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Memorandum 71-6
April 1971

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ENERGETICOS, A Process Analysis of Mexico's Energy Sectors*

Contents:

		Page Number
1.	Introduction	1
2.	Simplifications--time and space	3
3.	Structure of the programming model	5
4.	Iron and steel industry	10
5.	Electricity industry	13
6.	Petroleum and gas industry	24
7.	Sensitivity analyses--discount rate and foreign exchange premium	33
	Appendix. Project indivisibilities	39

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* This will form a chapter in a forthcoming book tentatively titled "Project Decisions and Multi-Level Planning: Case Studies in Mexico". The research was supported by the Comisión Federal de Electricidad and by the International Bank for Reconstruction and Development. The specific facts, methods of analysis, and conclusions are the sole responsibility of the individual authors.

** The authors are much indebted to: Ing. Raúl Meyer (Petróleos Mexicanos), Ings. Victor L. Games and Carlos Velasco (Instituto Mexicano del Petróleo); Lic. Gerardo Bueno and Ing. Fernando González Vargas (Nacional Financiera, S.A.); Ings. Juan Eibenschutz, Roberto Fabre, and José Alberto Valencia, and Lic. Bosco Muro (Comisión Federal de Electricidad); Mr. Richard Inman (Stanford University).

1. Introduction.

ENERGETICOS is one in a series of multi-level planning models. In its level of aggregation, it forms an intermediate stage between INTERCON (which is geographically disaggregated within the electric power industry) and DINAMICO (a dynamic input-output model of the entire Mexican economy). Together, this series is intended to define the information flows necessary for consistency between project, sectoral and macroeconomic decisions.

The sectoral model ENERGETICOS allows explicitly for substitution between alternative processes within Mexico's energy-producing industries: petroleum, gas, and electricity.* Also included are alternative energy-using activities for iron and steel manufacturing. Except for process substitution within the steel industry, we have not attempted to quantify the elasticity of demand with respect to the price of energy products. It is supposed that the energy sectors are price-takers with respect to their inputs (e.g., foreign exchange, labor, capital equipment and nuclear fuel), but that they are output-takers with respect to deliveries elsewhere within the Mexican economy.

Here, in projecting the exogenous final demands for each item, trend extrapolation methods were used. These projections were intended to be consistent with a 6.5% annual future growth rate of Mexico's gross domestic product. The demand projections have been compared against the results of DINAMICO. Because of the input-output categories employed in the more aggregative model (e.g., "basic metals", not "steel ingots"), it was not possible

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*The scope of ENERGETICOS is similar to that of an energy sector model developed by the Ministry of Power, United Kingdon. See Forster and Whitting (1968).

to arrive at a fully satisfactory cross-check between the two levels of planning.

For ENERGETICOS, the optimization is phrased as one of choosing investments in alternative processes so as to meet the output targets at minimum discounted costs, taking account of intersectoral flows. Each solution generates a set of incremental costs for supplying each product.

With this formulation, the principal source of interdependence arises from the intersectoral flows of industrial fuel. (By "industrial fuel", we mean natural gas and residual fuel oil converted into their calorific equivalents.) Both gas and residual fuel oil are supplied under conditions of increasing marginal cost within the petroleum industry. There are tradeoff possibilities between these fossil fuels and the introduction of nuclear plants within the electric power industry. The rate of nuclear power development could make a significant difference in the efficiency price of fuel. Similarly, within the steel industry, there are process alternatives which could have a sizeable impact upon the energy sector.

Several institutional features are to be noted at this point: Both the fossil fuel producer (Petróleos Mexicanos) and the electric utility consumer (Comisión Federal de Electricidad) are publicly-owned enterprises. The transfer price of industrial fuel is determined administratively at a high political level, and is not closely linked to the international fuel price prevailing in the Carribean area. During the period since the petroleum expropriation of 1938, Mexico's policy has been to remain largely self-sufficient with respect to imports and exports of petroleum products, and to stimulate industrialization through low prices for energy products.

For investment planning within Mexico's energy sector, the major uncertainty appears to be the future of exploration and producing costs for domestic crude oil and natural gas. Now that domestic sources of petroleum are becoming increasingly costly, one of the key policy issues is a reexamination of the country's tradition of self-sufficiency. This in turn has macroeconomic implications with respect to the balance-of-payments, and could affect the efficiency price of foreign exchange.. Raising the price of energy products could slow down the growth of those demands that are here regarded as predetermined.

2. Simplifications--time and space.

Principally because of uncertainties in predicting the location of future domestic oil and gas production, ENERGETICOS does not contain a regional dimension. Each process is idealized as though it were located at a single representative point in space. This simplification has the further advantage that it reduces the size of computations, and it eliminates those detailed investigations that would be needed to correct the transport costs for anomalies within the existing structure of pipeline and railway rates. The simplification means, however, that the present model cannot be employed directly for the choice of optimal plant locations, and that there is a need for multi-level planning.

Consider, for example, the choice between a coke-intensive process versus an electricity-intensive process in the steel industry. At a specific project location, the analyst can readily allow for the fact that the relative advantage of these two alternative processes will depend upon the transport

costs from the coke producing areas of northeast Mexico. This is an obvious case in which geographical disaggregation can be more readily handled at the level of project selection than at a macroeconomic level. For a static model in which there is a simultaneous evaluation of site and process alternatives within the steel sector and an attempt to correct for rail rate anomalies, see C. Quintana, G. Bueno, and F. González Vargas (1968). Similarly, for a geographically disaggregated analysis of the electric power sector--one in which transmission projects are evaluated simultaneously with generating plants--see INTERCON, Memorandum 71-5.

The dimension of time plays a key role in any investment model. Here the initial year is chosen as 1974, the earliest date for which the installed capacities are not already (as of 1970) predetermined by past investment decisions and time lags. For the period 1974-80, material balances are computed year-by-year. In the electric power sector, three additional five-year time intervals are considered--the first extending over 1981-85, the second 1986-90, and the third 1991-95. This time horizon is sufficiently long to allow for the eventual success of breeder reactor technology, and hence a sharp future decrease in the worth of the initially installed nuclear plants. For the entire 15 year period 1981-85, it is supposed that the price of fossil fuel is a datum to the electric power sector. For the near future period 1974-80, the marginal cost of fossil fuel is one of the elements to be determined simultaneously with the other unknowns of the programming problem.

Annual disaggregation proves to be particularly useful for analyzing the temporary imports and exports which need to be synchronized with the installation of major indivisible units of capacity. See the Appendix, where

indivisibilities are introduced through mixed integer programming. Elsewhere, with linear programming as the solution technique, indivisibilities have been neglected.

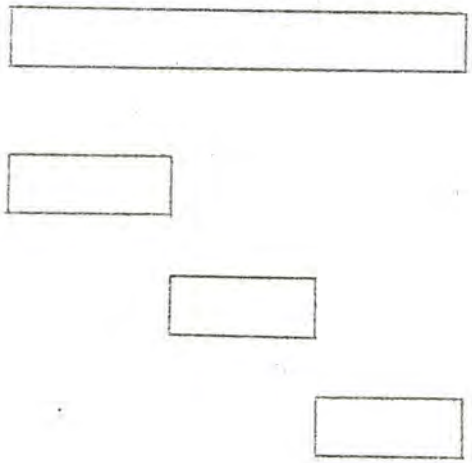
Because of the programming framework, a further difficulty arises. Throughout, we have employed "certainty equivalents". E.g., instead of regarding water inflows to the hydroelectric system as a probabilistic phenomenon, these are taken at their mean values. Similarly, instead of allowing for the influence of plant size upon the electricity generating system's reliability with respect to unscheduled downtime, we have taken an arbitrary downtime factor of 12% for reserve capacity. For studies of the Mexican electric power sector which handle one or another of these stochastic phenomena explicitly, see Stanford Research Institute (1968), and Rogers (1970). The computational techniques employed in these references cannot be readily extended to multi-sector optimal investment planning--hence the use of deterministic programming methods here.

3. Structure of the programming model.

From Figure 1, it will be seen that the coefficients of ENERGETICOS may be arranged in block-angular form--one block corresponding to each of the three industries: petroleum and gas (PE), electricity (EL), and iron and steel (SD). The only intersectoral flows consist of three items: industrial fuel, peak power and electrical energy. Once the interfaces were defined through these common items (prefix CM), it proved possible to construct and

Figure 1. Block-angular form of coefficients matrix for ENERGETICOS

Block	Number of rows in block	Prefix employed for row identification
Common items	27	CM
Petroleum and gas	67	PE
Electricity	48	EL
Iron and steel	71	SD
Total	213	



to test the sub-models independently.* The following sections describe the sub-models in increasing order of their complexity--first steel, then electricity and finally petroleum.

Table 1 identifies the 213 rows that appear within ENERGETICOS.** In addition, there are 15 rows that have been employed for convenience in converting future costs into their present value, and for converting foreign exchange costs into their domestic value. Table 2 gives the structure of this cost definition matrix, and indicates the numerical values adopted for the cost parameters: a 10% annual discount rate*** and a 15% level of protection against imports, converting foreign exchange into domestic currency at the conventional rate of 12.5 pesos per U. S. dollar. These cost parameters are similar to those currently employed for project evaluation by public sector enterprises. One of the later sections of this report

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*Since the energy sector was viewed as a price-taker for capital, foreign exchange, etc., the cost of these items entered into the objective function, but not into the common (CM) rows.

**A detailed computer listing is available for inspection at the office of either of the authors and also at the IBRD Development Research Center. A simplex solution required 2 minutes on the IBM 360/67 at Stanford University.

***If capital were allocated throughout the Mexican economy so that the rate of return was identical in all sectors, the macroeconomic model DINAMICO suggests that 15% would probably be more appropriate than a 10% discount rate. Given the present structure of Mexico's financial institutions, however, the cost of capital to public sector enterprises in the petroleum, electricity and steel industries is lower than that to private enterprises in such sectors as commerical agriculture, tourism, and motor vehicle manufacturing.

Table 1. Row identification for ENERGETICOS

Sector	Number of rows in sector	Prefix of sector employed for row identification	Suffix of row identification (omitting time index t)	Description of row	Unit of measurement (annual flow)	Exponent of 10 for unit of measurement
Common items	27 = 1x7 +2x10	CM	BTU	c/ industrial duel	BTU	12
		CM	GW	c/ peak power	GW	0
		CM	TWH	c/ electrical energy	TWH	0
Petroleum and gas	67a/ = 7x7 +3x6	PE	GSC	dry gas (fuel oil equivalent)	barrel	6
		PE	CRD	crude oil	barrel	6
		PE	GSR	refinery gas (fuel oil equivalent)	barrel	6
		PE	CPL	c/ liquefied gas	barrel	6
		PE	GSL	c/ gasoline	barrel	6
		PE	KD	c/ kerosine and diesel oil	barrel	6
		PE	COM	residual fuel oil	barrel	6
		PE	CPj	capacity irreversibility - process j (j=C,H,V)	barrel	6
Electricity	48b/ = 4x10 +1x8	EL	TRA	requirements for additional transmission capacity	GW	0
		EL	RQF	industrial fuel requirements - electricity sector	BTU	12
		EL	CNV	convex curve - initial fossil	-	-
		EL	LMF	energy limit - new fossil	TWHR	0
		EL	LMN	energy limit - early nuclear	TWHR	0
Iron and steel	71 = 5x7 +6x6	SD	RQF	industrial fuel requirements - steel	BTU	12
		SD	MNH	iron ore	ton	6
		SD	CHAT	ferrous scrap	ton	6
		SD	COQ	coke	ton	6
		SD	LIN	c/ steel ingots	ton	6
		SD	CPj	capacity irreversibility - process j (j=1...6)	ton	6

time index t	0	1	2	3	4	5	6	11	16	21
year	'74	'75	'76	'77	'78	'79	'80	'85	'90	'95

Notes: a/ Excludes upper bounds on individual unknowns referring to domestic availability of dry gas and crude oil at three progressively higher levels of marginal cost.

b/ Excludes lower bounds on individual unknowns referring to installation of fossil units in geographically isolated systems.

c/ Demand projected exogenously - either in part or in total

Table 2. Cost definition matrix

(at 1970 prices and conventional exchange rate of 12.5 pesos per U.S. dollar)

Row description	Unit of measurement	Row identification	costs incurred in year t CSt	discounted value of import costs CSMVA	import component of costs incurred in year t CSMCORt
minimand	10^6 pesos discounted to year 0 (1974) at 10% per year	CSVA	$(.909)^t$	.15	
costs, year t	10^6 pesos, year t	CSCORt	1.000		
discounted value of import costs	10^6 pesos discounted to year 0 (1974) at 10% per year	CSMVA		-1.000	$(.909)^t$
import component of costs incurred in year t	10^6 pesos, year t	CSMCORt			1.000

contains a sensitivity analysis with respect to the discount rate and the shadow price of foreign exchange.

To avoid "horizon effects", initial investment costs have been converted into annual flows through a capital recovery factor determined by the service life of the capital equipment and the 10% opportunity cost of capital. Beginning with the year in which capacity is first available, the discounted annual flows are then summed over the balance of the planning horizon.

4. Iron and steel industry.

Within the iron and steel industry, the principal opportunities for substitution between energy sources occur prior to the rolling stage. It is for this reason that the rolling processes have been omitted from the optimization here, and that steel ingots are viewed as the end product. Following Bueno and González Vargas (1969), it is estimated that the demand for ingots will increase at the compound annual rate of 9.1% over the period 1974-80.*

To satisfy the demands for new ingot capacity, six alternative processes are considered--each corresponding to an integrated plant size of 1.0 million tons per year. (See Table 3.) For each activity SDjt, note that the cost coefficients in the row CSCORt include not only current account expenses such as labor, but also a capital recovery factor of 13% for

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*In their demand estimates, Bueno and González Vargas made separate projections for flat and non-flat products and for seamless tubes, and then converted these into their ingot equivalents.

For 1974, the demand is taken at 6.237 million ingot tons. The existing plus planned production capacity in that year is taken at 5.180 million tons, excluding the project that is under consideration for Las Truchas.

Table 3. Detached coefficients matrix for iron and steel industry, year t (t = 0, 1,···,6) a/

row identification (see Tables 1 and 2 for units of measurement) \ activities	definition of fossil fuel and raw material inputs for new ingot capacity				inputs and outputs from six alternative integrated processes for 1.0 million tons/year of ingots						Unit cost of inputs 1977 shadow prices associated with 10% annual discount rate, 15% preference to domestic inputs, and conventional exchange rate of 12.5 pesos per U.S. dollar	inputs 1970 conventional prices
	SDRQFt	SDMNHt	SDCHATt	SDCOQt	SD1t	SD2t	SD3t	SD4t	SD5t	SD6t		
CSCORt b/		-80.	-500.	-300.	-595.	-459.	-536.	-550.	-418.	-416.	1.00	1.00
CSMCORt b/			-500.		- 51.	- 50.	- 61.	- 37.	- 55.	- 27.	.15	0
CMCWt					- .056	- .267	- .013	- .101	- .006	- .007	597.11	0 c/
CMFWt					- .718	- 2.000	- .100	- .760	- .047	- .500	37.35	100.00 c/
SDRQFt	1.0						- 4.509	- 2.950	- 1.640		3.93	5.00
SDMNHt		1.0			- 1.155	- 1.320	- 1.237	- 1.237	- 1.320		80.00	80.00
SDCHATt			1.0		- .325	- .275	- .275	- .275	- .275	- 1.050	675.	500.00
SDCOQt				1.0		- .330	- .400		- .650	- .050	300.	300.00
SDLINt					1.000	1.000	1.000	1.000	1.000	1.000		
PEGSGt					- 2.710						24.87	18.40
marginal cost of ingots (based on shadow prices for 1977) — pesos per ton					1033	1063	952	913	897	1097		
marginal cost of ingots (based on conventional input prices for 1970) — pesos per ton					972	1001	925	877	869	1006		
energy inputs, % of marginal cost (based on conventional input prices for 1970)					13%	30%	17%	10%	24%	6%		

Notes:

a/ Source: Ing. Fernando González Vargas, Nacional, Financiera, S.A.

b/ The unit costs (CSCORt) include a gross return on initial investment of 19%/year (allowing for depreciation and opportunity cost of capital at 13%, plus 6% for maintenance, overhead and insurance) plus labor costs at 96,360 pesos per man-year of direct labor (allowing for overhead, supervision etc. at 1.2 times the 1970 direct labor costs) plus miscellaneous inputs (e.g. limestone). The import component (CSMCORt) of unit costs is taken as 5%/year on the initial investment costs.

The six alternative integrated processes are defined as follows

		primary iron/scrap ratio
1.	sponge iron (HYLSA) + electric steel furnace	70/30
2.	electroreduction furnace + LD converter (COROX)	75/25
3.	prereduced pellets (SL-RN) + blast furnace + LD Converter (COROX)	75/25
4.	prereduced pellets (SL-RN) + electric steel furnace	75/25
5.	blast furnace + LD converter (COROX)	75/25
6.	non-integrated electric steel furnace	0/100

c/ The conventional electricity price of 100 pesos/10^3 KWH is not directly comparable with the separately calculated marginal costs for energy and peak power. In order to compare the two for a round-the-clock major industrial consumer operating 7.5 10^3 hours per year, divide the peak power costs by 7.5, and add to the electricical energy costs shown above. Thus for 1977, the full costs would be (597.11/7.5) + 37.35 = 116.96.

depreciation plus opportunity cost of capital. With capital costs converted into annual costs, in order to ensure the irreversibility of investment, the process choices in successive time periods are linked by the following constraints:

$$(SDCPjt) \quad - SDj,\ t+1 \geq SDj,\ t \qquad (t = 0,\ldots,5)$$

To simplify presentation, the intermediate iron product (either pig iron or sponge iron) has been netted out from Table 3, and the six alternative processes are each viewed as integrated from raw materials through the ingot state. It is supposed that the raw materials (iron ore, scrap and coke) will be available in unlimited quantities at a constant marginal cost. From the coefficients in the column SDCHATt, and row CSCORt, it will be seen that the input price for steel scrap (chatarra) is assumed to be 500 pesos per ton. Since Mexico will continue to be an importer of scrap during the 1970's, this entire amount is also viewed as a foreign exchange cost (row CSMCORt).

It is <u>not</u> assumed that industrial fuel will continue to be available to the steel industry at a constant marginal cost. The activities SDRQFt define the steel industry's fossil fuel requirements (either in the form of residual fuel oil or natural gas), translating these requirements into their British thermal unit equivalents. These are the activities that link process choices within the steel and the petroleum industries.

As things turn out, the optimal steel process choice is insensitive to the foreign exchange premium and the discount rate. In letting the discount rate range from 10% to 20% per year, and in letting the value of

foreign exchange vary between the conventional exchange rate of 12.5 and the shadow price of 20.0 pesos per U.S. dollar, the optimal investment choice within the steel industry remains process 5: blast furnace + LD converter (CONOX).

For comparative purposes, Table 3 indicates the marginal cost of ingot production based both upon conventional 1970 input prices and also based upon the 1977 dual variables. Note that there is only a small difference between the marginal costs of process 4 and 5, and that this difference is well within the range of likely errors in estimation. According to Table 3, there is a 12-15% difference between the marginal cost of ingots produced via the optimal process (#5) and the sponge iron process (#1). In order to reconcile this with the fact that a major new sponge iron project is under construction, one must recognize that the project has received investment inducements via low input prices for natural gas. One must also recognize that economies-of-scale are less pronounced with the sponge iron and with the electric steel furnace processes than with the others.

5. Electricity industry.

ENERGETICOS is based upon results from earlier, more detailed models of Mexico's electric power industry. (See Stanford Research Institute (1968), Fernández and Manne (1969), and Rogers (1970).) For example, the demand for electricity is characterized here in terms of just two parameters of the annual load-duration curve: the annual peak power and the annual

energy. This representation of demand is too aggregative to be altogether satisfactory for planning investments within the electric power sector. For example, in INTERCON (Memorandum 71-5), the decision problem entails tradeoffs between transmission and generating facilities. For evaluating these decisions, the annual load-duration curve was represented in terms of eight parameters: the peak power demand during each of two seasons plus the energy demand during six blocks of time. With interconnection between regions, it was then found that one of the two seasonal peaks dominated the other. Moreover, there was sufficient flexibility of hydroelectric operations so that--within a given region--the value of energy did not vary significantly between the six blocks of time.

On the basis of this prior information, no distinction is drawn here between time blocks within a year. Kilowatt-hours of electrical energy are aggregated into a single annual total. This simplifies the analysis of linkages between the electricity and the petroleum sector, but this also means that ENERGETICOS cannot solve those investment problems that relate to geographical tradeoffs between transmission and generating facilities. With the multi-level structure of INTERCON and ENERGETICOS, the facility location problem is handled as a separate sub-optimization.*

The more detailed models of the electric power industry have suggested that no great error would be associated with several further simplifications: regarding the water inflows to the hydroelectric system as deterministic rather than probabilistic quantities, the omission

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*Sub-optimization reduces the difficulties of data-gathering and of computations, but it is risky whenever there are significant interactions between the components of a system. Cf. Hitch (1953).

of gas turbines for any purpose other than reserve capacity, a factor of 12% for reserve capacity, and the predetermined sizes selected for fossil and nuclear generating units. Although such simplifications would be unsatisfactory for investment planning within the electric power industry itself, they appear defensible for examining the linkage between electricity, petroleum and steel investment decisions.

ENERGETICOS is constructed as though the only process choice for new electrical generating capacity were nuclear power versus fossil fuel plants, and as though the only fossil fuels were residual oil and natural gas. Diesel units, coal burning plants and geothermal plants each have their place in Mexico's power grid, but are not expected to supply a major percentage of the new capacity. We have omitted investment choices for new hydroelectric projects--partly because Mexico is close to exhausting her supply of low-cost hydroelectric sites and partly because hydro project costs have been consistently underestimated in the past.

The electricity sector's constraints are written out in Table 4.* The first three sets of constraints (CMGWt), (CMTWHt) and (ELTRAt) refer to satisfying the peak, energy and transmission demands, after deducting the supplies available from the initial hydro and fossil installations. (See Table 5 for the numerical values of the exogenously specified quantities.)

Not shown explicitly in Table 4 are the lower bounds imposed on the individual unknowns FOSt. These lower bounds insure that fossil capacity

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*This programming model resembles a number of those that have been constructed at Electricité de France. (See Massé and Gibrat (1957) and Bessière (1959).)

Table 4. Constraints for electric power industy

($t = 0, 1, \cdots, 6, 11, 16, 21$)

(CMGWt) cumulative new fossil capacity + cumulative nuclear capacity

$$\sum_{\tau=0}^{t} FOS_\tau + \sum_{\tau=2}^{t} NUC_\tau \geq \text{peak final demand} - \text{initial fossil capacity} - \text{hydro capacity}$$

(CMTWHt) generation from fossil: initial + new, minimum + new, above minimum; generation from nuclear: early, minimum + early, above minimum + late at maximum

$$\sum_{k=1}^{5} (\alpha_k)\, INT\,kt + 1.971 \sum_{\tau=0}^{t} FOS_\tau + XF_t + 1.861 \sum_{\tau=2}^{\min(t,6)} NUC_\tau + XN_t + 7.446 \sum_{\tau=11}^{t} NUC_\tau \geq \text{energy demand} - \text{hydro energy}$$

(ELTRAt) requirements for additional transmission capacity = $TRA_t = FOS_t + NUC_t$

(ELLMFt) energy generated in new fossil plants, above minimum = $XF_t \leq 5.913 \sum_{\tau=0}^{t} FOS_\tau$

(ELLMNt) energy generated in early nuclear plants, above minimum = $XN_t \leq 5.585 \sum_{\tau=2}^{\min(t,6)} NUC_\tau$

(ELCNVt) convex curve - initial fossil $\sum_{k=1}^{5} INT\,kt = 1$

(ELRQFt) industrial fuel requirements = $RQF_t = \sum_{k=1}^{5} (\beta_k)\, INT\,kt + \left(\text{heat rate, new fossil}\right)\left(1.971 \sum_{\tau=0}^{t} FOS_\tau + XF_t\right)$

Notes: Nuclear fuel inputs enter as cost coefficients for unknowns XNt , NUCt
Prefix EL precedes identification of all unknowns shown above.

Table 5. Predetermined demands and capacities, electricity industry

period t	year	annual peak (GW)				annual energy (TWH)		
		peak final demand[a]	initial fossil capacity	hydro capacity	cumulative requirements for new peak capacity[b]	final demand[a]	hydro energy	demand for non-hydro
0	1974	7.74	4.04	3.88	0.67	39.00	14.25	24.75
1	1975	8.52	4.04	4.20	1.16	43.40	14.70	28.60
2	1976	9.48	4.04	4.35	1.99	47.50	16.40	31.10
3	1977	10.52	4.04	4.35	3.03	53.00	16.23	36.77
4	1978	11.65	4.04	5.27	3.31	59.50	20.96	38.54
5	1979	12.78	4.04	5.27	4.47	66.00	20.96	45.04
6	1980	14.17	4.04	5.27	5.86	74.00	20.96	53.04
11	1985	23.13	4.04	5.27	14.82	152.00	20.96	131.04
16	1990	36.52	4.04	5.27	28.21	208.00	20.96	187.04
21	1995	57.39	4.04	5.27	49.06	330.00	20.96	309.04
%/year growth of demand, 1974-80		10.6%				11.3%		

[a] Neglecting endogenous electricity demands.

