)</td>
</tr>
<tr>
<td>Obs.</td>
<td>12881</td>
<td>11124</td>
</tr>
<tr>
<td>( \Delta s ), ( \Delta y_{-i} ) -</td>
<td>0.0205</td>
<td>0.0307</td>
</tr>
<tr>
<td></td>
<td>(-0.0078, 0.0487)</td>
<td>(-0.0055, 0.0668)</td>
</tr>
<tr>
<td>Obs.</td>
<td>11705</td>
<td>10448</td>
</tr>
<tr>
<td>( \Delta s ), ( \Delta p_{-i} ) +</td>
<td>0.4377</td>
<td>0.0737</td>
</tr>
<tr>
<td></td>
<td>(0.3483, 0.5271)</td>
<td>(0.0224, 0.1250)</td>
</tr>
<tr>
<td>Obs.</td>
<td>13160</td>
<td>11390</td>
</tr>
<tr>
<td>( \Delta s ), ( \Delta i_{-i} ) +</td>
<td>0.3220</td>
<td>0.0599</td>
</tr>
<tr>
<td></td>
<td>(0.2325, 0.4114)</td>
<td>(0.0123, 0.1074)</td>
</tr>
<tr>
<td>Obs.</td>
<td>13372</td>
<td>11362</td>
</tr>
<tr>
<td>( \Delta s ), ( \Delta i_{-i} ) -</td>
<td>0.1362</td>
<td>0.0528</td>
</tr>
<tr>
<td></td>
<td>(0.0730, 0.1994)</td>
<td>(0.0089, 0.0967)</td>
</tr>
<tr>
<td>Obs.</td>
<td>12929</td>
<td>11362</td>
</tr>
</tbody>
</table>

Table A6 shows the estimates of the extreme linkage between domestic currency depreciations and lagged macroeconomic variables, and asymptotic 95% confidence intervals (in parentheses) using 2.5% of the data. The purpose is to demonstrate that the results of extreme linkages are influenced by crisis episodes. The left column (1) shows the estimates using the entire observations, while the right column (2) shows the results when excluding crisis episodes, dubbed freely falling regime, classified in Ilzetzki, Reinhart and Rogoff (2008). The variables are the exchange rate returns \( \Delta s \) relative money supply growth \( \Delta m_{-i} \) relative real income growth \( \Delta y_{-i} \), relative inflation \( \Delta p_{-i} \) and changes in the interest rate differential \( \Delta i_{-i} \). The first column shows the pair of variables under investigation. Positive and negative signs indicate positive and negative relations between the two variables. The depreciation side is on the right tail of the exchange rate returns distribution. Hence, \( \Delta s \), \( \Delta m_{-i} \) + represents the linkage between the right tails of the distributions of exchange rate returns and lagged money supply growth, i.e. between large depreciations of the domestic currency and increases in domestic money supply relative to foreign money supply. For all three groups, i.e. Europe, Asia and Latin America, the US dollar (USD) is the base currency and the domestic fundamentals are relative to the US. However, for the European countries, we also consider the variables relative to Germany, and to the eurozone (after the introduction of the euro in 1999) for the countries which do not adopt the euro. The Deutsche mark (DM), and subsequently the euro (EUR), are the base currency.
Figure A1: Conditional Probability of Large Currency Depreciations

$$P(r > q | p, \Delta x > p)$$ (on the Y-axis) as a function of $$\Delta p$$, with $$p$$ the percentile of the variable $$\Delta x$$. That is when the shock becomes large $$q(p) \rightarrow \infty$$, on the X-axis $$1-p \rightarrow 1$$. The macroeconomic variables are relative money supply growth $$\Delta m$$, relative real income growth $$\Delta y$$, relative inflation $$\Delta p$$ and changes in the interest rate differential $$\Delta i$$. Positive and negative signs indicate positive and negative relations between the two variables.