[b] Requirements for new peak capacity = final demand - $(\frac{1}{1.12})$ (initial fossil + hydro capacity)

will be installed in geographically isolated regions that are not interconnected with major grids.

Most of the details of Table 4 are connected with the phenomenon of service shifting. That is, over time it can be anticipated that new thermal plants will be added to the system, and that these will have progressively lower energy costs. When a thermal unit enters into service, it is likely to operate as a base-load unit (i.e., at full capacity). The fluctuations in the daily load curve will be absorbed by older thermal plants or by hydro units. As time passes, it is typical that a thermal unit (whether a fossil or a nuclear plant) will be utilized at successively lower levels of capacity.

One formulation that provides for optimal service shifting may be illustrated in the case of new fossil fuel capacity. For this process choice, it is supposed that there are two types of unknowns: FOSτ and XFt. The former denotes the peak capacity installed in period τ, and the latter denotes the above-minimum energy to be produced by new fossil plants in period t. Plants of this type cannot be operated at more than 90% of the 8,760 hours in the year. In order for the boilers to be fired up so as to meet the annual and daily peak, these units cannot be operated for less than 25% of the time. This accounts for the energy coefficient of FOSτ shown in the constraints CMTWHt. ((8.760)(.90)(.25) = 1.971 thousand hours per year.) During the remaining hours that these fossil units are available, one kilowatt of capacity cannot produce more than 5.913

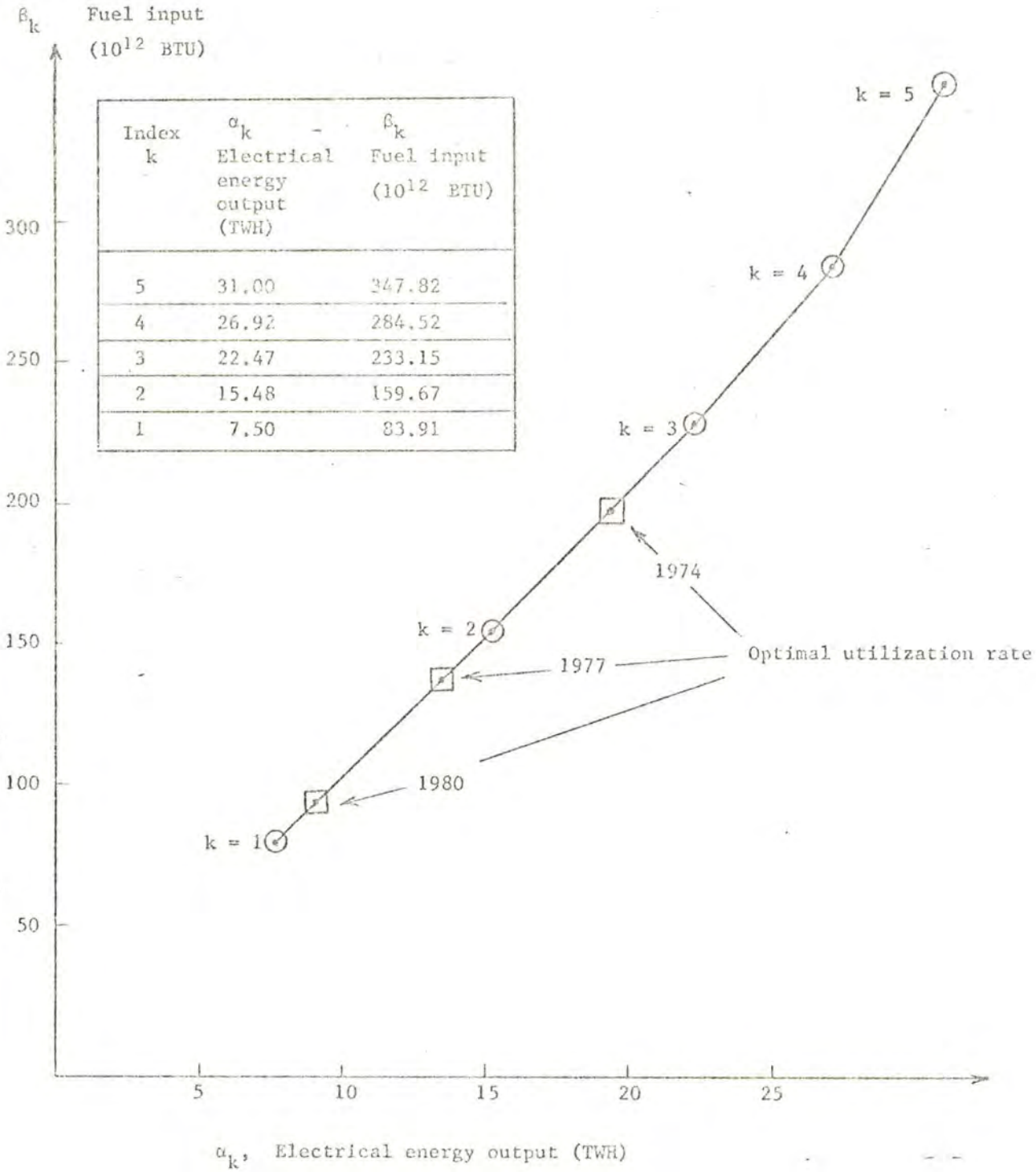
thousand kilowatt-hours of energy--hence the coefficient of FOSτ in the energy limit ELLMFt.

For early nuclear power plants, a similar provision is made for service shifting. ("Early" means those nuclear plants installed between years 2 and 6, 1976-80). The only difference between the coefficients for new fossil and early nuclear plants is that the latter are taken to have an annual availability factor of 85% rather than 90%. For the late nuclear plants (i.e., those installed between 1981 and 1995), it is supposed that these would be operated only for base-load purposes, and that service shifting may be neglected.

To describe the operations of the initially available fossil fuel plants, a preliminary sub-optimization was employed to reduce the number of rows in ENERGETICOS. Because of differences between the thermal efficiency of these units, there is a technologically determined "merit order" for their use. In order to minimize fossil fuel consumption--without increasing any other component of costs--kilowatt-hours would first be produced in the most efficient units up to their maximum capacity, then in the next most efficient units, and so on. Since the capacities of these units are already known, this leads to a predetermined relationship between the energy output and the fossil fuel input. (See Figure 2.) To summarize this convex relationship, five levels of output are specified, and these are identified through the index $k = 1, 2, \ldots, 5$.

From Figure 2, it can be seen that for each value of k there is a predetermined value of the total energy output (the coefficient α_k) and of the total fuel input (the coefficient β_k). The linear programming

Figure 2. Fuel input versus electrical energy generated by initially available fossil power stations



Index k	α_k - Electrical energy output (TWH)	β_k Fuel input (10^{12} BTU)
5	31.00	347.82
4	26.92	284.52
3	22.47	233.15
2	15.48	159.67
1	7.50	83.91

unknowns are not these inputs and outputs directly, but rather the "interpolation weight" variables INT kt. Each unknown INT kt appear in three constraints: (CMTWHt), (ELRQFt), and (ELCNVt). The convex curve constraints (ELCNVt) ensure that the interpolation unknowns will add up to unity for each time period t.* During year 3 (1977), for example, it is optimal for the initial plants to produce 13.64 TWH. This is obtained by setting the unknowns INT 13 = .23 and INT 23 = .77. The corresponding energy output is interpolated through the coefficients α_1 and α_2 in the energy constraint (CMTWH3). Similarly, the fossil fuel input requirement is interpolated through the coefficients β_1 and β_2 in the fuel constraint (ELRQF3). On Figure 2, note the decline in the optimal utilization rate for the older fossil units as lower-cost plants become available between 1974 and 1980.

In order to complete the input data for the electric power industry, cost coefficients must be calculated for each activity. These have been derived from the project characteristics given in Table 6, interpolating linearly for investment and operating costs within each of the three successive five-year intervals 1980-85, 1985-90, and 1990-95.

With these numerical assumptions on costs, demands, utilization rates, etc., the optimal installation program for generating capacity is given in Table 7. As is characteristic of linear programming, the solution

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*The process of optimization, together with the convex shape of Figure 2, ensures that it will never be optimal to assign positive intensities to two non-adjacent interpolation variables. E.g., it will not be optimal to set INT 31 > 0 and simultaneously INT 51 > 0. A reduction in fuel inputs could then be achieved by increasing the intensity of INT 41, and reducing both INT 31 and INT 51.

Table 6. Costs in the electricity sector

		Generating Plants:					
	Unit size	Fossil .300 GW	Fossil 1.000 GW	Nuclear .600 GW	Nuclear 1.000 GW	Nuclear Breeder 1.000 GW	Transmission + gas turbines for reserve capacity + overhead costs per GW of transmission capacity
I	Initial investment						
I.1	Total (M pesos)	426	1200[a/]	1374[b/]	2000[b/]	2000[c/]	1454
	annual change (%)	-0.5[a/]	-0.5[a/]	-1.0[b/]	-1.0[b/]	-2.0[c/]	0.0
I.2	Unit cost (M pesos/GW)	1426	1200	2290	2000	2000	1454
I.3	Domestic cost component (%)	65.0	50.0	25.0[b/]	25.0[b/]	25.0[c/]	70.0[a/]
	Annual change (%)	0.0	0.0	1.0[b/]	1.0[b/]	1.0[c/]	1.5[a/]
II	Unit fuel cost (M pesos/TWH)	35.55[d/]	35.33[d/]	16.25	15.75	8.37	
	Annual change (%)	0.0	0.0	-1.0	-1.0	-2.0	
III	Fixed operating costs (M pesos/year)	9.40	18.00	22.20	25.50	25.50	283.7
	Domestic cost component (%)	100	100	100	100	100	100
IV	First year available	1974	1981	1976	1981	1991	1974

Notes: [a/] From 1974
[b/] From 1976
[c/] From 1986
[d/] Valuing fossil fuel at 3.95 pesos per MBTU from 1981

Abbreviation: M = 10^6

Table 7. Optimal program for installation of generating capacity (unit: GW installed per year)

year	unit size	Fossil .300 GW	Fossil 1.000 GW	Nuclear .600 GW	Nuclear 1.000 GW	Nuclear Breeder 1.000 GW
1974		.676	n.a.	n.a.	n.a.	n.a.
1975		.493	n.a.	n.a.	n.a.	n.a.
1976		.050[a/]	n.a.	.784	n.a.	n.a.
1977		.026[a/]	n.a.	1.018	n.a.	n.a.
1978		.043[a/]	n.a.	.241	n.a.	n.a.
1979		.058[a/]	n.a.	1.107	n.a.	n.a.
1980		.064[a/]	n.a.	1.331	n.a.	n.a.
1981 - 85		n.a.	0	n.a.	1786	n.a.
1986 - 90		n.a.	0	n.a.	2678	n.a.
1991-95		n.a.	1.133	n.a.	n.a.	3.037

Notes:

n.a. = not available

[a/] Predetermined lower limit to ensure that fossil capacity will be installed in geographically isolated regions that are not interconnected with the major grids for the south - central and the northern systems.

tends to be of a bang-bang form: fossil units installed at their lower limit during 1976-80 and nuclear units for the balance of the capacity requirements between 1976 and 1990. During the period in which breeders first become available (1991-95), it is optimal to install a mix of fossil and breeder units. (If pumped-storage activities had been included explicitly within ENERGETICOS, it is likely that it would be optimal to install these rather than fossil units at the same time as breeders.) During 1976-80, the period of immediate interest, note that the annual capacity increments would be of the order of one or two nuclear units, each of .600 GW size.

6. Petroleum and gas industry.

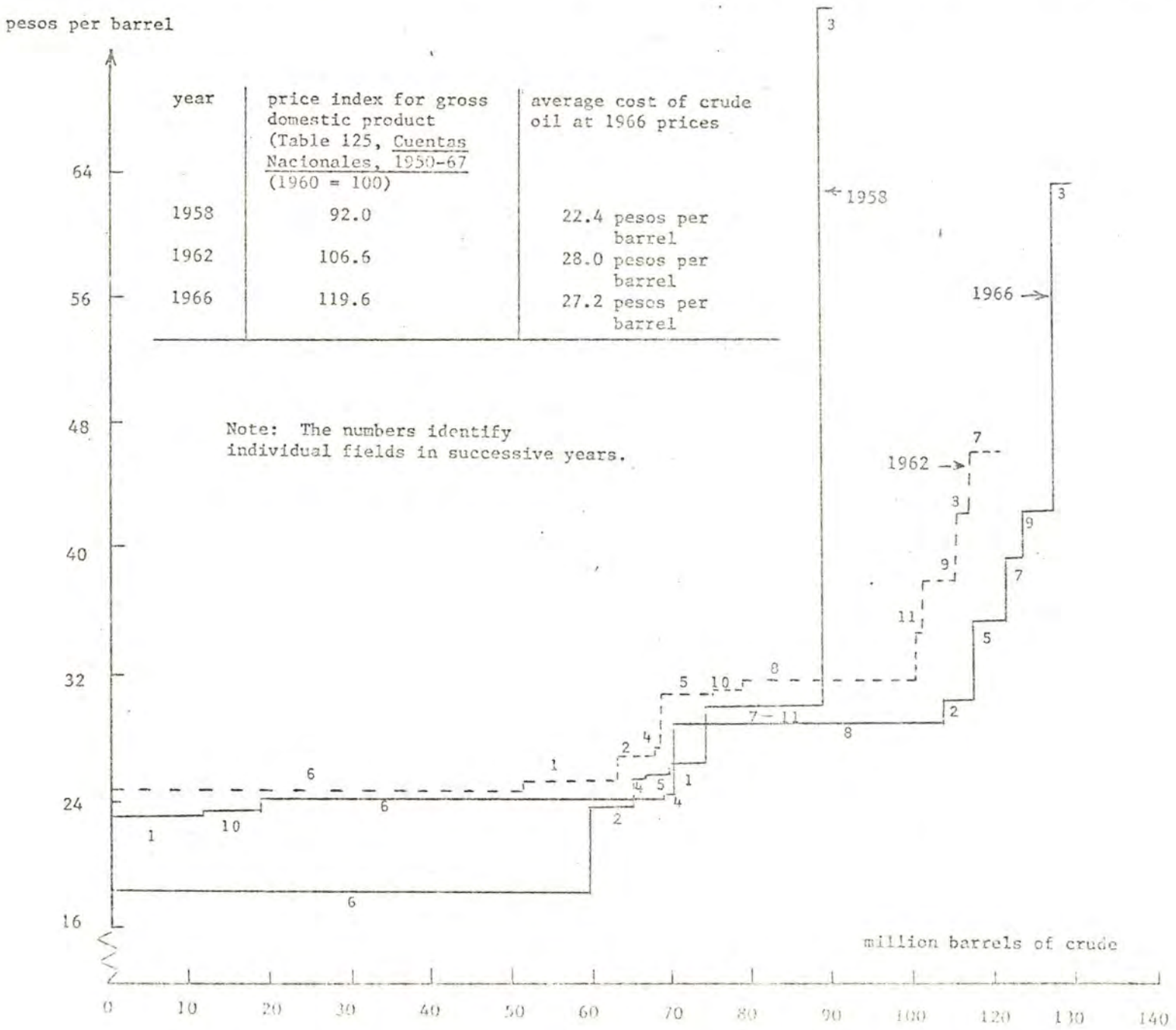
ENERGETICOS is stated as though the objective were to satisfy the domestic demands for petroleum products at minimum cost, exporting and importing whichever items are most advantageous. It is not stipulated that absolute priority be given to self-sufficiency, nor to employment within the petroleum and gas industry. Rather, we explore the tradeoffs between total costs and foreign exchange costs--allowing for a 15% rate of protection against imports.

In order to concentrate on the problem of intersectoral linkage, we have focussed on oil and gas production and on refining--excluding the transportation and marketing aspects. Within ENERGETICOS, the costs of domestic crude oil and gas production constitute the most problematical set of parameters. It is not at all clear what have been the past costs of domestic production--let alone what is appropriate in an extrapolation for a decade into the future.

Some clues on the future may be obtained from Figure 3, a cumulative distribution of curde oil costs by fields in 1958, 1962 and 1966. The historical figures suggest: (a) that there was a significant increase in the domestic cost of crude oil between 1958 and 1962, but not between 1962 and 1966 (after deflation by the gross domestic product price index); (b) that there is a significant difference between the costs of production in individual fields; (c) that the average field has costs that are competitive with an import price of 27.5 pesos per barrel = U.S. $2.20 per barrel; and (d) that there has consistently been a small volume of production from marginal fields with costs that lie well above the import price.

The classical academic impulse is to conclude that the high-cost fields ought to be shut down, that imported crude replace the domestic output, and that the necessary foreign exchange be earned through shifting the displaced petroleum workers into producing, say, fruits and vegetables for export. A more pragmatic reaction would be to question whether the historical accounting estimates bear any relation to the economic theorist's definition of opportunity costs. Underlying Figure 3 are arbitrary allocations between two joint products--crude oil and associated gas. There are quality differences between types of crude. There is the possibility that low-cost fields have not been exploited to their capacity in the past. There are arbitrary allocations of exploration and drilling costs, arbitrary estimates of proved reserves, and arbitrary allowances for the return on capital. Given this view of the data, it is hopeless to calculate an

Figure 3. Pemex Accounting Department estimates of crude oil costs by fields - 1958, 1962, 1966



year	price index for gross domestic product (Table 125, Cuentas Nacionales, 1950-67 (1960 = 100)	average cost of crude oil at 1966 prices
1958	92.0	22.4 pesos per barrel
1962	106.6	28.0 pesos per barrel
1966	119.6	27.2 pesos per barrel

optimal make-or-buy choice for crude oil. For the decision-maker who is subject to political pressure from petroleum workers, the line of least resistance is to adopt the criterion of self-sufficiency in raw materials--regardless of costs. (Consideration of this type are not unknown in the U.K. coal mines nor in the U.S. petroleum industry.)

Rather than abandon cost criteria completely, it seems preferable to make some assumptions on the supply curve for raw materials, and then to test how these assumptions influence the optimal choice of processes elsewhere within the system. For the decade of the 1970's, it can be supposed that the import price will remain constant at 27.5 pesos per barrel for crude oil and at 25.0 for the fuel oil equivalent of dry gas. (See Table 8.*) Given the uncertainties in cost extrapolations--and given the inevitable political pressures against imports--we have attempted to err in the direction of underestimating future domestic costs. For this reason, it is supposed that 85% of the future domestic supplies will be available at a cost substantially below that of imports, and that an additional 10% of the domestic supplies will be competitive if foreign exchange is valued at the conventional rate of 12.5 pesos per U.S. dollar. For the remaining 5% of potential oil and gas supplies, the cost parameters have been adjusted so that domestic material will cost more than imports even after allowance is made for a 15% rate of protection against imports. Typically, unless there is a premium value assigned to foreign exchange, it then turns out to be optimal to

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*For many of the estimates that appear in Tables 8-11, we are indebted to Ings. Raúl Meyer, Victor L. Games and Carlos Velasco.

Table 8. Projections of domestic availability and cost of oil, condensates and dry gas

	Availability (unit: 10^6 barrels)								Cost coefficients (row CSCORt) (unit: pesos per barrel) a/				
	'74	'75	'76	'77	'78	'79	'80	1974-80 %/ year	% of domestic supplies available at stated cost	85%	10%	5%	import price
time period t	0	1	2	3	4	5	6	growth rate					
Crude oil	172.2	176.1	191.4	209.3	228.3	234.5	255.0	6.8%		22.0	27.0	35.0	27.5
Condensates	31.9	34.5	37.2	39.6	42.2	44.9	47.5	6.8		c/	c/	c/	-
Dry gas b/	118.0	131.3	144.1	157.3	171.4	188.4	204.2	9.6		20.0	24.5	31.3	25.0 d/

Note:

a/ Foreign exchange component (row CSMCORt) it taken at 10% of domestic costs and 10% of import price.

b/ Fuel oil equivalent of dry gas at 6.321 10^6 BTU/barrel of fuel oil.

c/ Since the quantity of condensates is projected exogenously, no cost coefficients need to be estimated for this item.

d/ Import price of fuel oil.

exploit just 95% of the potentially available domestic supplies, and to import the balance of the raw material requirements for the petroleum refining industry.

For alternative refining processes, it is straightforward to estimate physical inputs and outputs. (See Table 9.) Each of the three alternative activities for new refining capacity is viewed as though it were vertically integrated from crude oil through refined products, after netting out intermediate materials such as vacuum gas oil. If crude oil were the only cost element and if the refining industry's objective were to maximize the output of gasoline (the highest-priced product) it would be straightforward to conclude that maximum hydrogen processing (process PEHt) is always optimal. Here, with a more complex objective function--satisfying demands at minimum costs--the optimal process choice is not maximum hydrogen processing. The process choice is sensitive to the refining cost coefficients shown in rows CSCORt and CSMCORt.*

Table 10 provides the marginal costs and optimal material balances for a representative year, 1977. For each year within the planning horizon of 1974-80, similar costs and material balances are available. Typically, as shown in Table 11, the optimal solution entails small-scale imports of crude oil and liquefied petroleum gas, and virtual self-sufficiency in other items. With other numerical assumptions (e.g., a slower rate of introduction of nuclear power), the solution will change. In particular,

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*The numerical values adopted for these cost coefficients are based upon the rather shaky estimate that annual refinery costs-including capital charges, labor, overheads and refinery inputs other than oil and gas--will be 40% of the initial investment costs for an integrated plant.

Table 9. Matrix of coefficients for petroleum refining

Row (for identification and units of measurement, see Tables 1 and 2)	Activity	Initial (1973) refining capacity	New refining capacity [b]			Outputs per million barrels of domestic condensates	Industrial fuel equivalents			
			conventional	visbreaking	maximum hydrogen processing					
		PECP73[a]	PECt	PEVt	PEHt	PEDCONt [c]	PEQGSCt	PEQGSRt	PEQCPLt	PEQCOMt
cost inputs	CSCORt		-6.2	-10.8	-16.4					
	CSMCORt		- .62	- 1.64	- 1.08					
physical inputs (-) and outputs (+); unit: 10^6 barrels, except for row CMBTUt	CMBTUt						6.321	6.321	4.173	6.321
	PEGSCt			- .0181	- .0328		-1.0			
	PECRDt	-170.0	-1.0000	-1.0000	-1.0000					
	PEGSRt	5.9	.0240	.0080	.0289			-1.0		
	PECPLt	12.5	.0280	.0602	.0836	.5005			-1.0	
	PEGSLt	71.5	.2580	.4876	.5859	.3900				
	PEKDt	52.6	.2770	.2546	.2863	.1095				
	PECOMt	35.0	.2870	.2218	.1282					-1.0

Notes:

[a] Initial capacity activity is operated at level of 1.0 during all time periods.

[b] All three processing sequences are based upon an integrated new refinery with 115 TB/D of primary distillation capacity, 50 TB/D of vacuum distillation capacity, and a 10 TB/D gasoline reformer. Other capacities would be as follows (in TB/D = thousand barrels per stream day);

	conventional	visbreaking	maximum hydrogen processing
H-oil	0	0	25
Isomax	0	30	30
Visbreaking	10	25	0
Catalytic cracking	27	0	0
Hydrogen (10^6 scf/day)	0	50	100

[c] Domestic condensates activity is operated at predetermined level during all time periods. See Table 8 for this availability level.

Table 10. Marginal costs and optimal material balances, 1977 - petroleum industry

(omitting inputs and outputs where optimal activity level is zero)

(unit: millions of barrels/year - except for industrial fuel)

Description of activity / Activity identification / Item	Refining capacity			Domestic gas a/		Domestic condensates	Domestic crude b/		Fuel oil equivalents			Imports			Intersectoral flows		Final demand	Marginal cost of supplying item in 1977 (pesos/barrel-except for industrial fuel)
	PEC P73	PEC 3	PEV 3	PEDGSC3	PEDGSCA3	PEDCON3	PEDCRD3	PEDCRDA3	PEQCSC3	PEQGSS3	PEQCOR3	PEMCRD3	PEMGPL3	PEMKD3	ELRQF3	SDRQF3		
CMBTU3 c/									905.4	39.6	223.8				-231.7	-138.7	798.4	3.93
PEGSC3			- .4	133.7	9.9				-143.2								0	24.87
PECRD3	-170.0	-7.7	-22.7				177.9	20.9				1.6					0	31.63
PEGSR4	5.9	.2	.2							- 6.3							0	24.87
PEGPL3	12.5	.2	1.4			19.8							6.1				40.0	33.12
PEGSL3	71.5	2.0	11.1			15.4											100.0	44.06
PEKD3	52.6	2.1	5.8			4.3								.5			65.3	54.40
PECOM3	35.0	3.0	5.0								- 35.4						7.6 d/	24.87

Notes:

a/ Activity PEDGSC3 denotes domestic gas obtainable at cost of 20.0 pesos/barrel; PEDGSCA3 denotes domestic gas obtainable at 24.5 pesos/barrel.

b/ Activity PEDCRD3 denotes domestic crude obtainable at cost of 22.0 pesos/barrel; PEDCRDA3 denotes domestic crude obtainable at 27.0 pesos/barrel.

c/ Unit: 10^{12} BTU/year.

d/ Includes asphalts only.

Table 11. Product demands; optimal imports and exports

(unit: 10^6 barrels/year - except for industrial fuel)

		export/ import price (pesos/ barrel)	year	'74	'75	'76	'77	'78	'79	'80	1974-80 %/ year growth rate of final demand	Identification of activity	Identification of item
			period t	0	1	2	3	4	5	6			
industrial fuel [a/]	final demand			616.5	672.0	732.5	798.4	870.3	948.6	1034.0	9.0%		
	intersectoral flows, electricity			250.0	285.2	249.9	231.7	231.6	215.3	197.3		ELRQFt	CMBTUt
	intersectoral flows, steel [b/]			117.2	124.2	130.9	138.7	147.1	156.4	166.5		SDRQFt	
liquefied petroleum gas	final demand	-		33.8	35.9	38.0	40.0	42.0	44.0	45.8	5.2		PEGPLt
	imports	28.8		(5.3)	(6.0)	(6.0)	(6.1)	(6.0)	(5.8)	(5.5)		PEMGPLt	
gasoline	final demand	-		80.0	86.3	93.0	100.0	107.4	115.1	123.1	7.4		
	exports	31.5		3.9	0	0	0	0	0	0		PEXGSLt	PEGSLt
	imports	47.3		0	0	0	0	0	0	0		PEMGSLt	
kerosine and diesel oil	final demand	-		55.1	58.4	61.8	65.3	68.9	73.4	76.7	5.7		
	exports	31.5		1.0	0	0	0	0	0	0		PEXKDt	PEKDt
	imports	47.3		0	(.6)	(.4)	(.5)	0	(.7)	(.1)		PEMKDt	
residual fuel oil	final demand [c/]	-		5.9	6.5	7.0	7.6	8.2	8.9	9.6	8.5		
	exports	21.2		0	0	0	0	0	0	1.3		PEXCOMt	PECOMt
	imports	25.0		(8.5)	(9.8)	0	0	0	0	0		PEMCOMt	
crude oil	imports	27.5		(6.4)	(7.9)	(6.1)	(1.6)	0	(5.8)	(.6)		PEMCRDt	PECRDt

Notes:

a/ Unit: 10^{12} BTU/year,

b/ Total industrial fuel requirements for steel industry includes rolling stage. This excludes coke requirements, and refers only to fuel oil plus natural gas.

c/ Includes asphalts only.

note that the optimality of the solution depends upon the import and export prices assumed for the various raw materials and products. To the extent that we have overstated the transport and marketing cost differentials between the import and the export price of each petroleum product, the results have been biased in favor of self-sufficiency.

7. Sensitivity analysis--discount rate and foreign exchange scarcity premium.

Up to this point, all results have been based upon those project evaluation criteria that we believe to be typical within the energy sector. It has been supposed that the public enterprises within this sector evaluate alternative projects as though capital were available at a 10% annual discount rate, as though foreign exchange were valued at the conventional rate of 12.5 pesos per U.S. dollar, and as though there were a 15% protection rate for domestic as against imported items.*

With respect to import substitution choices, a 15% protection rate has effects that are identical to a 15% scarcity premium imputed to foreign exchange. With respect to export promotion, however, the two criteria differ. E.g., to stipulate a zero scarcity premium on foreign exchange and a 15% protection rate implies that the energy sector would prefer to save $1.00 of foreign exchange through import substitution rather than earn up to $1.15 of foreign exchange through exports. A domestic protection rate, then, is to be interpreted as a price that Mexico is willing to pay in order

*Throughout the energy sector, domestic inputs are given preference over imported ones. For heavy electrical equipment financed by World Bank loans, the standard current practice for bid evaluation is a 15% preference rate. Outside the electrical sector, the protection criteria are seldom stated in explicit form.

to insulate her economy from fluctuations transmitted through international trade.

An alternative interpretation of the 15% preference rate is the following: Even if absolute preference is to be given to domestic raw materials such as oil and gas, the consumers of these items should evaluate their own process alternatives as though fuel were not priced much above the international level. If marginal domestic cost pricing is applied in this "second-best" situation, there will be an uneconomic stimulus to economize on oil and gas, e.g., to speed up the introduction of nuclear power into the electricity industry and of hydrogen processing into petroleum refineries.

Table 12 summarizes how the optimal solution would shift under alternative assumptions with respect to the foreign exchange premium and the discount rate--holding the protection rate constant at 15% of import costs. Let p denote the scarcity premium imputed to foreign exchange. Then, recalling the definitions given in Table 2, the cost minimand represents the discounted sum of one-period costs. The costs incurred in period t would be evaluated as follows:

costs at conventional prices	+ .15 (import costs)	+ p (import costs - export earnings)
CSCORt	+ .15 (CSMCORt)	+ p(CSMCORt - CSXCORt)

As p ranges between 0 and 60% (i.e., as the value of foreign exchange ranges between the conventional rate of 12.5 and the shadow price of 20.0 pesos per U.S. dollar*), note that the discounted import costs less export earnings would decrease, and that total costs evaluated at conventional prices would rise by a comparatively small amount.

Underlying the aggregative results, there are detailed import--export choices--which in turn depend upon the selection of production processes. For simplicity in presentation, Table 12 summarizes the results for the single representative year, 1977. Within the steel industry, the optimal choice would remain process #5 (blast furnace + LD converter) --regardless of whether the discount rate is 10% or 20% per year and regardless of whether p = 0 or p = 60%.

Within the electric power and the petroleum sectors, the process choices are sensitive both to the discount rate and the foreign exchange premium. Some of these choices are self-evident, e.g., the shift to high-cost domestic crude as the foreign exchange scarcity premium increases. Not all the results are so clearcut. E.g., the higher the discount rate, the greater becomes the capital cost of the domestic fossil fuel supplied by the petroleum to the electricity industry. Nonetheless, in changing the annual discount rate from 10 to 20%, the lower becomes the optimal amount of nuclear electric generating capacity. The high capital cost component of nuclear capacity would more than offset the rise in fossil fuel costs.

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*Given the prevailing levels of effective tariffs on imports of consumers' durables, an efficiency price of 20.0 pesos is not altogether unrealistic. See, e.g., Bueno (1970, Table 8.7).

Table 12. Effects of foreign exchange premium and of discount rate for petroleum, electricity and steel industries

		Unit of measurement	p = foreign exchange scarcity premium, %						
			0	10	20	30	40	50	60
Total discounted costs at conventional prices + 15% protection on imports		10^9 pesos discounted to 1974 (year 0) at 10% per year a/	127.34	127.46	127.75	127.95	127.98	128.11	128.27
Total discounted import costs less export earnings, at conventional prices		10^9 pesos discounted to 1974 (year 0) at 10% per year a/	23.67	21.91	20.06	19.29	19.22	18.90	18.70
1977 process choices	Ingot output from 1974-77 capacity, steel process #5	10^6 ingot tons	2.9	2.9	2.9	2.9	2.9	2.9	2.9
	Crude petroleum, domestic output, 1977	10^6 barrels	195.8	198.8	201.9	201.9	201.9	203.8	209.3
	Crude petroleum, imports, 1977	10^6 barrels	1.6	3.0	0	0	0	0	0
	Crude input to 1974-77 refining capacity (conventional)	10^6 barrels	7.7	10.7	10.9	10.9	10.9	14.9	23.4
	Crude input to 1974-77 refining capacity (visbreaking)	10^6 barrels	22.7	21.1	21.0	21.0	21.0	18.9	15.9
	Fossil electric generating capacity installed, 1974-77	MW	1,245.	1,869.	1,921.	2,483.	2,614.	2,821.	3,047.
	Nuclear electric generating capacity installed, 1976-77	MW	1,802.	1,177.	1,145.	564.	433.	226.	0
Total discounted costs at conventional prices + 15% protection on imports		10^9 pesos discounted to 1974 (year 0) at 20% per year a/	85.34	85.50	85.50	85.67	85.95	86.20	86.20
Total discounted import costs less export earnings, at conventional prices		10^9 pesos discounted to 1974 (year 0) at 20% per year a/	18.35	13.35	13.35	12.60	11.89	11.30	11.30
1977 process choices	Ingot output from 1974-77 capacity, steel process #5	10^6 ingot tons	2.9	2.9	2.9	2.9	2.9	2.9	2.9
	Crude petroleum, domestic output, 1977	10^6 barrels	177.9	198.8	198.8	198.8	198.8	201.9	201.9
	Crude petroleum, imports, 1977	10^6 barrels	23.9	3.0	3.1	3.1	3.1	0	0
	Crude input to 1974-77 refining capacity (conventional)	10^6 barrels	10.7	10.7	10.9	10.9	10.9	10.9	10.9
	Crude input to 1974-77 refining capacity (visbreaking)	10^6 barrels	21.1	21.1	21.0	21.0	21.0	21.0	21.0
	Fossil electric generating capacity installed, 1974-77	MW	3046.	3046.	3046.	3046.	3046.	3046.	3046.
	Nuclear electric generating capacity installed, 1976-77	MW	0	0	0	0	0	0	0

annual discount rate = 10%

annual discount rate = 20%

Note :
a/ Discounted over 1974-80 horizon for petroleum and steel industries; over 1974-95 horizon for electricity industry.

Table 13. Macroeconomic implications of six alternative programs for petroleum, electricity and steel industries. (1975 prices)

Annual discount rate, %		10			20		
Foreign exchange scarcity premium, %		0	30	60	0	30	60
Investment expenditures	'74	3.39	3.94	4.08	3.09	3.39	3.94
	'75	2.83	2.99	2.87	2.03	2.83	2.99
for capacity	'76	5.25	4.56	4.54	4.18	4.59	4.54
	'77	6.01	5.62	5.61	5.25	5.39	5.22
first available	'78	3.36	3.20	3.09	3.25	3.33	3.30
	'79	6.28	6.13	5.95	5.45	5.63	5.97
in year (10^9 pesos)	'80	7.87	6.69	7.00	6.73	6.90	6.69
Total		34.99	33.13	33.14	29.98	32.06	32.65
Import costs	'74	1.12	.84	.78	2.00	1.32	1.05
	'75	1.45	1.09	1.09	2.40	1.68	1.33
less export	'76	1.48	1.16	1.15	2.58	1.79	1.47
	'77	1.57	1.39	1.23	2.86	2.00	1.76
earnings	'78	1.43	1.34	1.29	2.86	1.97	1.75
	'79	1.92	1.58	1.63	3.57	2.58	2.18
(10^9 pesos)	'80	1.87	1.89	1.82	3.94	2.87	2.59
Total		10.84	9.29	8.99	20.21	14.21	12.13
Export earnings	'74	.16	.16	.19	.16	.16	.16
	'75	0	0	.02	0	0	.01
(from petroleum	'76	0	0	0	0	0	0
	'77	0	0	.09	0	0	0
products only)	'78	0	.02	.06	0	0	.19
	'79	0	0	.03	0	0	0
(10^9 pesos)	'80	.03	.02	.11	0	0	.20
Total		.19	.20	.50	.16	.16	.56
Total discounted costs at conventional prices + 15% protection on imports (10^9 pesos discounted to 1974 at 10% per year) a/		127.34	127.95	129.27	128.62	132.43	132.68

Note: a/Discounted over 1974-80 horizon for petroleum and steel industries; over 1974-95 horizon for electricity industry

Table 13 contains the six alternative time-phased vectors of investment and of foreign exchange expenditures that correspond to these process analysis results. Elsewhere (Memorandum 71-7), we will employ these vectors to show that the energy sector's process decisions are likely to have negligible feedback effects upon the final demands for its outputs or upon the efficiency prices of the sector's inputs. Under these conditions, a sectoral sub-optimization need not diverge from a global optimization.

APPENDIX: PROJECT INDIVISIBILITIES

Model Formulation:

To allow for indivisible projects within ENERGETICOS, the IPE mixed integer programming algorithm (Memorandum 71-3) was employed. Three mutual exclusivity sets were defined: one each for projects in the steel, petroleum and electricity sector. Mutual exclusivity between 6 steel projects is ensured through the row MXSD; between 9 petroleum projects through the row MXPE; and between 12 electricity projects through the row MXEL. (See Tables 14-16.) Altogether, the number of logically possible project combinations is $N = 6 \cdot 9 \cdot 12 = 648$. Each of these alternative combinations is identified through a sequence of 3 numbers. E.g., (6,4,7) would identify the combination consisting of steel project 6, petroleum project 4, and electricity project 7.

In defining the individual projects, we made use of whatever simplifications were suggested by the original linear programming solutions. This helped to narrow down both the process alternatives and the dates at which new capacity might be installed. Linear programming indicated, for example, that 2.9 million tons of steel capacity and output were to be added cumulatively between years 0 and 3, and that it was optimal to install ingot process 5 (blast furnace + LD converter) regardless of whether the foreign exchange premium was 0 or 60%. For purposes of mixed integer programming with the IPE algorithm, we therefore focussed on steel process 5--assuming that it could be installed only in indivisible plants of either 1.5 or 3.0 million tons.

(These sizes are within the range of the alternatives that have been proposed for the Las Truchas mill.) Since processes 2-4 also require large mills, these were omitted from the IPE model. Processes 1 (sponge iron) and 6 (electric furnace) would not necessarily require large plants, and these activities were therefore retained as continuous variables.

With this background on the steel sector, six alternatives appeared to be worth investigation through the IPE algorithm. Table 14 provides the time-phasing and process definitions for these alternatives. It also indicates how the six zero-one project variables are linked to the continuous variables SD5t through the rows SDROWt and the direction of the inequality for that row. If, for example, project Y5SD = 1.0, this would mean that no additional large steel mill capacity was available during year 0, that up to 3.0 million tons could be utilized in years 1-3, and that at least 3.0 million tons were to be utilized in subsequent years.

Table 15 shows a similar structure for the 9 mutually exclusive petroleum projects. Again, the definition of these projects was based partly upon the previous linear programming calculations. These indicated that it was non-optimal to adopt refining process PEHt (maximum hydrogen processing). The IPE computations were therefore confined to processes PECt (conventional processing) and PEVt (visbreaking). It was supposed that no more than one refinery would be added between years 2 and 5, and that this unit would have an annual crude intake identical to that being considered for the Tula plant: 50 million barrels of crude. The nine mutually exclusive refining alternatives are defined so that imports or exports of petroleum products will offset any temporary deficits or surpluses of installed capacity. If, for example,

Table 14. Tableau for linking continuous variables to indivisible projects in steel industry[a]

Row identification	Column identification	Continuous variables (cumulative ingot capacity added between year 0 and t, and utilized in year t, process 5)					Mutually exclusive steel projects[b]						
		SD50	SD51	SD52	SD53	SD54	Y1SD	Y2SD	Y3SD	Y4SD	Y5SD	Y6SD	
ingot capacity, process 5, year t (millions of tons)	SDROW 0	-1.						1.5	1.5			3.0	≥ 0
	SDROW 1		-1.					1.5	1.5		3.0	3.0	≥ 0
	SDROW 2			-1.				1.5	3.0	3.0	3.0	3.0	≥ 0
	SDROW 3				-1.			1.5	3.0	3.0	3.0	3.0	≥ 0
	SDROW 4					-1.		1.5	3.0	3.0	3.0	3.0	≤ 0
	MXSD						1.	1.	1.	1.	1.	1.	= 1.

Notes:

[a] Cost coefficients in rows CSCORt and CSMCORt were extrapolated from those shown in Table 3 for a 1.0 million ton plant, ingot process 5. The coefficient in row CSCORt (418 pesos per ton) covers the gross return on capital costs plus labor inputs. That in row CSMCORt (55 pesos per ton) covers the import component of capital costs. To estimate the economies of scale for a plant of 1.5 or of 3.0 million ingot tons, a scaling exponent of .7 was then applied to these coefficients. That is:

$$\frac{\text{cost for x million tons}}{\text{cost for 1 million tons}} = x^{.7}$$

Note that $(1.5)^{.7} = 1.33$, and that $(3.0)^{.7} = 2.16$. This would mean that for a plant designed to produce 3.0 million ingot tons, the total annual equivalent of capital and labor costs would be only 2.16 times the amount required for a plant designed for 1.0 million tons.

[b] The six alternative projects may be interpreted as follows:

Y1SD Add no integrated plants until after year 3; supply any capacity deficits from either process 1 (sponge iron) or process 6 (electric furnaces).

Y2SD Add an integrated 1.5 million ton mill in year 0; supply any capacity deficits from processes 1 or 6.

Y3SD Add an integrated 1.5 million ton mill in year 0, and add another in year 2.

Y4SD Add an integrated 3.0 million ton mill in year 2

Y5SD Add an integrated 3.0 million ton mill in year 1

Y6SD Add an integrated 3.0 million ton mill in year 0

Table 15. Tableau for linking continuous variables to indivisible projects in petroleum industry [a]

Row identification \ Column identification		Continuous variables, refining processes (cumulative capacity added between year 0 and t, and utilized in year t)								Mutually exclusive refinery projects [b]										
		visbreaking				conventional					visbreaking				conventional					
		PEV2	PEV3	PEV4	PEV5	PEC2	PEC3	PEC4	PEC5	Y1PE	Y2PE	Y3PE	Y4PE	Y5PE	Y6PE	Y7PE	Y8PE	Y9PE		
visbreaking year t (millions of barrels)	PEVROW 2	-1.												50.					≤	0
	PEVROW 3		-1.										50.	50.					≤	0
	PEVROW 4			-1.								50.	50.	50.					≤	0
	PEVROW 5				-1.						50.	50.	50.	50.					≤	0
conventional processing, year t (millions of barrels)	PECROW 2					-1.												50.	≤	0
	PECROW 3						-1.										50.	50.	≤	0
	PECROW 4							-1.								50.	50.	50.	≤	0
	PECROW 5								-1.						50.	50.	50.	50.	≤	0
	MXPE									1.	1.	1.	1.	1.	1.	1.	1.	1.	=	1.

Notes:

[a] Cost coefficients in rows CSCORt and CSMCORt were extrapolated from those shown in Table 9, neglecting any economies-of-scale. Here, all refinery projects are taken to have the identical crude throughput capacity: 50 million barrels per year. This capacity is 37% larger than the unit on which Table 9 was based.

[b] The nine alternative projects may be interpreted as follows:

Y1PE	Add no new refining capacity until after year 5; supply any deficits from imports of petroleum products.
Y2PE	Add 50 million barrels of crude capacity (visbreaking process) in year 5; offset any temporary deficits or surpluses through imports or exports of petroleum products
Y3PE	Same as Y2PE, except that the new refinery is to be brought onstream in year 4.
Y4PE	Same as Y2PE, except that the new refinery is to be brought onstream in year 3.
Y5PE	Same as Y2PE, except that the new refinery is to be brought onstream in year 2.
Y6PE-Y9P3	Same as Y2PE-Y5PE, except that the new refinery would employ conventional processing.

Table 16. Tableau for linking continuous variables to indivisible projects in electricity industry [a/]

Row identification \ Column identification		Continuous variables (nuclear capacity added in year t)					Mutually exclusive nuclear generating projects (size of each unit: .6 GW)											
		ELNUC2	ELNUC3	ELNUC4	ELNUC5	ELNUC6	Y1EL	Y2EL	Y3EL	Y4EL	Y5EL	Y6EL	Y7EL	Y8EL	Y9EL	Y10EL	Y11EL	Y12EL	
nuclear generating capacity added in year t (GW)	ELROW 2	-1.										.6	.6	.6	.6	.6	1.2	.6	= 0
	ELROW 3		-1.								.6	.6	.6	.6	.6	1.2	.6	.6	= 0
	ELROW 4			-1.						.6	.6	.6	.6	.6	1.2	.6	.6	1.2	= 0
	ELROW 5				-1.				.6	.6	.6	.6	.6	1.2	.6	.6	.6	1.2	= 0
	ELROW 6					-1.		.6	.6	.6	.6	.6	1.2	1.2	1.2	1.2	1.2	1.2	= 0
	MXEL						1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	= 1.

Note:

[a/] The continuous variables ELNUCt refer to the nuclear capacity added in year t, and the cost coefficients for these variables are based upon a unit size of .6 GW. (See Table 6.) No cost coefficients need therefore be added to rows CSCORt and CSMCORt for the 12 project variables.

Y4PE = 1.0, this would mean that a 50 million barrel plant (visbreaking process) is to go onstream in year 3. At the import and export prices stipulated for ENERGETICOS (Table 11), it then turns out to be optimal for gasoline to be imported in years 1 and 2, and to be exported in years 3 and 4 when refining capacity exceeds the domestic demands.

The structure of Table 16 is similar to the two preceding ones--except that the continuous variables ELNUCt refer to the nuclear generating capacity added in year t, not to the cumulative amount added between year 0 and t. It is supposed that the size of each nuclear power unit would be .6 GW, and that either zero, one or two of these units would be added annually. Any deficits in nuclear capacity are offset through substituting the fossil process in its place. In the electricity sector--unlike petroleum refining--temporary imports and exports are neglected.

Results:

Results have been computed for two alternative levels of the foreign exchange premium--holding the discount rate constant at 10% per year. The two IPE calculations may be summarized as follows:

Foreign exchange premium	60%	0
Optimal project combination	(6,5,1)	(6,4,7)
Total discounted costs + 15% protection on imports + foreign exchange premium (billions of pesos)	139.67	127.65
Number of steps required by IPE algorithm	11	26

In the steel sector, the optimal project choice is Y6SD-the installation of a 3.0 million ton mill in year 0. The cost of the initial excess capacity is more than offset by the economies-of-scale in subsequent years. Just as in the linear programming sensitivity analysis (Table 13), it turns out that the choice of steel projects is insensitive to the foreign exchange premium.

In the petroleum sector also, the foreign exchange premium would have a minor effect. The optimal petroleum project choice is either Y5PE or Y4PE--which means only a one-year difference in the onstream date of a 50 million barrel plant (visbreaking process). The choice of electricity projects is quite sensitive to the foreign exchange premium. If this were as high as 60%, the optimal project would be Y1EL. That is, it would be non-optimal to install nuclear generating units until some time well after year 6 (1980). With a zero foreign exchange premium, on the other hand, it would be optimal to install at least one nuclear unit each year.

What conclusions may be drawn about the extent of interdependence between project decisions in the steel, petroleum and electricity sectors? The number of steps required for convergence of the IPE algorithm may be an indicator of the strength or looseness of the coupling. Here the required number of steps is a small fraction of the logically possible maximum number (N = 648). If the project choices were highly interdependent, the efficiency prices would alter radially from one IPE step to the next, and many such steps would be required.* With perfect decomposability, on the other hand,

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*For an example of closely complementary project decisions, see the steel-machinery example, Memorandum 71-3.

the efficiency prices would not change at all, and a globally optimal combination could be located after just a single step. The IPE experiment suggests that ENERGETICOS provides an instance of near-decomposability.

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Banque Docs

Memorandum 71-5
March 1971

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MULTI-LEVEL PLANNING FOR ELECTRIC POWER PROJECTS*

Contents

		Page
1.	Introduction.	1
2.	A single-plant comparison of nuclear versus fossil fuel projects	2
3.	Algebraic formulation of static multi-region model. . .	5
4.	Numerical data.	15
5.	Mixed integer programming formulation	22
6.	Recursive dynamic optimization.	27
7.	Numerical results	31
8.	Effects of aggregation--the implications for multi-level planning.	39

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* This research was supported by the Comisión Federal de Electricidad and by the International Bank for Reconstruction and Development. The specific facts, methods of analysis, and conclusions are the sole responsibility of the individual authors.

** For help in various phases of this work, the authors wish to acknowledge their indebtedness to Ings. Juan Eibenschutz and Roberto Fabre and Lic. Bosco Muro (Comisión Federal de Electricidad) and to Mr. Richard Inman (Stanford University).

1. Introduction

Until converted into investment decisions on specific projects, a macroeconomic development plan remains non-operational.* In principle, to be sure, an individual project cannot be evaluated without a clear view of the entire system within which it is to operate. In practice, this ideal is approached through identifying one-way linkages -- parameters that influence the project decision, but that are not significantly influenced by it.

Following the suggestive terminology of Simon (1962, pp. 468 and 474), a project is the "lowest level of elementary subsystem" in a hierarchically organized complex system. One-at-a-time decisions on individual projects are likely to be near-optimal if the projects belong to "nearly decomposable systems, in which the interactions among the subsystems are weak, but not negligible.... In the long run, the behavior of any one of the components depends in only an aggregate way on the behavior of the other components."

We begin by examining a typical case -- the evaluation of a nuclear versus a fossil fuel electricity plant in Mexico during the late 1970's. In this and accompanying papers, the fossil-nuclear choice is reexamined within systems of increasing complexity: INTERCON, a multi-region model that introduces tradeoffs between electricity generation and transmission; ENERGETICOS, a process analysis that introduces tradeoffs between investments in electric power versus those in the petroleum sector; and DINAMICO, a multi-sector macroeconomic model.

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*On the relations between the macro and the micro stages of development planning, see Tinbergen (1967, ch. 9).

If the identical system of aggregation were maintained as we moved from the micro toward the macro stage, project evaluation would become increasingly more realistic -- but increasingly less manageable from the viewpoint of data-gathering, computation and timeliness of results. Clearly, the identical system of aggregation is inappropriate at the project, sectoral and multi-sector stages of planning. We shall conclude this paper by speculating on what are likely to be the gains from evaluating electricity projects within models of ever-increasing complexity, what are likely to be the distortions associated with ever-increasing aggregation of information, and what purposes are served by multi-level planning.*

2. A single-plant comparison of nuclear versus fossil fuel projects

Traditionally, project analysis has taken some such form as that shown in Figure 1. A project (a nuclear power plant in a specific region) is evaluated against an alternative (a fossil fuel plant) converting all investment, operating and fuel costs into equivalent annual flows.** Typically,

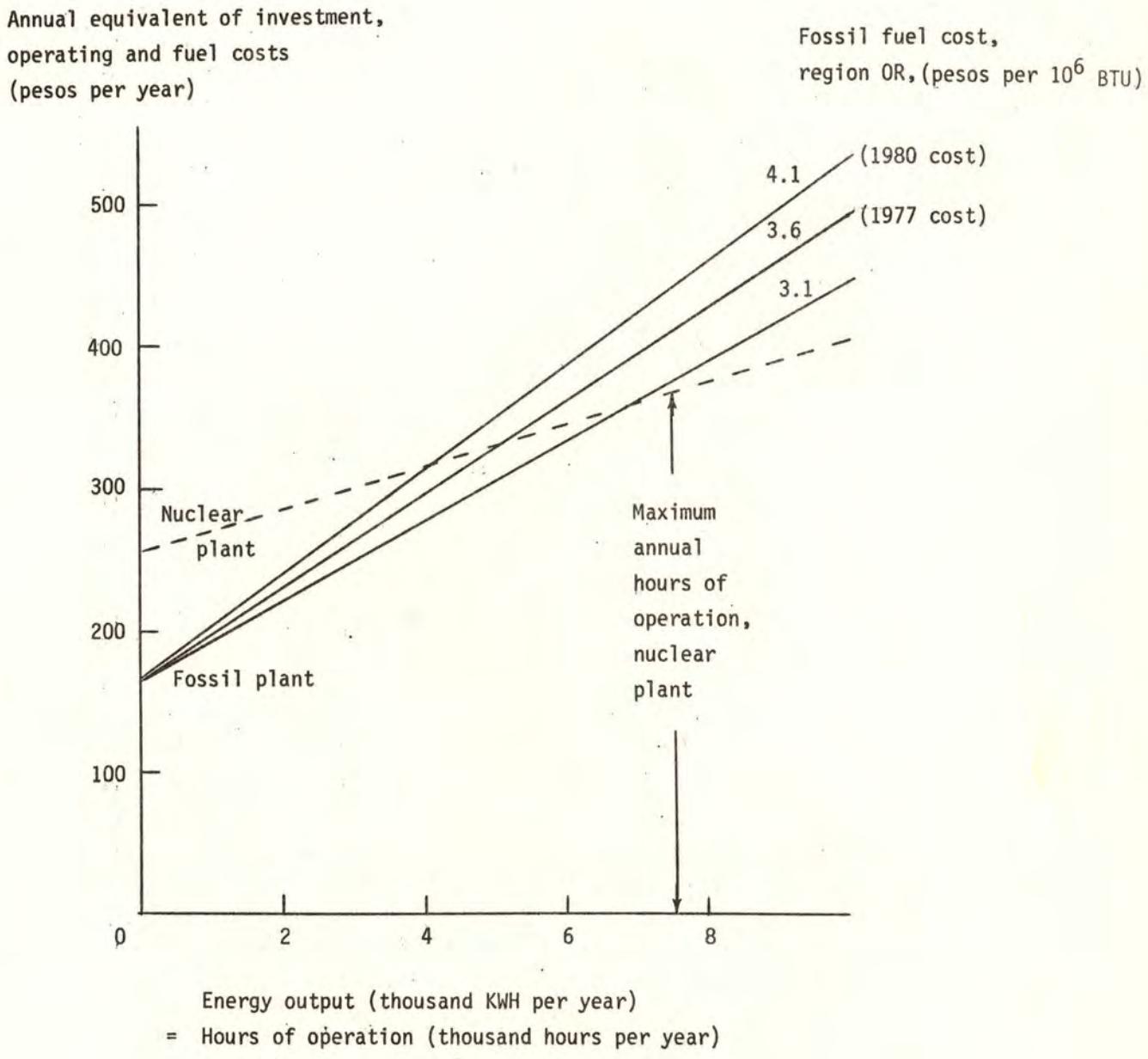
* On multi-level planning, see Kornai (1967, esp. ch. 25).

** At no point in this report do we attempt to quantify the environmental and pollution hazards associated with the two alternative sources of electricity. These hazards vary significantly from one site to another.

For the detailed numerical data that underlie Figure 1, see Table 6 below. The nuclear and fossil plant types are those identified as N1 and F2, respectively.

Of the 8 regions into which the Mexican electricity system has been divided, the OR (Eastern) region is the one in which fossil fuel costs are lowest. This then is the region least favorable to the introduction of nuclear power plants.

Figure 1. Single-plant cost comparison
(per KW of installed power capacity)



it is supposed that the project choice will exert no influence over such evaluation parameters as the cost of capital or of foreign exchange. Typically also, the project decision will be tested for sensitivity to a number of parameters. Here the single-plant comparison shows immediately that the optimal process decision is sensitive to the cost of fossil fuel (expected to rise during the 1970's) and to the number of hours of operation of the plant.*

The number of hours of operation of an individual power plant is a parameter that is not independent of the mix of other units within the system. For example, the Mexican grid contains a substantial number of hydroelectric units. These have a turbine/reservoir capacity ratio that permits the hydro plants to produce a large fraction of their energy output during the peak demand hours. This initially available plant-mix favors the introduction of nuclear units. Given the hydro plants in the system, nuclear units may be employed as "base-load" plants--- operated round-the-clock to take advantage of their low incremental costs per kilowatt-hour of energy.**

Ever since the pioneering study of Massé and Gibrat (1957)*** at Electricité de France, it has been recognized that linear programming provides a tool for embedding individual project choices within a power systems framework -- allowing, e.g., for complementarities between hydro and nuclear plants.

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* One kilowatt (KW) of power capacity operated for 1,000 hours will produce 1,000 kilowatt-hours (KWH) of electrical energy. The following units of measurement are also employed:

$$10^6 \text{ KW} = 1 \text{ GW}$$
$$10^9 \text{ KWH} = 1 \text{ TWH}$$

** For a more detailed exposition of this topic, see Berrie (1968, pp. 198-200).

*** For two surveys of applications within the Massé-Gibrat framework, see Bessière (1969) and Anderson (1970). For a mixed integer programming application, see Gateley (1970).

Each plant is then viewed as a source of joint products: electricity available at different points of time during the daily, weekly and seasonal cycle of operations. INTERCON is a model in the Massé-Gibrat tradition -- a model in which the number of hours of operation of each unit becomes an endogenous variable. This increase in generality is not costless. We must now collect data on components of the system other than the individual project itself.

3. Algebraic formulation of static multi-region model

Unlike a single-project analysis, INTERCON is disaggregated in time and space. The demands at different points of time and space are to be met at minimum cost. The greater the interregional diversity in the quantities of energy demanded (or of hydroelectric energy supplied) at different points of time, the greater will be the cost savings from interconnection through transmission lines.

The index t is employed to distinguish between two representative years: 1977 (t = 1), the first year in which a .600 GW nuclear plant might be in operation, and 1980 (t = 2), the first year for a 1.000 GW plant. Within each of these representative years, three four-month seasons are denoted by the subscript s. (Hydroelectric energy supplies differ markedly from one to another of these three seasons.) The 4 daily peak hours are distinguished from the 20 off-peak hours by the energy demand block subscript k. (See Table 1.)

The index r identifies the eight regions that might be interconnected by transmission lines. Each region is simplified as though it formed a single representative point. The distances for 12 possible transmission links -- and the fossil fuel costs within each region -- are shown on Figure 2. Isolated

Table 1. Identification of Indices

Class	Generic index for class	Specific values of index for members of class	
Plant type	i	I1	initially available gas turbine plants
		I2 and I3	initially available fossil plants
		HD	initially available hydroelectric plants
		F1	.075 GW gas turbine plants
		F2	.300 GW fossil plants
		F3	.600 GW fossil plants
		N1	.600 GW nuclear plants
		N2	1.000 GW nuclear plants
Transmission line voltage	j	1	230 kv (kilovolt) transmission lines
		2	400 kv (kilovolt) transmission lines
Region	r	TC	Torreón-Chihuahua
		FM	Falcón-Monterrey
		TP	Tampico
		SL	San Luis
		OR	Eastern
		CL	Central
		OC	Western
		MS	Mazatlan-Sonora
Demand block	k	1	peak hours, March-June (486.7 hours)[a]
		2	off-peak hours, March-June (2,433.3 hours)[b]
		3	peak hours, July-October (486.7 hours)[a]
		4	off-peak hours, July-October (2,433.3 hours)[b]
		5	peak hours, November-March (486.7 hours)[a]
		6	off-peak hours, November-March (2,4333 hours)[b]
Season	s	1	March-June
		2	July-October
		3	November-February
Representative year	t	1	March, 1977 - February, 1978
		2	March, 1980 - February, 1981

Notes:

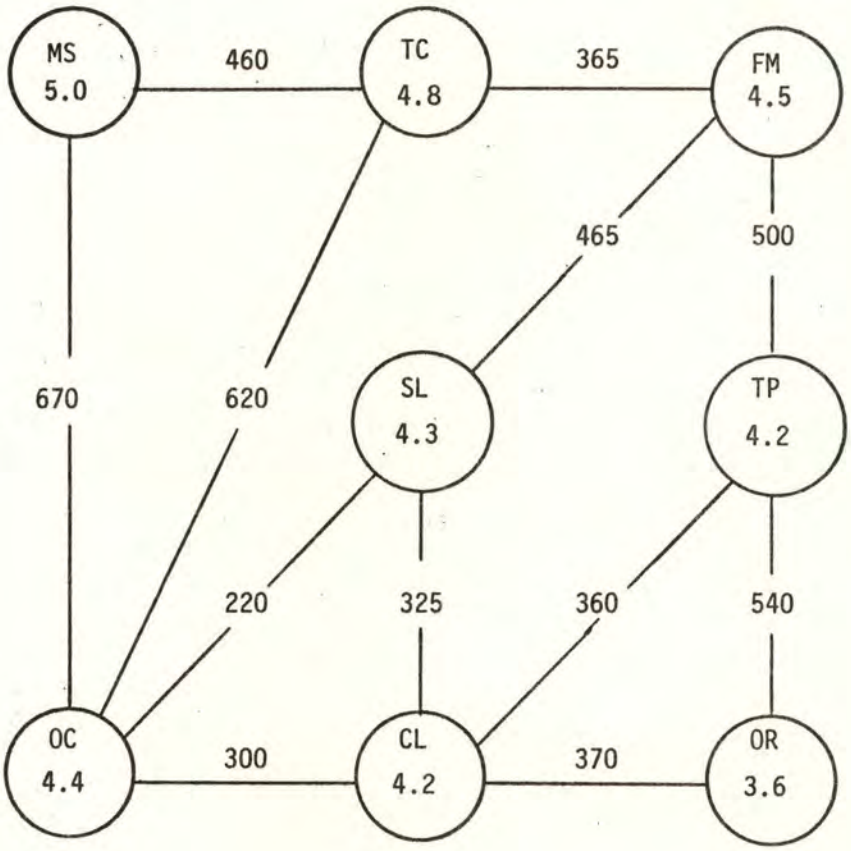
a/ For demand blocks $k = 1, 3, 5$, the time duration η_k is:

$$\eta_k = 486.7 \frac{\text{peak hours}}{\text{season}} = \left(\frac{4 \text{ peak hours}}{\text{day}}\right)\left(\frac{365 \text{ days}}{\text{year}}\right)\left(\frac{\text{year}}{3 \text{ seasons}}\right).$$

b/ For demand blocks $k = 2, 4, 6$, the time duration η_k is:

$$\eta_k = 2433.7 \frac{\text{peak hours}}{\text{season}} = \left(\frac{20 \text{ off-peak hours}}{\text{day}}\right)\left(\frac{365 \text{ days}}{\text{year}}\right)\left(\frac{\text{year}}{3 \text{ seasons}}\right).$$

Figure 2. Schematic diagram of distances (km.) and 1977 fossil fuel prices[a/] (pesos per million BTU)



Note:

[a/] For 1980, fossil fuel prices are taken to increase by .5 pesos per million BTU in each region.

Table 2. Number of Constraint Rows, Single-Period Models

Row description	Unit of measurement (annual flow)	Row identifying letter(s)	Row indices	Year t = 1 Region r = TC	FM	TP	SL	OR	CL	OC	MS	rr'	Total number of rows	Year t = 2 Region r = TC	FM	TP	SL	OR	CL	OC	MS	rr'	Total number of rows
Peak power demand	GW	GW	rst	2	2	2	2	2	2	2	2		16	2	2	2	2	2	2	2	2		16
Energy demand	TWH	TWH	rkt	6	6	6	6	6	6	6	6		48	6	6	6	6	6	6	6	6		48
Peak power generation capacity	GW	G	irt	4	4	3	0	4	4	4	4		27	4	4	3	0	4	4	4	4		27
Energy generating capacity	TWH	E	irkt	24	30	24	15	30	30	30	30		213	24	30	36	21	42	42	42	30		267
Fossil fuel requirements	10^{12} BTU	CF	rt	1	1	1	1	1	1	1	1		8	1	1	1	1	1	1	1	1		8
Hydro turbine limits	TWH	EHD	rkt	6	6	6	0	6	6	6	6		42	6	6	6	0	6	6	6	6		42
Hydro reservoir limits	TWH	HD	rst	3	3	3	0	3	3	3	3		21	3	3	3	0	3	3	3	3		21
Peak power transmission	GW	U	jrr'st									32	32									32	32
Energy transmission	TWHR	V	jrr'kt									96	96									96	96
Project capacity, nuclear	GW	CPN	rt		1		1		1	1			4										0
Project capacity, transmission	GW	CPT	rr't									6	6										0
Mutual exclusivity	-	MUEX	-										4	4										0
Total number of rows				46	53	45	25	52	53	53	52	134	4	517	46	52	57	30	64	64	64	52	128	557

Table 3. Number of Activity Columns, Single-Period Models

Column description	Unit of measurement	Column identifying letter(s)	Column indices	Year t = 1 Region r = TC	FM	TP	SL	OR	CL	OC	MS	rr'	Total number of columns	Year t = 2 Region r = TC	FM	TP	SLL	OR	CL	OC	MS	rr'	Total number of columns
Peak power capacity utilized	GW	G	irt	6	7	6	3	7	7	7	7		50	6	7	7	4	8	9	9	7		57
Energy generation, above minimum	TWH	E	irkt	30	36	30	15	36	36	36	36		255	30	36	42	21	48	48	48	36		309
Fossil fuel requirements	10^{12} BTU	CF	rt	1	1	1	1	1	1	1	1		8	1	1	1	1	1	1	1	1		8
Peak power transmitted	GW	U	jrr'st									64	64									64	64
Energy transmitted	TWH	V	jrr'kt									192	192									192	192
Transmission capacity installed	GW	T	jrr't									12	12									12	12
Zero-one variables for transmission and nuclear generating projects	-	Y	-										22 22										0
Total number of columns				37	44	37	19	44	44	44	44	268	22 603	37	44	50	26	57	58	58	44	268	642

systems (e.g., those in Baja California) have been excluded from INTERCON.

Transmission costs and line losses depend upon the line voltage. To distinguish between 230 kv and 400 kv lines, we employ the transmission line voltage subscript j. Electric power generating plants are distinguished from one another by the plant type subscript i. (Again see Table 1.)

When formulated as a mixed integer programming model for the single representative year 1977 (t = 1), INTERCON contains 517 rows and 603 non-slack columns. Since additional types of generating units will be available in 1980 (t = 2), there are more rows and columns in the programming model for the later year -- 557 rows and 642 columns. See Tables 2 and 3 for a row and column summary. The unknowns refer to the installation and utilization of either generating or transmission capacity.

To simplify typesetting and to avoid subscripts, we have adopted the convention that the various indices (i, j, k, r, s and t) appear on the identical line as the remainder of the name of a row or a column. For example, the row (GW rst) refers to peak power in region r, season s, year t. The column G irt refers to peak power capacity of generating plant type i, utilized in region r, year t. Coefficients are identified by a lower-case Greek letter and then by the various indices. For example, the line loss coefficient α jr'r refers to the peak power received in region r per unit transmitted from region r', employing the line voltage j.

With these notational conventions, the peak power and the energy demand constraints (GW rst and TWH rkt) are written, respectively:

(GW rst)

peak power capacity utilized in region r + (peak power received in region r per unit transmitted from region r') (peak power transmitted to region r from region r') - (peak power transmitted to region r' from region r) ≥ (peak power demand, region r, season s, year t)

$$\sum_i G_{irt} + \sum_j \sum_{r' \neq r} (\alpha_{jr'r})(U_{jr'rst}) - \sum_j \sum_{r' \neq r} U_{jrr'st} \geq$$

(TWH rkt)

(minimum hours for energy generation, plant type i, block k) (peak power capacity utilized, plant type i) + energy generation, above minimum, plant type i - (energy received in region r per unit transmitted from region r') (energy transmitted to region r from region r')

$$\sum_i (\beta_{ik})(G_{irt}) + \sum_i E_{irkt} + \sum_j \sum_{r' \neq r} (\alpha_{jr'r})(V_{jr'rkt})$$

energy transmitted to region r' from region r

$$- \sum_j \sum_{r' \neq r} V_{jrr'kt} \geq$$

(energy demand, region r, block k, year t)

For the initially available plant types, the peak power capacity is a datum. Upper bounds on the individual unknowns are therefore written as follows:

(G irt)

peak power capacity utilized, plant type i

$$G_{irt} \leq \left(\begin{array}{l}\text{peak power}\\ \text{capacity available,}\\ \text{plant type } i\end{array}\right) \quad (i = I1, I2, I3, HD)$$

In order for the boilers of fossil and nuclear plants to remain fired up so as to meet the daily demand peaks, these units cannot be operated at power levels below 25% of their capacity. This minimum limit has already been introduced through the coefficients β_{ik} in rows (TWH rkt). The operating time available <u>above</u> these minimum levels is given by the coefficient γ_{ik}. This leads to the following upper bounds on above-minimum energy production for each plant type, region, block and time period:

(E irkt)

energy generation, above minimum $\left(\begin{array}{l}\text{hours for energy}\\ \text{generation above}\\ \text{minimum, plant}\\ \text{type } i, \text{ block } k\end{array}\right)\left(\begin{array}{l}\text{peak power}\\ \text{capacity}\\ \text{utilized,}\\ \text{plant type } i\end{array}\right)$

$$E_{irkt} \leq \gamma_{ik} \; G_{irt} \quad (i \neq HD)$$

The fossil fuel requirements (i.e., the fuel inputs for plants other than hydro and nuclear) are calculated through equations (CF rt)

(CF rt)

$$\begin{pmatrix}\text{fossil fuel}\\ \text{requirements}\end{pmatrix} = \begin{pmatrix}\text{fossil fuel requirement per GW for minimum annual energy generation}\end{pmatrix}\begin{pmatrix}\text{GW of peak power capacity utilized, fossil plant type } i\end{pmatrix} + \begin{pmatrix}\text{fossil fuel requirement per TWHR of energy generation above minimum}\end{pmatrix}\begin{pmatrix}\text{TWHR of energy generation, above minimum fossil plant type } i\end{pmatrix}$$

$$CF\ rt = \sum_i (\delta\ ir)\ (G\ irt) + \sum_i \sum_k \epsilon\ ir)\ (E\ irkt) \qquad (i \neq HD, N1, N2)$$

Turbine limits imply the following upper bounds on hydro operations:

(EHD rkt)

$$\begin{pmatrix}\text{hydroelectric energy generated, block } k\end{pmatrix} = EHD\ krt \leq \begin{pmatrix}\text{hydroelectric energy generation limit, block } k\end{pmatrix} \qquad (k = 1, 2, \ldots, 6)$$

The following constraints are derived from reservoir limits governing hydro energy output during season s.

No attempt has been made to allow for the stochastic character of hydro energy flows:

(HD rst)

$$\begin{pmatrix}\text{hydroelectric energy generated, blocks } k \text{ and } k+1\end{pmatrix} = EHD\ krt + EHD\ (k+1)rt \leq \begin{pmatrix}\text{hydroelectric energy available during season } s \text{ containing blocks } k \text{ and } k+1\end{pmatrix} \qquad (s = 1, 2, 3)$$

For each of the 12 possible pairs of points r and r', there are capacity constraints on the peak power and on the energy transmitted. These constraints are identified as: (U jrr'st) and (V jrr'kt) respectively:

(U jrr'st) power transmission capacity available between regions r and r', year t

peak power transmitted between regions r and r', season s

$$U_{jrr'kt} + U_{jr'rkt} \leq \begin{pmatrix} \text{initially} \\ \text{available} \\ \text{capacity} \end{pmatrix} + T_{jrr't}$$

capacity installed between initial date and year t

(V jrr'kt)

energy transmitted between regions r and r', block k

$$V_{jrr'kt} + V_{jrr'kt} \leq (\eta_k) \begin{pmatrix} \text{hours per} \\ \text{year,} \\ \text{block k} \end{pmatrix} \begin{pmatrix} \text{power transmission capacity} \\ \text{available between region r} \\ \text{and r', year t} \end{pmatrix} \leq \begin{pmatrix} \text{initially} \\ \text{available} + T_{jrr't} \\ \text{capacity} \end{pmatrix}$$

For the time duration coefficients η_k, refer back to Table 1.

4. Numerical data

Tables 4-7 contain most of the numerical data for INTERCON.* At the time the model was constructed (1970), these data represented the most plausible estimates available within the Comisión Federal de Electricidad. Inevitably, they are subject to revision as newer information becomes available. These are approximately -- but not altogether -- consistent with the data employed within the energy sector model ENERGETICOS and the macroeconomic model DINAMICO.

The INTERCON programming problem is stated as one of supplying the electricity demands at minimum cost to the electricity producing enterprise.** This criterion does not necessarily coincide with minimizing the costs to the Mexican economy as a whole. Here the evaluation of investments is based upon a 10% annual rate of return on capital, converting domestic currency costs into foreign exchange at the conventional rate of 12.5 pesos per U.S. dollar. (For a sensitivity analysis of the fossil-nuclear choice with respect to the discount rate and the foreign exchange scarcity premium, see Table 13, memorandum 70-10.) In order to arrive at a static approximation to a dynamic investment planning problem, the initial investment costs have been converted to annual flows through "capital recovery factors."

Some features of the solution may be anticipated directly from the basic data. According to Table 4, the countrywide peak power demand occurs

* On the assumption that these were of limited general interest, we have not reproduced the region-by-region distribution of either the energy demands for each time block or the hydro supplies for each season. A sample of these data (for the OR region) appears in Table 13 below. For the complete details, worksheets are available for inspection in the authors' offices.

** We have reproduced only the basic data from which the cost coefficients were derived. These basic data are more directly understandable than the list of coefficients associated with more than 600 individual activities.

during season 3 (November-February). The only regions that constitute an exception to the countrywide pattern are the three northern ones (TC, FM and MS). There, the irrigation and air conditioning loads shift the annual peak into season 2 (July-October). Once that transmission links are established between the north and the south, the July-October peak power demand constraints are non-binding in any region, and can be satisfied at zero incremental cost.

The lack of diversity between regional demands may be quantified through a calculation for year $t = 1$ (1977). With no transmission between regions, the regional seasonal peak demands would lead to requirements totalling 11.768 GW of installed generating capacity. If all regions were interconnected and there were no transmission losses, the countrywide peak would occur during season 3, and these demands would lead to a total of 11.418 GW of capacity. (See Table 4.) In other words, interconnection would reduce the capacity requirements by at most .350 GW = 11.768 - 11.418 GW. This saving (3% of installed generating capacity) could be a significant gain to the electricity producing enterprise. From the viewpoint of constructing a model of the energy sector or of the economy as a whole, however, this calculation immediately suggests that it would be an acceptable simplification to allow for an annual peak, and to neglect the peak that occurs during each individual season. If seasonal disaggregation of the peak were to enter into macroeconomic planning, this would also imply the need for geographical disaggregation of power demands -- e.g., a distinction between the demand in the north and that in the south.

Table 4. Peak Power Demands[a] (Unit: GW)

Year t	1977			1980			
	1			2			
Season s / Region r	2	3	Regional seasonal peak	2	3	Regional seasonal peak	% per year increase, 1977-80
TC	.563	.506	.563	.690	.620	.690	7.0%
FM	1.297	1.228	1.297	1.931	1.829	1.931	14.2
TP	.175	.181	.181	.277	.285	.285	16.4
SL	0	0	0	0	0	0	-
OR	2.687	2.769	2.769	4.097	4.223	4.223	15.1
CL	4.174	4.450	4.450	5.617	5.987	5.987	10.6
OC	1.581	1.698	1.698	2.240	2.403	2.403	12.3
MS	.810	.586	.810	1.190	.862	1.190	13.7
Total	11.287	11.418	11.768	16.042	16.209	16.709	12.4

Table 5. Characteristics of Initially Available Generating Plants[a]

Region r \ Plant identification index i	Fossil fuel requirements, ϵ_{ir} (10^{12} BTU per TWH of energy)				Combined capacity of all units initially available (GW)					Regional demands and capacity deficits (GW), year t = 1	
	I1	I2	I3	HD	I1	I2	I3	HD	Total capacity	Regional seasonal peak demand	Capacity deficit
TC	15.9252	14.5446	11.3772	0	.158	.063	.216	.038	.475	.563	.088
FM	15.9931	11.2834	9.6236	0	.056	.610	.150	.071	.887	1.297	.410
TP	15.9931	9.6236	-	0	.014	.300	0	.018	.332	.181	0
SL	-	-	-	-	0	0	0	0	0	0	0
OR	16.542	12.9469	10.7149	0	.062	.066	.117	1.428	1.673	2.769	1.096
CL	15.6641	11.0920	9.4377	0	.152	.226	1.050	2.060	3.488	4.450	.962
OC	16.1381	13.0048	9.6236	0	.064	.040	.750	.572	1.426	1.698	.272
MS	15.9408	13.3228	10.8342	0	.095	.098	.232	.192	.617	.810	.193
			Total		.601	1.403	2.515	4.379	8.898	11.768	3.021

Note:

[a] Since the historical investment cost of these facilities is a datum, these costs are omitted from the minimand. Future operating and maintenance costs are not bygones, and these are therefore included.

Table 6. Characteristics of Generating Plants Available by 1980

Plant type			Fossil			Nuclear	
Identification index i			F1[a]	F2	F3	N1	N2
Capacity of single unit (GW)			.075	.300	.600	.600	1.000
Thousands of hours per year available for energy generation[b]	minimum, β_{ik}	k = 1,3,5	0	.1095	.1095	.1034	.1034
		k = 2,4,6	0	.5475	.5475	.5171	.5171
	above minimum, γ_{ik}	k = 1,3,5	.3942	.3285	.3285	.3102	.3102
		k = 2,4,6	0	1.6425	1.6425	1.5512	1.5512
Fossil fuel requirements[c] (10^{12} BTU)	per GW for minimum annual energy, δ_{ir}		0	18.7048	17.7390	0	0
	per TWH of energy generation above minimum, ϵ_{ir}		15.000	9.490	9.000	0	0
Operating and investment costs (10^6 pesos)	nuclear fuel cost per TWH[c]		0	0	0	15.66	15.20
	fixed annual operating and maintenance costs of single unit[c]		1.5	9.4	13.0	20.5	25.5
	initial investment cost for single unit[d]		75.0	428.0	780.0	1380.0[e,f]	2160.0[e]
Year t in which plant type may first be installed in region r, 1977 (t=1), or 1980 (t=2)	r =	TC	1	1	n.a.	n.a.	n.a.
		FM	1	1	n.a.	1	n.a.
		TP	1	1	2	n.a.	2
		SL	1	1	n.a.	1	2
		OR	1	1	2	n.a.	2
		CL	1	1	2	1	2
		OC	1	1	2	1	2
		MS	1	1	n.a.	2	n.a.

Notes on Table 6:

a/ F1 index refers to gas turbines. It is non-optimal to employ these as "base-load" units, i.e., during the off-peak time blocks 2, 4 and 6. It may, however, be optimal to employ these for peak hours and/or reserve power capacity.

b/ Calculated through the following coefficients:

plant type	gas turbine	other fossil	nuclear
identification index i	F1	F2 and F3	N1 and N2
operating hours available, fraction of year (time other than that required for scheduled maintenance and for unscheduled repairs)	.81	.90	.85
minimum time for energy production, fraction of operating hours (time required for boilers to remain fired up so as to meet daily peak)	0	.25	.25

E.g., since there are .4867 thousand hours per year in time blocks, 1, 3 and 5, the β ik factor for these blocks and for i = F2 is:

$$\beta \text{ F2 k} = (.90)\ (.25)\ (.4867) = .1095 \text{ thousand hours}\ .$$

Similarly, since there are 2.4333 thousand hours per year in time blocks 2, 4 and 6, the γ ik factor for these blocks and for i = F2 is:

$$\gamma \text{ F2 k} = (.90)\ (.75)\ (2.433) = 1.6425 \text{ thousand hours}\ .$$

c/ For fuel and operating costs, the present value factor is $(1/1.10)^{.5}$, as though these costs were paid at the middle of year t.

d/ The initial investment costs have been converted into equivalent constant annual flows through "capital recovery factors", For INTERCON, these factors are based upon a 10% annual cost of capital. The service life is taken to be 20 years for gas turbines (plant type F1) and 30 years for all other generating and transmission facilities. Assuming that the first annual payment is made on January 1 of year t, the capital recovery factor is therefore 10.68% per year for gas turbines and 9.64% for all other facilities.

e/ The nuclear plant cost estimates refer to all regions other than CL and OC. For these two regions, the following is assumed:

region r	investment cost for single nuclear unit (million pesos)		reason for additional investment costs in region r
	.600 GW unit	1.000 GW unit	
CL	1460	2240	additional transmission line, 200 kilometers
OC	1440	2260	additional cooling tower

f/ For constructing two nuclear plants (each .600 GW) at a single location for use in 1977 (t = 1), it is supposed that there would be a saving of 20% (276 million pesos) in the investment cost for the second unit.

n.a./ Not available.

Table 7. Characteristics of Transmission Lines (Zero Compensation)

Line number	Region r'	Region r	Distance (kilometers)	Line voltage: 230 kv (j = 1)		Line voltage: 400 kv (j = 2)			
				Initially available capacity (GW)	Power received in region r per unit transmitted from region r', $\alpha_{1r'r}$	Initially available capacity (GW)	Power transmission capacity for new line (GW)	Capital investment cost for new line (million pesos)	Power received in region r per unit transmitted from region r', $\alpha_{2r'r}$
1	SL	OC	220	0	-	0	.846	79.2	.9794
2	CL	OC	300	.300	.9519	.687	.687	108.0	.9719
3	SL	CL	325	0	-	0	.650	117.0	.9696
4	TP	CL	360	0	-	0	.613	129.6	.9660
5	TC	FM	365	.300	.9458	0	.605	131.4	.9658
6	OR	CL	370	.300	.9453	.602	.602	133.2	.9653
7	TC	MS	460	0	-	0	.513	165.6	.9570
8	FM	SL	465	0	-	0	.512	167.4	.9560
9	FM	TP	500	0	-	0	.471	180.0	.9532
10	TP	OR	540	.300	.9295	0	.443	194.4	.9495
11	TC	OC	620	0	-	0	.403	223.2	.9420
12	OC	MS	670	0	-	0	.380	241.2	.9373

From the basic data, one may observe not only the lack of demand deversity between regions but also the size of the regional capacity deficits. In Table 5, we have erred on the side of overestimating these deficits. That is, transmission is neglected, and it is supposed that each region would require sufficient capacity to meet its own seasonal peak demand. On that basis, between 1975 and 1977 (year t = 1), it would be necessary to add 3.021 GW to the installed capacity. This magnitude is equivalent to five nuclear plants, each unit .600 GW in size.* If the five construction projects were all completed on schedule, there could be substantial amounts of temporary excess capacity. More likely -- with five hastily constructed nuclear plants -- there would occur delays which could easily wipe out any cost savings to be obtained from a nuclear program. Because of the likelihood of these construction difficulties, it has therefore been supposed that at most two nuclear units (each .600 GW in capacity) could be available by 1977. For capacity added in subsequent years, this construction scheduling limit has been removed.

Among the process options listed in Table 6, note that no new hydroelectric generating projects are listed. This simplification was adopted -- partly because Mexico is close to exhausting her supply of low-cost hydro sites, and partly because hydro project costs have been consistently underestimated in the past.

5. Mixed integer programming formulation

Divisibility is one of the axioms that underlies the theory of linear programming. This assumption creates no special difficulties with fossil fuel plants installed prior to 1977. Moreover, for all types of capacity added subsequent to 1977, the economies-of-scale are not large in

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* After allowing for transmission and for the gains from regional diversity, it turns out that the required capacity increment would be equivalent to four rather than five units. See the INTERCON results reported in Table 11.

relation to the annual increases in power demand. For transmission links and nuclear generating plants installed prior to 1977, there are difficulties with divisibility. When INTERCON is solved as a linear program for this year, it turns out that the optimal capacity additions consist, for the most part, of fractional nuclear plants and fractional transmission lines (Table 8).

To avoid solutions that call for fractional units, a mixed integer programming model was employed. So as to apply the IPE algorithm (described in memoranda 71-2 and 71-3), all zero-one variables have been defined as "projects" and grouped into "mutual exclusivity sets". Table 9 describes the relations between these project variables (which may take on the value of either zero or one) and the continuous linear programming unknowns.

In order to reduce the computer time spent on mixed integer programming, the original formulation was simplified in several ways. One such simplification consisted of dropping four regions as possible locations for nuclear power plants. These four appeared to be unpromising locations -- either because of low capacity deficits (regions TC and TP), or because of low fossil fuel prices (region OR), or because of an isolated position in the transmission network (region MS).

For each of the four regions that remain as candidates for the installation of nuclear capacity (that is, for r = FM, SL CL and OC), Table 9 contains an equation identified as (CPN r). These four equations link the zero-one project variables to the continuous linear programming unknowns G Nlrl, which define the nuclear capacity additions in the four regions. (For the four non-candidate regions, the continuous unknowns G Nlrl are removed from the matrix at this point.)

Table 8. Optimal Linear Programming Solution to Single-Period Model for t = 1 (1977)

Region r	Nuclear generating capacity added in region r; level of linear programming variables G N1r1	Fraction of one .600 GW plant
FM	.629 GW	1.05
SL	.004 GW	.01
CL	.104 GW	.17
OC	.252 GW	.42
MS	.211 GW	.35
Total	1.200 GW	2.00

Regions r and r'	400 kv transmisson capacity added between regions r and r'; linear programming variables T 2rr'1	400 kv power transmission capacity for one line added between regions r and r'	fraction of one 400 kv line between regions r and r'
SL-OC	.096 GW	.846 GW	.11
TP-CL	.003 GW	.613 GW	.01
FM-SL	.097 GW	.512 GW	.19
FM-TP	.159 GW	.471 GW	.34
OC-MS	.144 GW	.380 GW	.38

Total annual costs: 1976 million pesos

Table 9. Tableau for Linking Continuous Variables to Indivisible "Projects"

Row identification \ Column identification		Continuous variables, nuclear generating capacity				Mutually exclusive projects, nuclear generating capacity, set 1																
		GN1FM1	GN1SL1	GN1CL1	GN1OC1	Y 1,1	Y 2,1	Y 3,1	Y 4,1	Y 5,1	Y 6,1	Y 7,1	Y 8,1	Y 9,1	Y 10,1	Y 11,1	Y 12,1	Y 13,1	Y 14,1		
nuclear generating capacity (GW)	CPNFM	-1.0					.6				.6	.6	.6							=	0.
	CPNSL		-1.0					.6			.6			.6	.6		1.2			=	0.
	CPNCL			-1.0					.6			.6		.6		.6		1.2		=	0.
	CPNOC				-1.0					.6			.6		.6	.6			1.2	=	0.
	MUEX1					1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	=	1.0

Row identification \ Column identification		Continuous variables, transmission capacity						Mutually exclusive transmission projects											
								set 2			set 3			set 4					
		T2SLOC1	T2SLCL1	T2TPCL1	T2FMSL1	T2FMTP1	T2OCMS1	Y 1,2	Y 2,2	Y 3,2	Y 1,3	Y 2,3	Y 3,3	Y 1,4	Y 2,4				
transmission capacity (GW)	CPTSLOC	-1.0							.846	.846						=	0.		
	CPTSLCL		-1.0							.650						=	0.		
	CPTTPCL			-1.0									.613			=	0.		
	CPTFMSL				-1.0				.512	.512						=	0.		
	CPTFMTP					-1.0						.471	.471			=	0.		
	CPTOCMS						-1.0								.380	=	0.		
	MUEX2							1.0	1.0	1.0						=	1.0		
	MUEX3										1.0	1.0	1.0			=	1.0		
	MUEX4													1.0	1.0	=	1.0		

14 mutually exclusive "projects" for nuclear generating capacity have been defined. (Mutual exclusivity is ensured through the constraint MUEX 1.) If the first of the 14 projects is adopted (i.e., if Y 1,1 = 1.0), no nuclear capacity is added. If project 2 is adopted (i.e., if Y 2,1 = 1.0), one plant of .600 GW will be added, and this unit will be located in region FM. Similarly, if Y 14,1 = 1.0, two units totalling 1.200 GW will be located in region OC. Note that the 14 alternatives have been defined so that no matter which of them is adopted, no more than 1.200 GW of nuclear capacity will be added to the system prior to year t = 1.

As another step in economizing upon the mixed integer computations, we reduced the number of transmission links from 12 to 6 possibilities during the initial time period. For each of these 6 possibilities, there is a continuous linear programming unknown T 2rr'1 which defines the transmission capacity to be added between regions r and r' -- employing a 400 kv. line and installing this capacity prior to year 1. Table 9 shows that these 6 continuous variables are linked to the zero-one "projects" through the 6 equations (CPT rr').

The transmission links have been grouped into three mutual exclusivity sets. Mutual exclusivity is ensured through the three constraints MUEX 2,3,4. Two of the three sets are based upon geographical complementarity between transmission links. Within a set, individual projects are distinguished by the index p. For the three projects identified as Y p,2 (p = 1, 2, 3), SL is employed as the junction point between the north and the south. For projects Y p,3 (p = 1, 2, 3), TP is the junction point. Projects Y p,4 (p = 1, 2) refer to an either-or choice on a single transmission link, OC-MS.

Within the columns defining the transmission projects, the coefficients refer to the capacity of a 400 kv. line. E.g., from Table 9, it can be seen that if project Y 2,2 is operated at unit intensity, this would imply the construction of two links joining at SL. The longer of these (FM-SL) would have a transmission capacity of .512 GW, and the shorter (SL-OC) would have .846 GW.

To summarize the definitions presented in Table 9: There are 14 mutually exclusive nuclear generating projects, 3 mutually exclusive transmission projects with SL as the junction point, 3 projects with TP as the junction point, and 2 projects for the OC-MS link. This leads to $14 \cdot 3 \cdot 3 \cdot 2 = 252$ logically possible alternative combinations. Hereafter, these 252 combinations will be identified through a four-number sequence identifying which of the projects has been selected within each set. E.g., combination (12,2,1,2) refers to adopting project number 12 in the first mutual exclusivity set, number 2 in the second, 1 in the third, and 2 in the fourth.

6. Recursive dynamic optimization

INTERCON could have been handled directly as a dynamic investment planning problem. In addition to the multi-region static model already described in Sections 3 and 5, the following irreversibility constraints would be imposed -- both for generating and for transmission capacity other than the initially available units. These multi-period linkage rows are the only ones in which variables with the time index t would appear simultaneously with others having the time index t+1:

$$G_{ir\ t+1} \geq G_{ir\ t}$$

$$T_{jrr'\ t+1} \geq T_{jrr'\ t}$$

If the regional and seasonal disaggregation of INTERCON were retained, a dynamic problem would require upwards of 500 rows for each year distinguished over the planning horizon. As an alternative to this, we have attempted to economize on computer time through a recursive optimization procedure.* First, the static version for year t = 1 was solved as a mixed integer programming problem. Among other results, this indicated the capacity levels G_{ir1} and $T_{jrr'1}$. These values were then substituted as constants into the capacity irreversibility constraints and added to the static problem for year t = 2. Since these are lower bounds on individual unknowns, this adds a negligible amount to the time required for solving the static model.

Clearly, the recursive procedure produces a feasible solution to the multi-period problem. Equally clearly, since future-period data do not influence the capacity mix chosen in year 1, this may be a far from optimal solution. Because it fails to anticipate the upward trend in fossil fuel prices, the procedure would be biased in favor of adding fossil capacity during the initial year. On the other hand, to the extent that it eliminates any anticipation of the improved nuclear plants that will become available subsequently, the procedure is biased in favor of the initial nuclear process. (See the numerical data in Figure 2 and Table 6.) In order to overcome these objections -- and simultaneously perform a sensitivity analysis -- we have applied the identical recursive procedure to four alternative combinations of generating and transmission projects. The results are summarized in Table 10.

---

* Each static linear programming version of INTERCON required less than 10 minutes of execution time on the IBM 360/67 of Stanford University. A two-period dynamic version (solving simultaneously for the 1977 and 1980 capacity mix) required approximately two hours on this same machine.

Table 10. Summary of Single-Period, Recursive Optimizations

project combination			(1,1,2,2)	(7,1,2,2)	(8,1,2,2)	(12,2,1,2)
total annual costs	t = 1	(1977)	2,086.	2,006.	2,003.	2,002.
(unit: million pesos)	t = 2	(1980)	3,358.	3,266.	3,271.	3,260.
cumulative nuclear capacity installed[a] t = 1 (1977) (unit: GW)	region r =	FM	0	.600	.600	0
		SL	0	0	0	1.200
		CL	0	.600	0	0
		OC	0	0	.600	0
		Total	0	1.200	1.200	1.200
cumulative nuclear capacity installed[b] t = 2 (1980) (unit: GW)	region r =	TC	n.a.	n.a.	n.a.	n.a.
		FM	1.065	1.093	1.084	1.019
		TP	.476	.501	.511	.077
		SL	0	0	0	1.200
		OR	.487	1.361	1.311	1.334
		CL	2.274	2.268	2.187	2.274
		OC	.856	.856	.945	.124
		MS	.569	.567	.567	.571
		Total	5.727	6.646	6.605	6.599
increment in 400 kv transmission capacity (1975-77) (unit: GW)[c]	regions rr'	SL-OC	0	0	0	.846
		SL-CL	0	0	0	0
		TP-CL	0	0	0	0
		FM-SL	0	0	0	.512
		FM-TP	.471	.471	.471	0
		OC-MS	.380	.380	.380	.380
increment in 400 kv transmission capacity (1975-80)[d]	regions rr'	SL-OC	0	0	0	.950
		CL-OC	0	0	0	0
		SL-CL	0	0	0	0
		TP-CL	0	0	0	0
		TC-FM	.179	.110	.101	.085
		OR-CL	0	0	0	0
		TC-MS	.006	0	.008	.003
		FM-SL	0	0	0	.512
		FM-TP	.471	.471	.471	0
		TP-OR	0	0	0	0
		TC-OC	0	0	0	0
		OC-MS	.380	.380	.380	.380
annual fossil fuel requirements[e]	t = 1	(1977)	368.4	278.2	277.9	282.8
(unit: 10^{12} BTU)	t = 2	(1980)	172.8	108.4	101.2	112.7

Notes:

[a] Equal to level of activity GN1r1

[b] Equal to level of activities GN1r2 + GN2r2

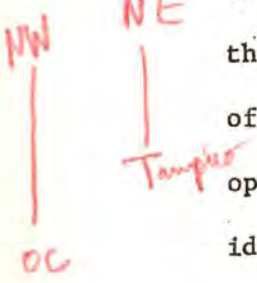
[c] Equal to level of activity T 2rr'1

[d] Equal to level of activity T 2rr'2

[e] Equal to level of activities $\sum_{r}$ CFrt (t = 1,2)

For the static problem, project combination (12,2,1,2) minimizes the total annual equivalent of investment plus operating costs during year t = 1. This solution -- obtained through mixed integer programming -- calls for the installation of two nuclear units at location SL, together with the following transmission lines: SL-OC, FM-SL and OC-MS.

Three alternatives to this solution were also calculated -- arbitrarily* selecting one of the mutually exclusive generating projects and then applying mixed integer programming to select the least-cost combination of transmission projects. E.g., if no nuclear plant is to be built, the optimal transmission links would be: FM-TP and OC-MS. This combination is identified as (1,1,2,2). If either nuclear project 7 or 8 were adopted, these transmission links would also be optimal. The two middle columns of Table 10 are therefore identified as combinations (7,1,2,2) and (8,1,2,2).



Among the three combinations that call for the installation of two nuclear units with a capacity total of 1.200 GW, the total costs are virtually indistinguishable. These differ by less than any plausible margin for error in the basic data. Each is 4% less costly than combination (1,1,2,2), the program in which no nuclear capacity is installed during the initial year. Of the four alternative solutions, note that (12,2,1,2) is least costly during both planning periods. The empirical results suggest that this is a "robust" combination, and that little would have been learned by formulating INTERCON as a two-period dynamic problem.

---

*These alternatives were not chosen in an altogether arbitrary way. Most of the preliminary site investigations for nuclear plants were based on an investment plan that resembled nuclear generating "project" 7.

7. Numerical results

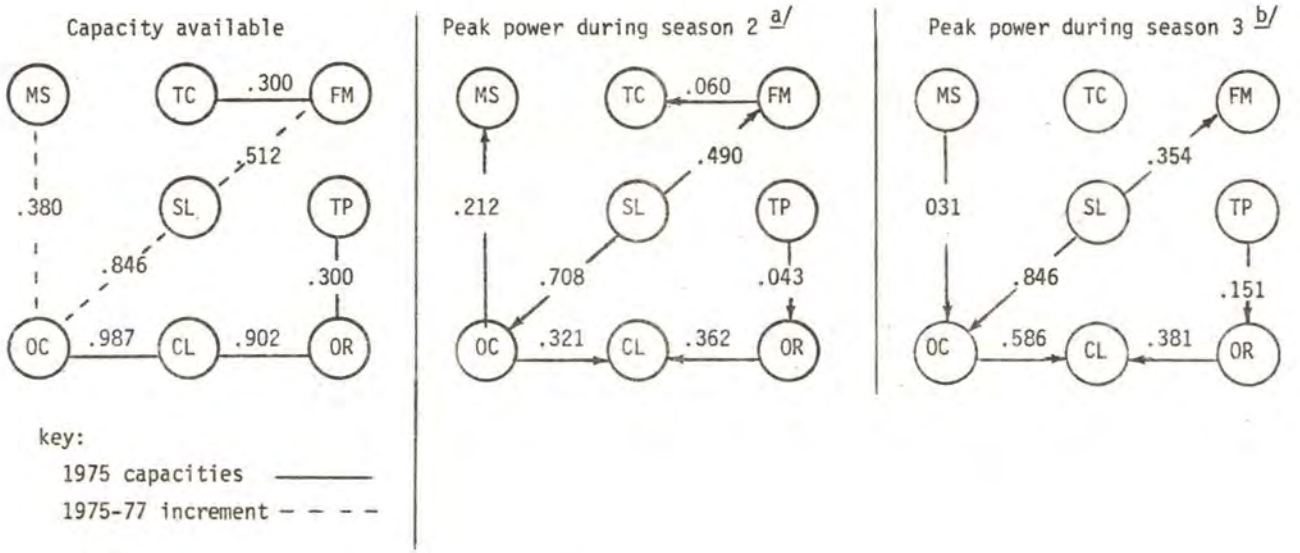
Figures 3, 4 and Tables 11-13 contain numerical results on project combination (12,2,1,2), the least-cost solution obtained through the recursive optimization procedure. These are typical of the results obtained for the other project combinations. The details should not be taken too literally, but they help to illustrate how the grid's operations are simulated by INTERCON.

Table 11 summarizes the mix of generating capacities available for operation during year t = 1. The bulk of the 1975-77 capacity increase is concentrated at two locations: SL (1.200 GW of nuclear capacity) and OR (1.336 GW of fossil capacity). For each of the static multi-region cases that we have run, the OR region has proved to be the most preferable location for installing fossil capacity. The result is immediately understandable as soon as one recalls that this is the region where fossil fuel costs are lowest.

The countrywide total of installed generating capacity lies between the countrywide peak and the regional seasonal peak demands already shown in Table 4 (11.418 and 11.768 GW respectively). Somewhat unexpected is the result that the optimal amount of installed capacity is 11.493 GW -- almost as low as the countrywide peak. Apparently, the relative costs of transmission and of generating capacity are such that it is optimal* to

* As a byproduct of the IPE mixed integer programming computations, there is readily available a numerical example of the tradeoffs between transmission and generating costs. If no nuclear plants are to be added to the system, the optimal project combination is (1,1,2,2). For year t = 1, this combination has an annual cost of 2086 million pesos for generation and transmission. If the FM-TP and OC-MS transmission lines are eliminated, but we maintain the constraint that no nuclear plants are to be added to the system, this combination of projects would be identified as (1,1,1,1). For this combination, total costs would increase to 2123 million pesos. There would be a saving of 41 million pesos on the FM-TP and OC-MS transmission lines, but an increase of 78 in the generating costs.

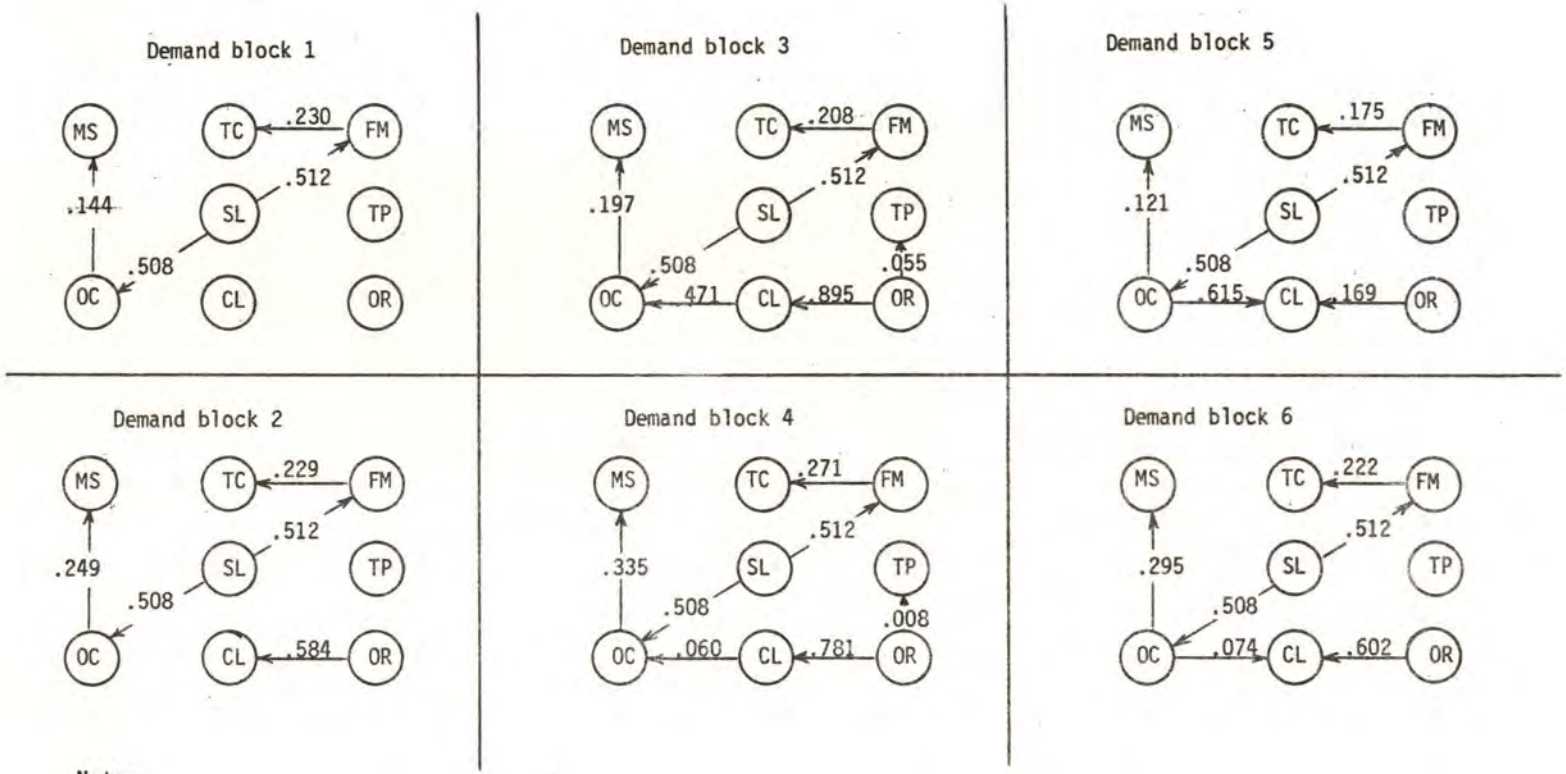
Figure 3. Schematic diagram of power transmission (unit: GW) - year t = 1 (1977); project combination (12, 2, 1, 2)



Note:

a/ Equal to level of activities $\sum_{j=1}^{2} U_{j\,rr'\,sl}$ for season s.

Figure 4. Schematic diagram of energy transmission [a/], year t = 1 (1977); project combination (12, 2, 1, 2); TWH converted into GW equivalents by time factors η_k



Note:

[a/] Equal to level of activities $\sum_{j=1}^{2}$ V j rr' k1 for block k.

Table 11. Cumulative Capacity Installed,[a] t = 1 (1977), Project Combination (12,2,1,2) (unit GW)

Region r \ Plant identification index i	Initially available fossil and hydro	Fossil		Nuclear	Total
	I1 + I2 + I3 + HD	F1	F2	N1	
TC	.475	0	.031	n.a.	.506
FM	.887	0	.003	0	.890
TP	.332	0	0	n.a.	.332
SL	0	0	0	1.200	1.200
OR	1.673	0	1.336	n.a.	3.009
CL	3.488	.025	0	0	3.513
OC	1.426	0	0	0	1.426
MS	.617	0	0	n.a.	.617
Total	8.898	.025	1.370	1.200	11.493

Notes:

[a] Equal to level of activity G irl

n.a. Not available.

interconnect each region to at least one other. Transmission then permits the northern and the southern regions to interchange power during their respective peak periods, seasons 2 and 3 respectively.

The transmission linkages also help in understanding the set of optimal marginal costs. In Table 12, it can be seen that there is a zero marginal cost of meeting power demands during season 2. For season 3 (the countrywide peak demand period), the marginal annual costs vary from one region to another, but lie within the range of 102.09 to 125.85 pesos per kilowatt.

These marginal costs for power and for energy lie somewhat below the values suggested by the single-plant analysis for a fossil generating unit. According to Figure 1, the annual costs of such a plant would be 167.40 pesos per kilowatt of peak power.* For energy produced by this unit -- even when fossil fuel is priced as low as 3.6 pesos per million BTU -- the costs are estimated at 49.31 pesos per thousand kilowatt-hours.

The formulation of INTERCON is such that the marginal costs of supplying energy will vary from one region r and one demand block k to another. (The marginal costs for energy in year 1 are equal to the dual variables associated with the 48 constraint rows TWH rk1.) According to Table 12, there are interregional differences of up to 30% in the marginal cost of energy. E.g., during time blocks 1 and 2, the cost per thousand KWH

* These numbers are not directly comparable with the marginal costs for peak power that emerge from the sectoral model ENERGETICOS (memorandum 70-10, Tables 3 and 10). The sectoral model has no regional dimension -- hence contains no tradeoffs between generating and transmission costs. Instead, the investment and operating costs for transmission lines are taken to be directly proportional to the demand for peak power. With this definition of "peak power", it is to be expected that the marginal costs of meeting the demand will appear considerably higher in ENERGETICOS than in INTERCON.

Table 12. Marginal Cost of Supplying Electricity, t = 1 (1977), Project Combination(12, 2, 1, 2)

Region r	Pesos per KW of peak power demand[a]		Pesos per thousand KWH of energy demand,[b]					
	season s =		block k =					
	2	3	1	2	3	4	5	6
TC	0	106.32	51.18	51.18	51.18	51.18	51.18	51.18
FM	0	106.79	48.41	48.41	48.41	48.41	48.41	48.41
TP	0	112.92	38.54	38.54	38.43	38.43	38.54	38.54
SL	0	102.09	39.54	39.54	38.08	38.08	39.54	39.54
OR	0	121.48	39.39	39.39	35.72	35.72	40.10	40.10
CL	0	125.85	40.80	40.80	37.79	37.79	41.54	41.54
OC	0	122.31	40.37	40.37	38.88	38.88	40.37	40.37
MS	0	114.64	43.07	43.07	41.48	41.48	43.07	43.07

Notes:

[a] Equal to dual variable, constraint row GW rs 1

[b] Equal to dual variable, constraint row TWH rk1

ranges from 39.39 pesos in region OR (where fossil fuel costs are low) up to 51.18 pesos in region TC (where fossil fuel costs are high). Within each of the regions, the marginal cost of energy does not vary widely from one demand block to another. The largest differences occur in region OR. We now take a closer look at the annual cycle in the operation of each type of plant within this one region.

For Table 13, the plants of the OR region have been ranked in ascending order of their marginal fuel cost. These costs are zero not only for the energy produced by hydroelectric units, but also for the minimum quantities of energy produced in fossil units so that the boilers will remain fired up to meet the daily demand peaks. Within the OR region, the low-cost plant type (F2) is operated at its maximum level throughout the year, and the high-cost types (I2 and I1) are operated at their minimum levels. The marginal plants are those identified as I3. These units would be operated at their minimum rates during time blocks 3 and 4, and at their maximum rate during the other four blocks. Note that the cost of operating these marginal units is 36.78 pesos per thousand KWH. According to Table 12, this lies above the marginal cost of supplying energy in the OR region during blocks 3 and 4 (35.72 pesos) and below the marginal costs during the other four time blocks (39.39 or 40.10 pesos).

One detail of the annual operating cycle is not recorded explicity in these tables -- the fact that the hydroelectric turbine capacity constraints are non-binding. Indeed, these constraints (EHD krt) are non-binding in any of the regions during year t = 1. This means that hydroelectric energy can be shifted freely from the off-peak to the peak time blocks within each season, subject only to the reservoir constraints (HD rst). It is for this

Table 13. Generation and Transmission of Energy, OR Region, t = 1 (1977), Project Combination (12, 2, 1, 2)

Block k		1	2	3	4	5	6	Fossil fuel energy cost, plant type i (pesos per thousand KWH)[a]
				(unit: TWH)				
Generation minimum	$\sum_i \beta$ ik Gi OR1	.167	.832	.167	.832	.167	.832	0
Generation, above minimum, E i ORk1 activities	i = HD	.086	1.639	.634	2.291	.219	1.931	0
	i = F2	.439	2.195	.439	2.195	.439	2.195	32.57
	i = I3	.038	.192	0	0	.038	.192	36.78
	i = I2	0	0	0	0	0	0	44.44
	i = I1	0	0	0	0	0	0	56.78
Transmission from region OR	V2ORCLk1	0	-1.421	- .293	-1.465	- .082	-1.465	
	V1ORCLk1	0	0	- .146	- .179	0	0	
	V1ORTPk1	0	0	- .027	- .019	0	0	
Energy demand, region OR, block k, year 1		.730	3.436	.774	3.655	.781	3.685	
Marginal cost of supplying energy (pesos per thousand KWH)		39.39	39.39	35.72	35.72	40.10	40.10	

Note:

[a] Fossil fuel energy cost (region OR, year 1) = $(\epsilon$ i OR$)$ (3.6) $\left(\frac{10^6 \text{ BTU}}{10^3 \text{ kwhr}}\right)\left(\frac{\text{pesos}}{10^6 \text{ BTU}}\right)$ $\left(\frac{1}{1.10}\right)^{.5}$ (present value factor--as though fuel costs were paid at midyear)

reason that Table 12 indicated the marginal cost of energy to be identical during each of the two time blocks that form a season. Even during year t = 2 (1980), when substantial amounts of nuclear capacity have been added to the system and are operated as base-load units, the hydroelectric turbine capacity constraints remain non-binding in virtually all of the regions.*

8. Effects of aggregation -- the implications for multi-level planning

From this case study, what lessons have we learned about aggregation? Are kilowatt-hours interchangeable between different points of time and space? Regrettably, INTERCON provides no all-purpose answer to these questions. This will depend upon the policy decision to which the model is addressed, how quickly an answer is needed, and what are the data-gathering and computational costs. E.g., if an investment decision entails significant tradeoffs between generating versus transmission facilities, it is clear that spatial disaggregation must be introduced into the evaluation of the project. Moreover, since one of the purposes of transmission is to take advantage of regional diversities in the timing of electricity supplies and demands, it follows that seasonal disaggregation is needed -- say, six demand blocks within an annual cycle of operations.

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* TP is the only region in which the turbine constraints are operative during 1980. This might therefore be the first location in Mexico at which it would be optimal to install pumped storage facilities. These facilities consume energy during the off-peak hours of the daily cycle, and produce it during the peak hours. In order for such investments to be worthwhile, there must be a sizeable differential between the marginal cost of energy during the two blocks of time.

Alternatively, if a model is addressed to the problems of linkage between the electricity sector and the balance of the economy, it is much less clear that seasonal and spatial disaggregation will prove to be worth the costs of data-gathering and computation.* Consider the ENERGETICOS model. There, instead of taking the fossil fuel price as an exogenously given parameter, we attempt to embed the electricity sector's decisions within a broader framework -- linking these decisions to those in the oil and gas sector. For ENERGETICOS, no regionally disaggregated investment planning model of the petroleum sector was readily available. Even for the drilling department of the petroleum enterprise itself, it would be no easy task to estimate the regional distribution of future oil and gas wells.

All would be lost if each level in a problem-solving hierarchy insisted upon halting its work until some other level had produced a completely satisfactory set of disaggregated data. Typically, this is the point where aggregation enters into multi-level planning -- as a substitute for missing pieces of information. Here aggregation is employed because too few data are available. This is a more difficult problem to formalize than the aggregation problem typically described in the econometric literature -- the case where too many data are available, and the aim is then to reduce the volume of numerical computations. Cf. Fisher (1969).

INTERCON, which contains both a spatial and a seasonal dimension, helps to place bounds on the possible errors created by aggregation. It suggests that: (1) There will be virtually no error if we neglect the regional diversity of power demands and characterize the annual peak in

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* For understanding a system, the extent of disaggregation need not be as detailed as for understanding the components of that system. Here is a dictum in search of a theory! Would the theory depend upon the finiteness of the information processing capability of a single human mind?

terms of the countrywide total. (2) Within a region, there will be negligible daily and seasonal variations in the marginal cost of electrical energy. (3) From one region to another, the marginal cost of electrical energy will not differ by more than 30%.

Aggregation rules -- together with error bounds -- are not readily transferable from one nation or decade to another. They are, however, among the vital information flows which enable one level to assist the next in a hierarchy for solving complex problems.* Each of the simplifications that emerges from INTERCON can be useful in constructing ENERGETICOS, a planning model with a broader scope. In turn, the aggregated results of ENERGETICOS may be embedded within the macroeconomic model DINAMICO to check whether the energy sector's investment decisions are likely to alter the economy-wide efficiency prices of, say, foreign exchange, capital and skilled labor. As things turn out (memorandum 70-11), the evidence indicates that the energy sector exerts a negligible influence upon these macroeconomic efficiency prices. This series of multi-level planning studies suggests that there are diminishing returns from systems analysis. Beyond a certain point, the analyst can neglect the feedback effects upon the system within which the individual project decision is embedded.

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* Cf. K. Boulding (1966, p. 8): "There is a great deal of evidence that almost all organizational structures tend to produce false images in the decision-maker, and that the larger and more authoritarian the organization, the better the chance that its top decision-makers will be operating in purely imaginary worlds. This perhaps is the most fundamental reason for supposing that there are ultimately diminishing returns to scale."

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Memorandum 71-3
February, 1971

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A Mixed Integer Algorithm for Project Evaluation*

Contents

Page number

1. Relation to other mixed integer algorithms 1
2. The IPE lemma. 2
3. An investment planning example: interdependence between projects in two sectors. 6
4. Remarks on writing a computer program for IPE. 12
5. An investment planning example: four locations and two time periods 13
6. Effects of external prices upon project evaluation 19

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* This research was supported by the Development Research Center, International Bank for Reconstruction and Development. The specific facts, methods of analysis, and conclusions are the sole responsibility of the individual author.

** The author is indebted to Richard Inman, who wrote the computer programs and performed the numerical analysis reported here. See his accompanying paper, Inman (1971).

Thanks also go to Arthur Geoffrion, who reviewed an earlier draft of this paper, and pointed out the connection of the IPE algorithm to Benders' partitioning procedure. In addition, he noted that it would economize on computer capacity to store the list of glb's (greatest lower bounds) in implicit rather than explicit form.

1. Relation to other mixed integer algorithms

This algorithm is a specialized application of Benders' partitioning procedure for solving mixed integer programming problems.* It is visualized that the principal application would be for the evaluation of indivisible and interdependent investment projects. Hence we shall refer to the zero-one unknowns as "project decision variables", to the remaining nonnegative unknowns as "continuous variables", and to the algorithm itself as IPE. Toward the end of this paper, we shall examine the relation between IPE and the project evaluation criterion proposed by Little and Mirrlees (1968).

IPE (or any other variant upon Benders' partitioning procedure) is an implicit enumeration algorithm. It resembles branch-and-bound in that it proceeds by solving successive linear programming problems - at each step placing bounds upon the value of the objective function attainable with alternative combinations of the project decision variables. Both methods are similar in that there are only a finite number of combinations to be considered, a given combination is examined only once, and convergence is assured with only a finite number of steps.

IPE differs from branch-and-bound in that: (1) At each step, all of the zero-one variables are set at integer values. Hence, each step provides a locally optimal integer-feasible solution for an as yet untried combination of the zero-one variables. (2) Dual variables and reduced costs are generated at each step. These are employed to provide lower bounds on the value of the global minimand attainable with each of the remaining combinations of the project decision variables. Thus, each step provides both upper and lower bounds relevant to the global optimization - rather than bounds that are restricted to the particular "branch" being explored.

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* A general formulation of the partitioning procedure was first stated in Benders (1962). Also see Benders, Catchpole and Kuiken (1959), Balinski (1965, pp. 271-274), Buzby (1966), Hu (1969, pp. 259-265), and Geoffrion (1970).

Underlying Benders' partitioning procedure is the assumption that it is moderately expensive to solve an individual linear programming problem - and prohibitively expensive to solve one such problem for each of the N logically possible combinations of the zero-one variables. IPE differs from other forms of Benders' procedure in that the "master" all-integer problem is solved through complete enumeration of the N possible combinations. From an esthetic viewpoint, enumeration is inelegant. However, since IPE takes advantage of mutual exclusivity constraints* between projects, this is not an obviously inefficient approach for problems where N is a moderate-sized number, say less than a million.

As yet, we have insufficient computational experience for comparing the efficiency of the proposed technique versus that of alternative mixed integer algorithms. To date, our experience points to only one reasonably certain conclusion - that with economically meaningful investment planning models, it is unnecessary to calculate explicit linear programming solutions for more than a small fraction of the logically possible total number of project combinations.

2. The IPE lemma

Denote the continuous decision variables by x_j, and denote the kth project decision variable within set s as y_{ks}. Within each of the S mutual exclusivity sets, exactly one of the K alternative projects is to be selected. Hence:

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* For another mixed integer algorithm that takes advantage of mutual exclusivity constraints, see Healy (1964).

$$(1) \qquad \sum_{k=1}^{K} y_{ks} = 1 \qquad (s = 1 \cdots S)$$

$$(2) \qquad y_{ks} = 0 \text{ or } 1 \qquad (\text{all } k \text{ and } s)$$

$$(3) \qquad x_j \geq 0 \qquad (j = 1 \cdots J)$$

Given the coefficients ($a_{ij}, c_j, \alpha_{iks}$ and γ_{ks}) and the right-hand side constants b_i, the mixed integer programming model consists of assigning values to the x_j and y_{ks} so as to minimize (5) subject to constraints (1)-(4):

$$(4) \qquad \sum_{j=1}^{J} a_{ij} x_j + \sum_{s=1}^{S} \sum_{k=1}^{K} \alpha_{iks} y_{ks} = b_i \qquad (i = 1 \cdots I)$$

$$(5) \qquad \min z = \sum_{j=1}^{J} c_j x_j + \sum_{s=1}^{S} \sum_{k=1}^{K} \gamma_{ks} y_{ks}$$

According to (1) and (2), the number of logically possible project combinations is* $(K)^S = N$. (In actual applications, it is likely that specific arguments could be employed to reduce N. E.g., a blast furnace would not be installed unless a steel mill or an iron foundry were also available at the identical location.) Now suppose that a particular combination of values has been assigned to the unknowns y_{ks}, and that these values satisfy constraints (1) and (2). Let the index n be employed to identify this specific combination.

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* Here S denotes a power of K. At all other points, a raised index will denote a superscript.

Given the combination n, the problem has been reduced to conventional linear programming format - minimization of (5) subject to constraints (3) and (4) on the continuous variables x_j, regarding the zero-one unknowns as parameters provisionally fixed at the specific values y^n_{ks}. From the simplex solution for combination n, we obtain the locally optimal values of the primal variables x^n_j and of the minimand z^n. For each of the constraints (4), there are locally optimal dual variables: $u^n_i, (i = 1 \cdots I)$. From the dual variables, in turn we obtain the "reduced costs", δ^n_{ks}. These quantities may be interpreted as the cost increase or decrease associated with project ks at the efficiency prices prevailing for combination n:

$$\delta^n_{ks} = \gamma_{ks} - \sum_{i=1}^{I} \alpha_{iks} u^n_i$$

We shall omit the details on three special cases: (a) an unbounded solution - which would then be the global minimum, (b) the possibility that there is no feasible solution for combination n, and (c) the possibility that there remain zero-intensity artificial activities in the optimal basis - hence no dual variables for one or more of the I rows in (4). Cases (b) and (c) may be avoided by formulating the model so as to include high-cost positive and negative unit vectors for each of the I rows.

Now consider a move from combination n to some other combination m - but without explicitly solving the linear programming model for combination m. Then the key to the IPE algorithm is the following lemma:

$$z^m \geq z^n + \sum_{s=1}^{S} \sum_{k=1}^{K} \delta^n_{ks} \, (y^m_{ks} - y^n_{ks}) \tag{6}$$

Proof of lemma: Retaining the nonnegativity constraints (3) on the continuous primal unknowns x_j, rewrite (4) and (5) as the following parametric linear programming problem. The symbol θ denotes a scalar parameter, with $0 \leq \theta \leq 1$. When θ is varied from zero to unity, the simplex method will produce the local minima z^n and z^m respectively. The values y^n_{ks} and y^m_{ks} are known constants, and so the right most double-sum terms of (4a) and (5a) are also constants:

$$\text{(5a)} \qquad \min z = \sum_{j=1}^{J} c_j x_j + \sum_{s=1}^{S} \sum_{k=1}^{K} [\gamma_{ks}(y^m_{ks}-y^n_{ks})]\theta + \sum_{s=1}^{S} \sum_{k=1}^{K} \gamma_{ks} y^n_{ks}$$

$$\text{(4a)} \qquad \sum_{j=1}^{J} a_{ij} x_j + \sum_{s=1}^{S} \sum_{k=1}^{K} [\alpha_{iks}(y^m_{ks}-y^n_{ks})]\theta = b_i - \sum_{s=1}^{S} \sum_{k=1}^{K} \alpha_{iks} y^n_{ks}$$

$$(i = 1..I)$$

Now interpret θ as the intensity of a new activity being introduced into the locally optimal solution for combination n, with $0 \leq \theta \leq 1$. The conventional "reduced cost" criterion for the introduction of this new activity is $\sum_{s=1}^{S} \sum_{k=1}^{K} \delta^n_{ks}(y^m_{ks}-y^n_{ks})$. This implies the weak inequality (6) - which was to be proved.

Remarks:

(a) Note that z^n is associated with an integer-feasible solution, and immediately provides an <u>upper</u> bound on the value of the global minimand z.

(b) The key lemma (6) establishes a <u>lower</u> bound on z^m - a bound which may be calculated solely with the linear programming results for project combination n: z^n, u^n_i, and δ^n_{ks}. If $\sum_{s=1}^{S} \sum_{k=1}^{K} \delta^n_{ks}(y^m_{ks}-y^n_{ks}) \geq 0$, combination m may be immediately rejected, and no linear programming solution need be calculated explicitly for that combination. The computational efficiency of IPE (or any other Benders-type implicit enumeration algorithm) will depend upon the frequency with which

this inequality permits us to reject combinations. Note that (6) is a criterion for rejecting project combinations - not individual projects by themselves.

(c) The differences in project intensities $(y^m_{ks}-y^n_{ks})$ are either +1, 0, or -1. Moreover, within mutual exclusivity set s, there will be at most one positive and one negative difference.

(d) If a locally optimal solution for combination n has been obtained through the simplex method, this immediately provides a dual-feasible initial basic solution for combination m. (Proof: All that has changed is a constant term in each of the rows (4a) and (5a).) For obtaining the optimal solution with combination m, it should be advantageous to apply the dual simplex technique, initiating the calculations from the basic solution for combination n.

3. An investment planning example: interdependence between projects in two sectors

Our first example is a simplified version of a model described in Chenery (1959). This example illustrates interdependence between investment decisions in the case of steel and machinery projects. A hypothetical Latin American country is concerned with choosing the level of imports and exports, simultaneously selecting among indivisible investment projects in these two sectors. Costs are to be minimized - subject to the constraint of delivering 1 MT/Y (million tons/year) of steel and 1 MT/Y of machinery to "final demands" outside the steel and machinery sectors. It takes one ton of steel to make one ton of machinery, but no machinery is needed as a current account input to produce steel.

The continuous decision variables are $x_1 \cdots x_4$, the rate of imports and exports of steel and machinery, respectively. Steel projects may be built in one of three mutually exclusive sizes: 0,2 and 4 MT/Y. Machinery projects may be built in one of three mutually exclusive sizes: 0,1 and 2 MT/Y. Assuming that no more than one project of each type is to be built, there are three decision variables for steel projects: y_{11}, y_{21}, and y_{31}, and three for machinery: y_{12}, y_{22} and y_{32}. The number of logically possible combinations is therefore $N = (K)^S = 3^2 = 9$.

The programming tableau appears in Table 1 - first the cost row, then the material balances, and then the mutual exclusivity rows. From the cost row, it can be seen that economies-of-scale are significant. E.g., in order to double the output of steel from 2 to 4 MT/Y, the annual costs would increase from 320 to 520 $ millions/year, an increase of approximately 60%. Export earnings enter as a negative item in the cost row. Note that the costs and scales of demand have been arranged so that it is cheaper to import both steel and machinery rather than to import one item and to produce the other domestically. The global optimum, however, is achieved through a coordinated "big push" in which both items are produced domestically, there are no imports or exports, and $y_{21} = y_{22} = 1$. With this combination of projects, the total costs are 650 $ million/year.

Below the cost row in Table 1, there are two material balance rows - for steel and machinery respectively. These correspond to equation group (4) above, with $I = 2$. Next come the two mutual exclusivity rows - one for the steel and one for the machinery sector. These correspond to equation group (1) above, with $S = 2$. It is understood that the unknowns are to satisfy the nonnegativity constraints (3) and the integer constraints (2).

Table 1

Detached coefficients tableau for steel and machinery example

	continuous variables				steel projects			machinery projects			
	imports, steel	imports, machinery	exports, steel	exports, machinery	size: 0MT/Y	size: 2MT/Y	size: 4MT/Y	size: 0MT/Y	size: 1MT/Y	size: 2MT/Y	
unknowns	x_1	x_2	x_3	x_4	y_{11}	y_{21}	y_{31}	y_{12}	y_{22}	y_{32}	
costs	200	500	-50	-250		320	520		330	530	= z$millions/ year
material balance-steel	1		-1			2	4		-1	-2	= 1 MT/Y
material balance-machinery		1		-1					1	2	= 1 MT/Y
mutual exclusivity-steel projects					1	1	1				= 1
mutual exclusivity-machinery projects								1	1	1	= 1

Table 2 traces out the successive step of IPE. Each column corresponds to one step - the results of the linear programming optimization for a particular combination n. Each column is divided into four parts: (a) the identification of n and the values of the dual variables π_i^n; (b) the values of the reduced costs δ_{ks}^n; (c) the locally optimal minimand z^n (denoted by an asterisk), together with the values of the glb (greatest lower bound) known for z^m (for all $m \neq n$); and (d) the global lub (least upper bound) and the global glb on z.

Each of the N combinations is identified here by a two-digit number - the first of which identifies the size index k of the steel project and the second the size index of the machinery project. Now suppose that the first step is taken arbitrarily, and that the project combination chosen is 11 - a zero size for both the steel and the machinery projects. Solving the continuous linear program for $n = 11$, it is observed that $z^{11} = 700$ \$ millions/year, and that the optimal dual values π_i^{11} are set by the import costs: \$200/ton of steel and \$500/ton of machinery. Using these prices to obtain the reduced costs δ_{ks}^{11}, we employ (6) to calculate lower bounds on costs with each of the alternative combinations m. E.g., for $m = 32$, we would have:

$$z^{32} \geq z^{11} + \sum_{s=1}^{2} \sum_{k=1}^{3} \delta_{ks}^{11} (y_{ks}^{32} - y_{ks}^{11})$$

$$z^{32} \geq z^{11} + [\delta_{11}^{11}(-1) + \delta_{21}^{11}(0) + \delta_{31}^{11}(1)] + [\delta_{12}^{11}(-1) + \delta_{22}^{11}(1) + \delta_{32}^{11}(0)]$$

$$z^{32} \geq 700 + [\ 0 + 0 - 280\ ] + [\ 0 + 30 + 0\ ]$$

$$z^{32} \geq 450$$

Table 2

Successive steps of IPE for steel and machinery example

step number			1	2	3	4	5
n,	identification of combination for linear programming optimization		11	33	21	13	22
π_1^n,	dual values for material balance - steel		200	50	50	200	200
π_2^n,	dual values for material balance - machinery		500	250	500	250	500
δ_{11}^n,	reduced costs for steel project 1		0	0	0	0	0
δ_{21}^n	reduced costs for steel project 2		-80	220	220	-80	-80
δ_{31}^n	reduced costs for steel project 3		-280	320	320	-280	-280
δ_{12}^n	reduced costs for machinery project 1		0	0	0	0	0
δ_{22}^n	reduced costs for machinery project 2		30	130	-120	280	30
δ_{32}^n	reduced costs for machinery project 3		-70	130	-370	430	-70
z^n or glb, z^m (for $m \neq n$)	(if asterisked)	project combination n or m					
		11	700*	700	700	700	-
		12	730	-	-	-	-
		13	630	630	630#	880*	-
		21	620	620#	770*	-	-
		22	650	650	650	650#	650*
		23	550	650	650	800	-
		31	420	620	870	-	-
		32	450	750	-	-	-
		33	350#	750*	-	-	-
	global lub on z		700	700	700	700	650
	global glb on z		350	620	630	650	650

Notes: * denotes locally optimal minimand z^n.

# denotes global glb on z.

A similar calculation is employed to determine a glb for each of the eight combinations $m \neq n$, and the result is entered in the glb list of Table 2. Since this first step indicates that $z^{11} = 700$, and that $z^{12} \geq 730$, the combination 12 may be dropped hereafter as a candidate for the global optimum. At the end of step 1, it is established that the optimal value of z must lie between the global lub of 700 and the global glb of 350. Since the lowest glb is associated with project combination 33, the second step consists of solving the continuous linear program for $n = 33$.

From Table 2, it will be observed that the locally optimal solution $z^{33} = 750$, that the glb from step 1 did not provide a sharp bound, and that combination 33 may be discarded hereafter. Furthermore, with these massive scales of investment, the π_i drop substantially, and are now equal to the export price of \$50/ton of steel and \$250/ton of machinery. Again applying the inequality (6) from the IPE lemma, the new dual values enable us to state that $z^{32} \geq 750$, and that combination 32 may be dropped. Furthermore, these dual values enable us to tighten the glb for combinations 23 and 31.

At the end of step 2, note that the global lub remains at 700, but that the global glb has been moved up to 620. For step 3, we therefore set $n = 21$, solve a third linear program, see that $z^{21} = 770$, and that $z^{31} \geq 870$. This proves that combinations 21 and 31 may be dropped hereafter.

Two more steps - or a total of 5 linear programming solutions out of the maximum possible of 9 - are required to discard all combinations but 22, and to verify that this is a globally optimal solution with min $z = z^{22} = 650$.

4. Remarks on writing a computer program for IPE

(a) If the computer's high-speed storage capacity were not a bottleneck, it would be advantageous to maintain an explicit list of the glb's - one for each of the N possible project combinations, as in Table 2. Alternatively, at the cost of an increase in arithmetic operations, this list may be maintained in implicit form. This is done through storing the quantities z^n and δ^n_{ks} produced at each of the explicit simplex solutions. At each step and for each combination in turn, the glb calculation (6) is repeated - applying the quantities z^n and δ^n_{ks} obtained from previous steps. In our computer program (written for an IBM 360/67), we have stored the bounds in implicit form. This will reduce the computer storage capacity requirements whenever:

$$(1 + \text{number of } y_{ks} \text{ variables}) \begin{pmatrix} \text{cumulative number of explicit} \\ \text{simplex solutions required} \\ \text{for IPE} \end{pmatrix} < N$$

(b) It is likely that the number of steps can be reduced through a judicious choice of starting-point(s). Clearly the initial lub can be lowered through a good guess at the least-cost combination. There is nothing in the basic logic of IPE that precludes two or more starting-points.

(c) The number of IPE iterations is influenced through a user-specified tolerance parameter $\varepsilon>0$. The algorithm is terminated whenever the global glb $+\varepsilon \geq$ global lub. The lower the value of ε, the larger the number of IPE steps, but the sharper become the bounds upon the minimand. Note that the algorithm may be initated with a high value of ε. After intermediate results have been examined, a lower value may be assigned to this tolerance parameter, and the calculations may be restarted.

5. An investment planning example: four locations and two time periods

Consider the problem of setting up new manufacturing capacity so as to minimize the cost of satisfying the demands for a single product at four distinct locations in two time periods. Denote the requirements (less the existing capacity) at location i, period t, by r_i^t ($i = 1 \cdots 4$; $t = 1,2$). The complete tableau for this dynamic model is shown in Table 3. The first four rows of this tableau (FRi) refer to the requirements for first-period capacity, and the second four rows (SRi) to the requirements for second-period capacity. Let the 24 continuous variables x_{ij}^t denote the quantities shipped from location i to j in period t.

Alternative test problems have been generated by stipulating two alternative sets of requirement vectors (RHS) and (RHS2). The first is based upon a symmetrical geographical distribution of demands and the other upon a more realistic non-symmetrical distribution. In both cases, it is supposed that demands are growing over time so that $\sum_{i=1}^{4} r_i^1 = 20$ units, and $\sum_{i=1}^{4} r_i^2 = 40$ units.

Suppose that capacity may be installed in only one of three mutually exclusive sizes: 0, 10, or 20 units. In general, these indivisibilities will lead to local deficits and surpluses of capacity. Suppose, however, that the system as a whole is to have no excess capacity during either time period. The assumption of indivisibilities - coupled with that of zero excess capacity - then restricts the possibilities to ten mutually exclusive "projects" for plant capacity initially available during the first time period. The mutual exclusivity row FMEX ensures that one and only one of these ten possibilities will be selected. E.g., the project variable y_6^1 enters into the mutual exclusivity constraints FMEX with a coefficient of 1. If the project is operated

at unit intensity, this means that 10 units of capacity are to be installed at locations 1 and 3 during the first time period (rows FR1 and FR3). The construction of these plants also implies that 10 units of capacity are to be available at each of these locations during the second time period (rows SR1 and SR3).

Two alternative minimand rows are specified in Table 3: COST (for which the cost of capital is taken to be 100% per period) and COST1 (with a capital cost of 40%). In both, the coefficients for each activity or project refer to the cumulative second period value of the costs.* The minimand coefficients for the project variables have been calculated by supposing that there is a "setup charge" of 3.5 units each period for each plant - independent of the amount of capacity installed.** Once the setup charge has been paid, the other manufacturing costs are taken to be proportional to the output - regardless of the location of that output.

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* E.g., for each unit shipped from location 1 to 2, the shipping costs are .3. For the second period shipping variable x_{12}^2, the coefficient of .3 therefore appears in both rows COST and COST1. For the shipping variable x_{12}^1, however, the first period cost of .3 cumulates to .6 in the second period when the cost of capital is 100% per period (row COST), but to only .42 when the cost of capital is 40% (row COST1).

** E.g., the project decision variables y_k^2 (k = 1,2,3,4) refer to setting up a single plant in period 2. The coefficients are therefore 3.5 both in rows COST and COST1. For the project decision variables y_k^1 (k = 1,2,3,4), the setup charge is to be paid both during periods1 and 2. Hence, for row COST1 (with a 40% cost of capital), the coefficient is 1.40(3.5) + 1.00(3.5) $\approx$ 8.5.

Table 3. Tableau for four location, two period investment planning model

		Continuous variables: x^1_{ij}, with ij = 12	13	14	21	23	24	31	32	34	41	42	43	Continuous variables: x^2_{ij}, with ij = 12	13	14	21	23	24	31	32	34	41	42	43	First period projects: y^1_k, with k = 1	2	3	4	5	6	7	8	9	10	Second period projects: y^2_k, with k = 1	2	3	4	5	6	7	8	9	10	Alternative requirements vectors		RHS	RHS2
First period requirements	FR1	-1	-1	-1	1			1			1															20				10	10	10														≥	5	10
	FR2	1			-1	-1	-1		1			1															20			10			10	10												≥	5	7
	FR3		1			1		-1	-1	-1			1															20			10			10		10										≥	5	3
	FR4			1			1			1	-1	-1	-1															20			10			10	10										≥	5	0
Second period requirements	SR1													-1	-1	-1	1			1			1			20				10	10	10				20				10	10	10				≥	10	15
	SR2													1			-1	-1	-1		1			1			20			10			10	10			20			10			10	10		≥	10	12
	SR3														1			1		-1	-1	-1			1			20			10			10		10		20			10			10		≥	10	8
	SR4															1			1			1	-1	-1	-1			20			10			10	10			20			10			10	10	≥	10	5
Mutual exclusivity	FMEX																									1	1	1	1	1	1	1	1	1	1											=	1	1
	SMEX																																			1	1	1	1	1	1	1	1	1	1	=	1	1
Alternative minimand vectors	COST (100% capital cost)	.6	.4	.8	.6	1.0	.6	.4	1.0	.8	.8	.6	.8	.3	.2	.4	.3	.5	.3	.2	.5	.4	.4	.3	.4	10.5	10.5	10.5	10.5	21	21	21	21	21	21	3.5	3.5	3.5	3.5	7	7	7	7	7	7				
	COST1 (40% capital cost)	.42	.28	.56	.42	.70	.42	.28	.70	.56	.56	.42	.56	.3	.2	.4	.3	.5	.3	.2	.5	.4	.4	.3	.4	8.5	8.5	8.5	8.5	17	17	17	17	17	17	3.5	3.5	3.5	3.5	7	7	7	7	7	7				

The numerical parameters have been chosen so that the optimal solution to this mixed integer program is not immediately obvious. The results depend upon the relative magnitudes of the transport costs, the economies-of-scale in the plant setup costs, and the cost of capital. This formulation produces four test problems - one for each of the possible combinations of the requirements vector (columns RHS and RHS2) and of the minimand vector (rows COST and COST1).

Table 4 lists the optimal values of the project variables and of the minimand for these four mixed integer problems. Also shown in Table 4 are the values obtained by neglecting the integer constraints on the unknowns y_k^t and solving as a conventional linear program. This provides a lower bound on the minimand for the mixed integer problem, but the bound is <u>not</u> sharp, E.g., for the test problem defined by the vectors RHS and COST, the minimand value is 14.0 cost units for the linear programming solution versus 28.0 for the mixed integer program. Note that each of these problems has been designed so that more than one simplex solution will be required by any type of implicit enumeration algorithm - regardless of whether that algorithm is based upon branch-and-bound or upon Benders partitioning.

In evaluating an algorithm such as IPE, the principal issue is the number of steps required - the number of explicit simplex solutions to be solved before reaching a globally optimal solution. With these four test problems, the number of such steps could conceivably be as large as $N = (10)^2 = 100$. In actual practice, the number has proved to be much lower, and to depend upon the particular test problem and starting-point.

Table 4. Optimal values, four test problems - four location , two period investment planning model

		optimal solution to mixed integer problem				linear programming solution, neglecting integer constraints on unknowns y_k^t			
test problem number		1	2	3	4	1	2	3	4
requirements vector		RHS	RHS	RHS2	RHS2	RHS	RHS	RHS2	RHS2
minimand vector		COST	COST1	COST	COST1	COST	COST1	COST	COST1
y_k^1, with k =	1	1.0	1.0	1.0	1.0	.25	.25	.50	.50
	2	0	0	0	0	.25	.25	.35	.35
	3	0	0	0	0	.25	.25	.15	.15
	4	0	0	0	0	.25	.25	0	0
	5	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0	0
	9	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0
y_k^2, with k =	1	0	0	0	0	.25	.25	.375	.375
	2	0	0	1.0	1.0	.25	.25	.300	.300
	3	0	0	0	0	.25	.25	.200	.200
	4	1.0	1.0	0	0	.25	.25	.125	.125
	5	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0	0
	9	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0
value of minimand		28.0	23.3	23.4	19.78	14.0	12.0	14.0	12.0

Table 5. Frequency distribution of IPE steps, four test problems - four locations, two period investment planning model (number of combinations N = 100)

test problem number requirements vector minimand vector		1 RHS COST	2 RHS COST1	3 RHS2 COST	4 RHS2 COST1
number of steps required for optimal solution via IPE - initiating algorithm from each of the 100 possible starting-points	5	0	0	2	0
	6	0	0	18	2
	7	0	0	54	62
	8	0	0	26	26
	9	3	0	0	6
	10	10	0	0	3
	11	24	6	0	1
	12	30	34	0	0
	13	30	59	0	0
	14	3	1	0	0
total number of starting-points		100	100	100	100

Table 5 contains a frequency distribution of the number of steps required to solve each of the four investment planning problems - initiating IPE from each of the 100 possible starting-points. It is encouraging that the most steps are required for the two least realistic test cases - those based upon the vector RHS, with identical demands at each of the four locations. It is also encouraging that the number of steps does not appear to go up in proportion to N, the number of combinations. Earlier, a similar series of numerical experiments had been performed on five single-period plant location problems, each with N = 35. For those problems, the number of IPE steps had ranged between 4 and 8. Here - with N increased by a factor of approximately 3 - the number of IPE steps now ranges between 5 and 14. For results on two larger and more realistic problems (N = 648), see Table 6 below.

6. Effects of external prices upon project evaluation

If there is little change in the dual variables π_i between combinations n and m - e.g., because international trade imposes narrow limits on the efficiency prices of tradable goods for a "small" country* - we would expect the inequalities (6) to provide a tight bound upon z^m. Under these circumstances, in order to arrive at an optimal selection of individual projects, it is unnecessary to construct a detailed model of the domestic economy within which these projects are to operate. The efficiency prices of that economy will coincide with the exogenously determined world prices. Little and Mirrlees (1968, ch. VII) have observed that it is then sufficient to evaluate domestic projects at the world prices.

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* Similarly, suppose that competition imposes narrow limits upon the efficiency prices within a single "small" enterprise. Projects may then be evaluated individually at the prices that are externally fixed for that enterprise.

Conversely, if external trade is limited and the individual projects are "large", the π_i can change radically from one combination to the next. E.g., for ENERGETICOS, a process analysis of Mexico's energy sector, it is supposed that there is a 37% difference between the import price and the export price of residual fuel oil.* When solving by IPE, we would not expect to reach the optimal solution in a single step. In this model - with the possibility of a 37% difference in the efficiency price of petroleum fuel from one step to the next - there may be a radical shift in the choice between electric power generating projects based upon nuclear fuel versus those based upon petroleum fuel. For electric power project decisions in Mexico, the Little-Mirrlees criterion is not a reliable guide. In order to allow for the interdependence between the efficiency price of petroleum fuel and the quantity of that item demanded by the electricity industry, multi-sector planning is needed - applying a model such as ENERGETICOS.

The IPE algorithm has been tested on one version of ENERGETICOS. This was a version in which investment projects were to be chosen so as to satisfy final demands at minimum discounted costs. The plant size, time-phasing and process choices were selected simultaneously so as to minimize the total cost of satisfying final demands. It was supposed that 27 projects were available, and that these were grouped into three mutual exclusivity sets - one set each for the steel sector (6 projects),

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* See Fernández de la Garza and Manne (1970, Table 10). Note that the 37% difference between the import and export prices includes not only an allowance for transport and marketing costs but also a 15% preference rate for domestically produced materials.

petroleum (9 projects), and electricity (12 projects). That is, $S = 3$, and the number of logically possible combinations $N = 6 \cdot 9 \cdot 12 = 648$. The number of constraint rows $I = 272$, and the number of continuous variables $J = 350$.

Two numerical cases were solved - one corresponding to a foreign exchange premium of 60%, and the other of zero. In terms of the number of IPE steps required, the algorithm proved to be reasonably efficient - 11 steps in the one case and 26 in the other. Even with 26 steps, this meant explicit linear programming solutions for only 4% out of the logically possible total of 648 combinations. Table 6 indicates the rate at which the gap was closed between the upper and lower bounds on the minimand. From this table, the reader can calculate the increase in the minimand that would have resulted from a numerical tolerance factor that cut off IPE computations prematurely. For futher details on computational efficiency, see Inman (1971).

We will conclude on a speculative note: Does the number of IPE iterations provide a useful measure of the extent of interdependence between projects? The greater the number of such iterations, the poorer are the results that may be anticipated from single-sector project selection rules of the Little-Mirrlees type. E.g., Westphal (1971) noted that the steel and petrochemical sectors of Korea were competing for a limited total of foreign capital available. Given that this scarce resource was to be rationed between the two sectors, the efficiency price of foreign capital was not an exogenous datum. For steel and petrochemical project evaluation, Westphal therefore found it worthwhile to embed these choices within a multi-sector model of the Korean economy.

Table 6. Computational summary - application of IPE for investment project evaluation in steel, petroleum and electricity sectors (number of combinations N = 648)

IPE step number	60% foreign exchange premium				zero foreign exchange premium			
	minimand, z (billions of pesos)		identification of project combination		minimand, z (billions of pesos)		identification of project combination	
	lub	glb	lub	glb	lub	glb	lub	glb
1	145.874	134.699	1,1,1	4,2,1	131.140	123.270	1,1,1	4,5,12
2	144.704	134.699	4,2,1	5,2,1	131.140	123.298	1,1,1	6,5,12
3	143.870	134.699	5,2,1	6,2,1	129.332	124.537	6,5,12	6,5,1
4	141.561	137.434	6,2,1	6,3,1	128.151	125.244	6,5,1	5,6,11
5	140.493	137.680	6,3,1	6,4,1	128.151	126.080	6,5,1	6,2,11
6	139.821	138.240	6,4,1	2,5,1	128.149	126.533	6,2,11	2,5,11
7	139.821	138.453	6,4,1	6,5,1	128.149	126.873	6,2,11	6,9,6
8	139.667	138.869	6,5,1	6,7,1	127.851	127.076	6,9,6	4,5,1
9	139.667	138.940	6,5,1	6,8,1	127.851	127.107	6,9,6	6,3,3
10	139.667	138.940	6,5,1	6,9,1	127.851	127.111	6,9,6	6,4,10
11	139.667	139.667	6,5,1	6,5,1	127.771	127.115	6,4,10	6,4,11
12					127.771	127.310	6,4,10	2,6,10
13					127.771	127.345	6,4,10	6,4,5
14					127.764	127.354	6,4,5	6,5,10
15					127.764	127.372	6,4,5,	6,5,11
16					127.764	127.413	6,4,5,	6,3,10
17					127.764	127.452	6,4,5,	6,3,8
18					127.764	127.464	6,4,5	6,8,3
19					127.764	127.519	6,4,5,	6,3,7
20					127.722	127.520	6,3,7	6,8,10
21					127.722	127.539	6,3,7	6,8,11
22					127.722	127.540	6,3,7	6,5,5
23					127.722	127.594	6,3,7	6,4,7
24					127.648	127.597	6,4,7	6,2,3
25					127.648	127.636	6,4,7	6,8,5
26					127.648	127.648	6,4,7	6,4,7

For Mexico, the capital rationing constraints appear to be less severe than those suggested by Westphal for Korea. The multi-sector model DINAMICO suggests that the efficiency price of foreign capital is virtually an exogenous datum to Mexico - the reservation price of the foreign private investor, roughly 15% per year. This means that even though several of the energy sector's projects are competing claimants for foreign capital, interdependencies between projects do not arise through capital rationing. Within Mexico's energy sector, however, the efficiency price of petroleum fuel is not an exogenous datum set by the international market. This then is a source of significant interdependencies between project decisions. Several iterations of information flow may be needed for project evaluation. There is two-way interdependence - no matter whether the energy sector's projects are evaluated by a planning agency or by an optimizing algorithm such as IPE.

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Memo 70-10

August 20, 1970

To: Participants in IBRD Basic Research Center's Project No. 1

From: Guillermo Fernández de la Garza and Alan S. Manne*

Subject: ENERGETICOS, A Process Analysis of Mexico's Energy Sectors

Contents:

		Page Number
1.	Introduction.	1
2.	Simplifications - time and space.	3
3.	Structure of the programming model; marginal costs of the common items	5
4.	Iron and steel industry	13
5.	Electricity industry.	16
6.	Petroleum and gas industry.	27
7.	Sensitivity analyses - discount rate and foreign exchange premium.	37

1. Introduction

ENERGETICOS is intended as a step toward multi-level planning, identifying the information flows necessary for consistency between project, sectoral and macroeconomic decisions. DINAMICO, a dynamic input-output model of the entire Mexican economy over the time horizon 1968-80, is being constructed in parallel.

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*For help in various phases of this work, the authors wish to acknowledge their indebtedness to: Ing. Raúl Meyer (Petróleos Mexicanos), Ings. Victor L. Games and Carlos Velasco (Instituto Mexicano del Petróleo); Lic. Gerardo Bueno and Ing. Fernando González Vargas (Nacional Financiera, S.A.); Ings. Juan Eibenschutz, Roberto Fabre, and José Alberto Valencia, and Lic. Bosco Muro (Comisión Federal de Electricidad).

ENERGETICOS allows explicitly for substitution between alternative processes within Mexico's energy-producing sectors: petroleum, gas, and electricity.* Also included are alternative energy-using activities within the iron and steel industry. Except for process substitution within the steel industry, we have not attempted to quantify the elasticity of demand with respect to the price of energy products. It is supposed that the energy sectors are "price-takers" with respect to their inputs (e.g., foreign exchange, labor, capital equipment and nuclear fuel), but that they are "output-takers" with respect to deliveries elsewhere within the Mexican economy. The linear programming optimization is phrased as one of choosing investments in alternative processes so as to meet the output targets at minimum discounted costs, taking account of intersectoral flows. Each solution generates a set of incremental costs for supplying each product.

With this formulation, the principal source of interdependence arises from the intersectoral flows of industrial fuel. (By "industrial fuel", we mean natural gas and residual fuel oil converted into their calorific equivalents.) Both gas and residual fuel oil are supplied under conditions of increasing marginal cost within the petroleum industry. There are tradeoff possibilities between these fossil fuels and the introduction of nuclear plants within the electric power industry. Hence, the rate of nuclear power development could make a significant difference in the efficiency price of fossil fuel.

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* The scope of ENERGETICOS is similar to that of an energy sector model developed by the Ministry of Power, United Kingdom. See Forster and Whitting (1968).

Several institutional features are to be noted at this point: Both the fossil fuel producer (Petroleos Mexicanos) and the electric utility consumer (Comisión Federal de Electricidad) are publicly-owned enterprises. The transfer price of industrial fuel is determined administratively at a high political level, and is not closely linked to the international fuel price prevailing in the Carribean area. During the period since the petroleum expropriation of 1938, Mexico's policy has been to remain largely self-sufficient with respect to imports and exports of petroleum products, and to stimulate industrialization through low prices for energy products. For investment planning within Mexico's energy sector, the major uncertainty appears to be the future of exploration and producing costs for domestic crude oil and natural gas. Now that domestic sources of petroleum are becoming increasingly costly, one of the key policy issues is a reexamination of the country's tradition of self-sufficiency. This in turn has macroeconomic implications with respect to the balance-of-payments, and could affect the efficiency price of foreign exchange. Raising the price of energy products could slow down the growth of those demands that are here regarded as predetermined.

2. Simplifications - time and space

Principally because of uncertainties in predicting the location of future domestic oil and gas production, ENERGETICOS does not contain a regional dimension. Each process is idealized as though it were located at a single representative point in space.

This simplification has the further advantage that it reduces the size of computations, and it eliminates those detailed investigations that would be needed to correct the transport costs for anomalies within the existing structure of pipeline and railway rates. The simplification means, however, that the present model cannot be employed directly for the choice of optimal plant locations, and that there is a need for multi-level planning.

E.g., consider the choice between a coke-intensive process versus an electricity-intensive process in the steel industy. At a specific project location, the analyst can readily allow for the fact that the relative advantage of these two alternative processes will depend upon the transport costs from the coke producing areas of northeast Mexico. This is an obvious case in which geographical disaggregation can be more readily handled at the level of project selection than at that of sectoral analysis. For a static model in which there is a simultaneous evaluation of site and process alternatives within the steel sector and an attempt to correct for rail rate anomalies, see C. Quintana, G. Bueno, and F. González Vargas, (1968).

The dimension of time plays a key role in an investment model such as ENERGETICOS. The initial year is chosen as 1974, the earliest date for which the installed capacities are not already (as of 1970) predetermined by past investment decisions and time lags. For the period 1974-80, material balances are computed year-by-year. It is anticipated that this annual disaggregation will prove particularly useful for analyzing those temporary imports and exports which would need to be synchronized with the installation of major indivisible

units of capacity.

For the electric power generating sector, because of the likelihood of success in breeder reactor technology, three additional five-year time intervals are considered - the first extending over 1981-85, the second 1986-90, and the third 1991-95. For the entire 15 year period 1981-85, it is supposed that the price of fossil fuel is a datum to the electric power sector. For the near future period 1974-80, the marginal cost of fossil fuel is one of the elements to be determined simultaneously with the other unknowns of the programming problem.

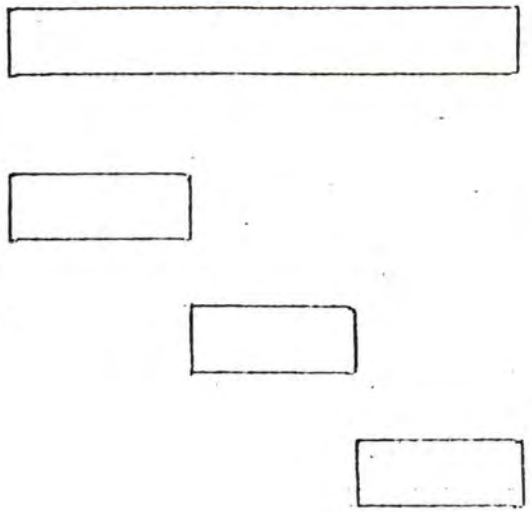
3. Structure of the programming model; marginal costs of the common items

From Figure 1, it will be seen that the coefficients of ENERGETICOS may be arranged in block-angular form - one block corresponding to each of the three industries: petroleum and gas (PE), electricity (EL), and iron and steel (SD). The only intersectoral flows consist of three items: industrial fuel, peak power and electrical energy. Once the interfaces were defined through these common items (prefix CM), it proved possible to construct and to test the sub-models independently. The following sections describe the sub-models in increasing order of their complexity - first steel, then electricity and finally petroleum.

Thus far, no attempt has been made to apply the Dantzig-Wolfe decomposition algorithm to take advantage of the block-angular

Figure 1. Block-angular form of coefficients matrix for ENERGETICOS

Block	Number of rows in block	Prefix employed for row identification
Common items	27	CM
Petroleum and gas	67	PE
Electricity	48	EL
Iron and steel	71	SD
Total	213	



structure. In the course of future research, we hope to do so - and also test out a mixed-integer programming algorithm for indivisible interdependent projects. By adopting a conventional linear programming formulation for the current version of ENERGETICOS, we have had to neglect indivisibilities, and have therefore had to predetermine the plant size that corresponds to each alternative process. With a mixed integer programming algorithm, the plant size and time-phasing can be optimized explicitly.

Because of the linear programming formulation, a further difficulty arises. We have been forced to employ "certainty equivalents". E.g., instead of regarding water inflows to the hydroelectric system as a probabilistic phenomenon, these are taken at their mean values. Similarly, instead of allowing for the influence of plant size upon the electricity generating system's reliability with respect to unscheduled downtime, we have taken an arbitrary downtime factor of 12% for reserve capacity. For studies of the Mexican electric power sector which handle one or another of these stochastic phenomena explicitly, see Stanford Research Institute (1968), and Rogers (1970). The computational techniques employed in these references cannot be readily extended to multi-sector optimal investment planning - hence the use of deterministic linear programming here.

Table 1 identifies the 213 rows that appear within ENERGETICOS.* In addition, there are 15 rows that have been employed for convenience in converting future costs into their present value, and for converting foreign exchange costs into their domestic value. Table 2 gives the structure of this cost definition matrix, and indicates the numerical values adopted for the cost parameters: a 10% annual discount rate** and a 15% level of protection against imports, converting foreign exchange into domestic currency at the conventional rate of 12.5 pesos per U.S. dollar. The concluding section of this report contains a sensitivity analysis with respect to the discount rate and the shadow price of foreign exchange.

To avoid "horizon effects", initial investment costs have been converted into annual flows through a capital recovery factor determined by the service life of the capital equipment and the 10% opportunity cost of capital. Beginning with the year in which capacity is first available, the discounted annual flows are then summed over the balance of the planning horizon.

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* A detailed computer listing is available for inspection at the office of either of the authors and also at the IBRD Basic Research Center. A simplex solution required 12 minutes on the IBM 360/50 of Comisión Federal de Electricidad and 2 minutes on the IBM 360/67 of Stanford University.

** If capital were allocated throughout the Mexican economy so that the rate of return was identical in all sectors, 15% would probably be more appropriate than a 10% discount rate. Given the present structure of Mexico's financial institutions, however, the cost of capital to public sector enterprises in the petroleum, electricity and steel industries is lower than that to private enterprises in such sectors as commercial agriculture, tourism, and motor vehicle manufacturing.

Table 1. Row identification for ENERGETICOS

Sector	Number of rows in sector	Prefix of sector employed for row identification	Suffix of row identification (omitting time index t)	Description of row	Unit of measurement (annual flow)	Exponent of 10 for unit of measurement
Common items	27 = 1x7 +2x10	CM	BTU	c/ industrial duel	BTU	12
		CM	GW	c/ peak power	GW	0
		CM	TWH	c/ electrical energy	TWHR	0
Petroleum and gas	67a/ = 7x7 +3x6	PE	GSC	dry gas (fuel oil equivalent)	barrel	6
		PE	CRD	crude oil	barrel	6
		PE	GSR	refinery gas (fuel oil equivalent)	barrel	6
		PE	GPL	c/ liquefied gas	barrel	6
		PE	GSL	c/ gasoline	barrel	6
		PE	KD	c/ kerosine and diesel oil	barrel	6
		PE	COM	residual fuel oil	barrel	6
		PE	CPj	capacity irreversibility - process j (j=C,V,H)	barrel	6
Electricity	48b/ = 4x10 +1x8	EL	TRA	requirements for additional transmission capacity	GW	0
		EL	RQF	industrial fuel requirements - electricity sector	BTU	12
		EL	CNV	convex curve - initial fossil	-	-
		EL	LMF	energy limit - new fossil	TWHR	0
		EL	LMN	energy limit - early nuclear	TWHR	0
Iron and steel	71 = 5x7 +6x6	SD	RQF	industrial fuel requirements - steel	BTU	12
		SD	MNH	iron ore	ton	6
		SD	CHAT	ferrous scrap	ton	6
		SD	COQ	coke	ton	6
		SD	LIN	c/ steel ingots	ton	6
		SD	CPj	capacity irreversibility - process j (j=1...6)	ton	6

time index t	0	1	2	3	4	5	6	11	16	21
year	'74	'75	'76	'77	'78	'79	'80	'85	'90	'95

Notes: a/ Excludes upper bounds on individual unknowns referring to domestic availability of dry gas and crude oil at three progressively higher levels of marginal cost.

b/ Excludes lower bounds on individual unknowns referring to installation of fossil units in geographically isolated systems.

c/ Demand projected exogenously - either in part or in total

Table 2. Cost definition matrix

(at 1970 prices and conventional exchange rate of 12.5 pesos per U.S. dollar)

Row description	Unit of measurement	Row identification	costs incurred in year t CSt	discounted value of import costs CSMVA	import component of costs incurred in year t CSMCORt
minimand	10^6 pesos discounted to year 0 (1974) at 10% per year	CSVA	$(.909)^t$	.15	
costs, year t	10^6 pesos, year t	CSCORt	1.000		
discounted value of import costs	10^6 pesos discounted to year 0 (1974) at 10% per year	CSMVA		-1.000	$(.909)^t$
import component of costs incurred in year t	10^6 pesos, year t	CSMCORt			1.000

Table 3. Marginal cost of supplying three energy products in year t (pesos per energy unit)

Row	Item	Energy unit	t	0	1	2	3	4	5	6	11	16	21	Conventional price paid in 1970 by a major industrial consumer
			year	'74	'75	'76	'77	'78	'79	'80	'85	'90	'95	
CMBTUt	industrial fuel a/	10^6 BTU		4.55	4.55	4.27	3.93	3.93	3.93	3.35	3.95 b/	3.95 b/	3.95 b/	5.
CMGWt	peak power	KW		563.90	627.20	549.33	597.11	590.88	581.91	683.01	636.61	347.77	609.07	c/
CMTWHt	electrical energy	10^3 KWHR		47.82	47.82	44.94	37.35	37.35	37.35	35.43	41.53	14.81	9.38	100. c/

Notes:

a/ The marginal costs for supplying industrial fuel are ex refinery, before transportation expenses to the industrial consumer.

b/ Exogenously specified fuel cost for 1981-95.

c/ The conventional electricity price of 100 pesos/10^3 KWHR is not directly comparable with the separately calculated marginal costs for energy and peak power. In order to compare the two for a round-the-clock major industrial consumer operating 7.5 10^3 hours per year, divide the peak power costs by 7.5, and add to the electrical energy costs shown above. Thus, for 1977, the full costs would be (597.11/7.5) + 37.35 = 116.96.

In projecting the exogenous final demands for each item, trend extrapolation methods were used. These projections were intended to be consistent with a 6.5% annual future growth rate of Mexico's gross domestic product, and have been cross-checked against the results of the dynamic input-output model DINAMICO.

Table 3 contains one portion of the solution to ENERGETICOS - the dual variables for the three intersectoral flow items. These results indicate the marginal cost of supplying each item in year t (after converting the 1974 "present" value back into the future value for year t). For comparative purposes, also shown are the conventional prices that would typically be paid in 1970 by a major industrial consumer such as a steel mill. Note the declining time trend in the costs for supplying electrical energy (a direct consequence of the technological trends postulated for nuclear fuel). On paper at least, there is also a declining time trend for industrial fuel, with low-cost domestic crude oil and gas reserves increasing at a sufficiently rapid rate after 1974 to offset the tendency toward rising marginal costs of exploration and production.

4. Iron and steel industry

Within the iron and steel industry, the principal opportunities for substitution between energy sources occur prior to the rolling stage. It is for this reason that the rolling processes have been omitted from the optimization here, and that steel ingots are viewed as the end product. Following Bueno and González Vargas (1969), it is estimated that the demand for ingots will increase at the compound annual rate of 9.1% over the period 1974-80.*

To satisfy the demands for new ingot capacity, six alternative processes are considered - each corresponding to an integrated plant size of 1.0 million tons per year. (See Table 4). For each activity SDjt , note that the cost coefficients in the row CSCORt include not only current account expenses such as labor, but also a capital recovery factor of 13% for depreciation plus opportunity cost of capital. With capital costs converted into annual costs, in order to ensure the irreversibility of investment, the process choices in successive time periods are linked by the following constraints:

$$(SDCPjt) \qquad SDj,\ t+1 \geq SDj,\ t \qquad (t = 0,\ldots,5)$$

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* In their demand estimates, Bueno and González Vargas made separate projections for flat and non-flat products and for seamless tubes, and then converted these into their ingot equivalents.

For 1974, the demand is taken at 6.237 million ingot tons. The existing plus planned production capacity in that year is taken at 5.180 million tons, excluding the 1.0 million ton project that is under consideration for Las Truchas.

To simplify presentation, the intermediate iron product (either pig iron or sponge iron) has been netted out from Table 4, and the six alternative processes are each viewed as integrated from raw materials through the ingot stage. It is supposed that the new materials (iron ore, scrap and coke) will be available in unlimited quantities at a constant marginal cost. From the coefficients in the column SDCHATt, and row CSCORt, it will be seen that the input price for steel scrap (chatarra) is assumed to be 500 pesos per ton. Since Mexico will continue to be an importer of scrap during the 1970's, this entire amount is also viewed as a foreign exchange cost (row CSMCORt).

It is <u>not</u> assumed that industrial fuel will continue to be available to the steel industry at a constant marginal cost. The activities SDRQFt define the steel industry's fossil fuel requirements (either in the form of residual fuel oil or natural gas), translating these requirements into their British thermal unit equivalents. These are the activities that link process choices within the steel and the petroleum industries.

As things turn out, the optimal steel process choice is insensitive to the foreign exchange premium and the discount rate. In letting the discount rate range from 10% to 20% per year, and in letting the value of foreign exchange vary between the conventional exchange rate of 12.5 and the shadow price of 20.0 pesos per U.S. dollar, the optimal investment choice within the steel industry remains process 5: blast furnace + LD converter (CONOX).

Table 4. Detached coefficients matrix for iron and steel industry, year t (t = 0, 1,···,6)

row identification (see Tables 1 and 2 for units of measurement) \ activities	definition of fossil fuel and raw material inputs for new ingot capacity				inputs and outputs from six alternative integrated processes for 1.0 million tons/year of ingots						Unit cost of inputs	
	SDRQFt	SDMNHt	SDCHATt	SDCOQt	SD1t	SD2t	SD3t	SD4t	SD5t	SD6t	1977 shadow prices associated with 10% annual discount rate, 15% preference to domestic inputs, and conventional exchange rate of 12.5 pesos per U.S. dollar	1970 conventional prices
CSCORt		-80.	-500.	-300.	-595.	-459.	-536.	-550.	-418.	-416.	1.00	1.00
CSMCORt			-500.		- 51.	- 50.	- 61.	- 37.	- 55.	- 27.	.15	0
CMGWt					- .096	- .267	- .013	- .101	- .006	- .067	597.11	0
CMTWHt					- .718	- 2.000	- .100	- .760	- .047	- .500	37.35	100.00
SDRQFt	1.0						- 4.589	- 2.950	- 1.640		3.93	5.00
SDMNHt		1.0			- 1.155	- 1.320	- 1.237	- 1.237	- 1.320		80.00	80.00
SDCHATt			1.0		- .325	- .275	- .275	- .275	- .275	- 1.050	575.	500.00
SDCOQt				1.0		- .330	- .400		- .650	- .050	300.	300.00
SDLINt					1.000	1.000	1.000	1.000	1.000	1.000		
PEGSCt					- 2.710						24.87	10.40
marginal cost of ingots (based on shadow prices for 1977)					1033	1063	952	913	897	1037		
marginal cost of ingots (based on conventional input prices for 1970)					972	1001	925	877	859	1006		
energy inputs, % of marginal cost (based on conventional input prices for 1970)					13%	30%	17%	10%	24%	6%		

Notes: Source: Ing. Fernando González Vargas, Nacional Financiera, S.A.
The unit costs (CSCORt) include a gross return on initial investment of 19%/year (allowing for depreciation and opportunity cost of capital at 13%, plus 6% for maintenance, overhead and insurance) plus labor costs at 96,360 pesos per man-year of direct labor (allowing for overhead, supervision etc. at 1.2 times the 1970 direct labor costs) plus miscellaneous inputs (e.g. limestone). The import component (CSMCORt) of unit costs is taken as 5%/year on the initial investment costs.
The six alternative integrated processes are defined as follows

		primary iron/scrap ratio
1.	sponge iron (HYLSA) + electric steel furnace	70/30
2.	electroreduction furnace + LD converter (CONOX)	75/25
3.	prereduced pellets (SL-RN) + blast furnace + LD Converter (CONOX)	75/25
4.	prereduced pellets (SL-RN) + electric steel furnace	75/25
5.	blast furnace + LD converter (CONOX)	75/25
6.	non-integrated electric steel furnace	0/100

For comparative purposes, Table 4 indicates the marginal cost of ingot production based both upon conventional 1970 input prices and also based upon the 1977 dual variables. Note that there is only a small difference between the marginal costs of process 4 and 5, and that this difference is well within the range of likely errors in estimation. According to Table 4, there is a 12-15% difference between the marginal cost of ingots produced via the optimal process (#5) and the sponge iron process (#1). In order to reconcile this with the fact that a major new sponge iron project is under construction, one must recognize that the project has received investment inducements via low input prices for natural gas. One must also recognize that economies-of-scale are less pronounced with sponge iron and with non-integrated electric steel furnace than with processes #2-5.

5. Electricity industry

In ENERGETICOS, we have taken advantage of results from earlier, more detailed models of Mexico's electric power industry. (See Stanford Research Institute (1968) and Rogers (1970).) For example, the demand for electricity is characterized here in terms of just two parameters of the annual load-duration curve: the annual peak power and the annual energy. This representation of demand is too aggregative to be altogether satisfactory for planning investments within the electric power sector. In Fernández and Manne (1969), the load-duration curve had been represented in terms of the annual peak plus three blocks of energy: the energy during the four peak hours of 250 working days, the energy during the 20 off-peak hours of 250

working days, and the energy during the remaining 2760 hours of the year. It was then found that for the central-east-west interconnected grid, there was sufficient flexibility of hydroelectric operations so that energy had the identical value during each of the three blocks of time through 1980. On the basis of this prior information, no distinction is drawn here between the three blocks of the annual load-duration curve.

The more detailed models of the electric power industry have suggested that no great error would be associated with several further simplifications: regarding the water inflow of the existing hydroelectric system as deterministic rather than probabilistic quantities, the omission of gas turbines for any purpose other than reserve capacity, a factor of 12% for reserve capacity, and the predetermined sizes selected for fossil and nuclear generating units. Although such simplifications would be unsatisfactory for investment planning within the electric power industry itself, they appear defensible for examining the linkage between electricity, petroleum and steel investment decisions.*

ENERGETICOS is constructed as though the only process choice for new electrical generating capacity were nuclear power versus fossil fuel plants, and as though the only fossil fuels were residual oil and natural gas. Diesel units, coal burning plants and

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* For understanding a system, the extent of disaggregation need not be as detailed as for understanding the components of that system. Here is a dictum in search of a theory! Would the theory depend upon the finiteness of the information processing capability of a single human mind?

geothermal plants each have their place in Mexico's power grid, but are not expected to supply a major percentage of the new capacity. We have omitted investment choices for new hydroelectric projects - mostly because Mexico is close to exhausting her supply of low-cost hydroelectric sites, but partly because hydro project costs have been consistently underestimated in the past.

Our constraints are written out in Table 5.* The first three sets of constraints (CMGWt), (CMTWHt) and (ELTRAt) refer to satisfying the peak, energy and transmission demands, after deducting the supplies available from the initial hydro and fossil installations. (See Table 6 for the numerical values of the exogenously specified quantities.)

Not shown explicitly in Table 5 are the lower bounds imposed on the individual unknowns FOSt. These lower bounds insure that fossil capacity will be installed in geographically isolated regions that are not interconnected with major grids.

Most of the details of Table 5 are connected with the phenomenon of service shifting. That is, over time it can be anticipated that new thermal plants will be added to the system, and that these will have progressively lower energy costs. When a thermal unit enters into service, it is likely to operate as a base-load unit (i.e., at full capacity). The fluctuations in the daily load curve will be absorbed by older thermal plants or by hydro units. As time passes, it is typical that a thermal unit (whether a fossil or a nuclear plant) will be operated at successively lower levels of capacity.

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* This programming model resembles a number of those that have been constructed at Electricité de France. (See Bessière (1969).)

Table 5. Constraints for electric power industy

$(t = 0, 1, \cdots, 6, 11, 16, 21)$

(CMGWt)

$$\underbrace{\sum_{\tau=0}^{t} FOS\tau}_{\text{cumulative new fossil capacity}} + \underbrace{\sum_{\tau=2}^{t} NUC\tau}_{\text{cumulative nuclear capacity}} \geq \underbrace{\text{peak final demand}}_{} - \underbrace{\text{initial fossil capacity}}_{} - \underbrace{\text{hydro capacity}}_{}$$

(CMTWHt)

$$\overbrace{\text{initial} + \text{new, minimum} + \text{new, above minimum}}^{\text{generation from fossil}} + \overbrace{\text{early, minimum} + \text{early, above minimum} + \text{late at maximum}}^{\text{generation from nuclear}}$$

$$\sum_{k=1}^{5} (\overline{TWHR}_k)\ INT\ kt + 1.971 \sum_{\tau=0}^{t} FOS\tau + XFt + 1.861 \sum_{\tau=2}^{\min(t,6)} NUC\tau + XNt + 7.446 \sum_{\tau=11}^{t} NUC\tau \geq \text{energy demand} - \text{hydro energy}$$

(ELTRAt)

$$TRAt = FOSt + NUCt$$

(ELLMFt)

$$XFt \leq 5.913 \sum_{\tau=0}^{t} FOS\tau$$

(ELLMNt)

$$XNt \leq 5.585 \sum_{\tau=2}^{\min(t,6)} NUC\tau$$

(ELCNVt)

$$\sum_{k=1}^{5} INT\ kt = 1$$

(ELRQFt)

$$RQFt = \sum_{k=1}^{5} (\overline{BTU}_k)\ INT\ kt + \begin{pmatrix}\text{heat}\\ \text{rate,}\\ \text{new}\\ \text{fossil}\end{pmatrix}\left(1.971 \sum_{\tau=0}^{t} FOS\tau + XFt\right).$$

Notes: Nuclear fuel inputs enter as cost coefficients for unknowns XNt , NUCt
Profit. Eq. precedes identification of all unknowns shown above.

Table 6. Predetermined demands and capacities, electricity industry

period t	year	annual peak (GW)				annual energy (TWHR)		
		peak final demand[b]	initial fossil capacity	hydro capacity	cumulative requirements for new peak capacity [a]	final demand[b]	hydro energy	demand for non-hydro
0	1974	7.74	4.04	3.88	0.67	39.00	14.25	24.75
1	1975	8.52	4.04	4.20	1.16	43.40	14.70	28.60
2	1976	9.48	4.04	4.35	1.99	47.50	16.40	31.10
3	1977	10.52	4.04	4.35	3.03	53.00	16.23	36.77
4	1978	11.65	4.04	5.27	3.31	59.50	20.96	38.54
5	1979	12.78	4.04	5.27	4.47	66.00	20.96	45.04
6	1980	14.17	4.04	5.27	5.86	74.00	20.96	53.04
11	1985	23.13	4.04	5.27	14.82	152.00	20.96	131.04
16	1990	36.52	4.04	5.27	28.21	208.00	20.96	187.04
21	1995	57.39	4.04	5.27	49.06	330.00	20.96	309.04
%/year growth of demand, 1974-80		10.6%				11.3%		

a/ Requirements for new peak capacity = final demand - $(\frac{1}{1.12})$ (initial fossil + hydro capacity)

b/ Excluding small amounts of endogenous electricity demands for new steel ingot capacity.

One formulation that provides for optimal service shifting may be illustrated in the case of new fossil fuel capacity. For this process choice, it is supposed that there are two types of unknowns: FOSτ and XFt. The former denote the peak capacity installed in period τ, and the latter denote the above-minimum energy to be produced by new fossil plants in period t. Plants of this type cannot be operated at more than 90% of the 8,760 hours in the year. In order for the boilers to be fired up so as to meet the annual and daily peak, these units cannot be operated at less than 8,760(.90)(.25) = 1,971 hours per year - hence the energy coefficient for FOSτ shown in the constraints CMTWHt. During the remaining hours that these fossil units are available, one kilowatt of capacity cannot produce more than 5.913 thousand kilowatt-hours of energy - hence the formulation of the energy limit ELLMFt with respect to the unknown XFt.

For early nuclear power plants, a similar provision is made for service shifting. ("Early" means those nuclear plants installed between years 2 and 6, 1976-80.) The only difference between the coefficients for new fossil and early nuclear plants is that the latter are taken to have an annual availability factor of 85% rather than 90%. For the late nuclear plants, (i.e., those installed between 1981 and 1995), it is supposed that these would be operated only for base-load purposes, and that service shifting may be neglected.

To describe the operations of the initially available fossil fuel plants, an alternative formulation was adopted in order to optimize service shifting. Because of differences between the thermal efficiency of these units, there is a convex relationship

between the energy output and the fossil fuel input. (See Figure 2.) To summarize this nonlinear relationship, five levels of output are specified, and these are identified through the index $k = 1,2..5$. From Figure 2, it can be seen that for each value of k there is a predetermined value of the total fuel input (the coefficient $\overline{BTU}_k$) and of the total energy output (the coefficient $\overline{TWHR}_k$). The linear programming unknowns are not these inputs and outputs directly, but rather the "interpolation weight" variables INT kt. The unknowns INT kt appear in these constraints: (CMTWHt), (ELRQFt), and (ELCNVt). The convex curve constraints (ELCNVt) ensure that the interpolation unknowns will add up to unity for each time period t. If during time period 1, for example, it were optimal for the initial plants to produce 28.96 TWHR per year (i.e., to operate midway* between points 4 and 5 on Figure 2), this could be achieved by setting the unknowns INT 41 = .5 = INT 51. The corresponding energy output would then be interpolated through the coefficients $\overline{TWHR}_4$ and $\overline{TWHR}_5$ in the energy constraint CMTWH1. Similarly, the fossil fuel input requirement would be interpolated through the coefficients $\overline{BTU}_4$ and $\overline{BTU}_5$ in the fuel definition constraint (ELRQF1).

In order to complete the input data for the electric power industry, cost coefficients must be calculated for each activity.

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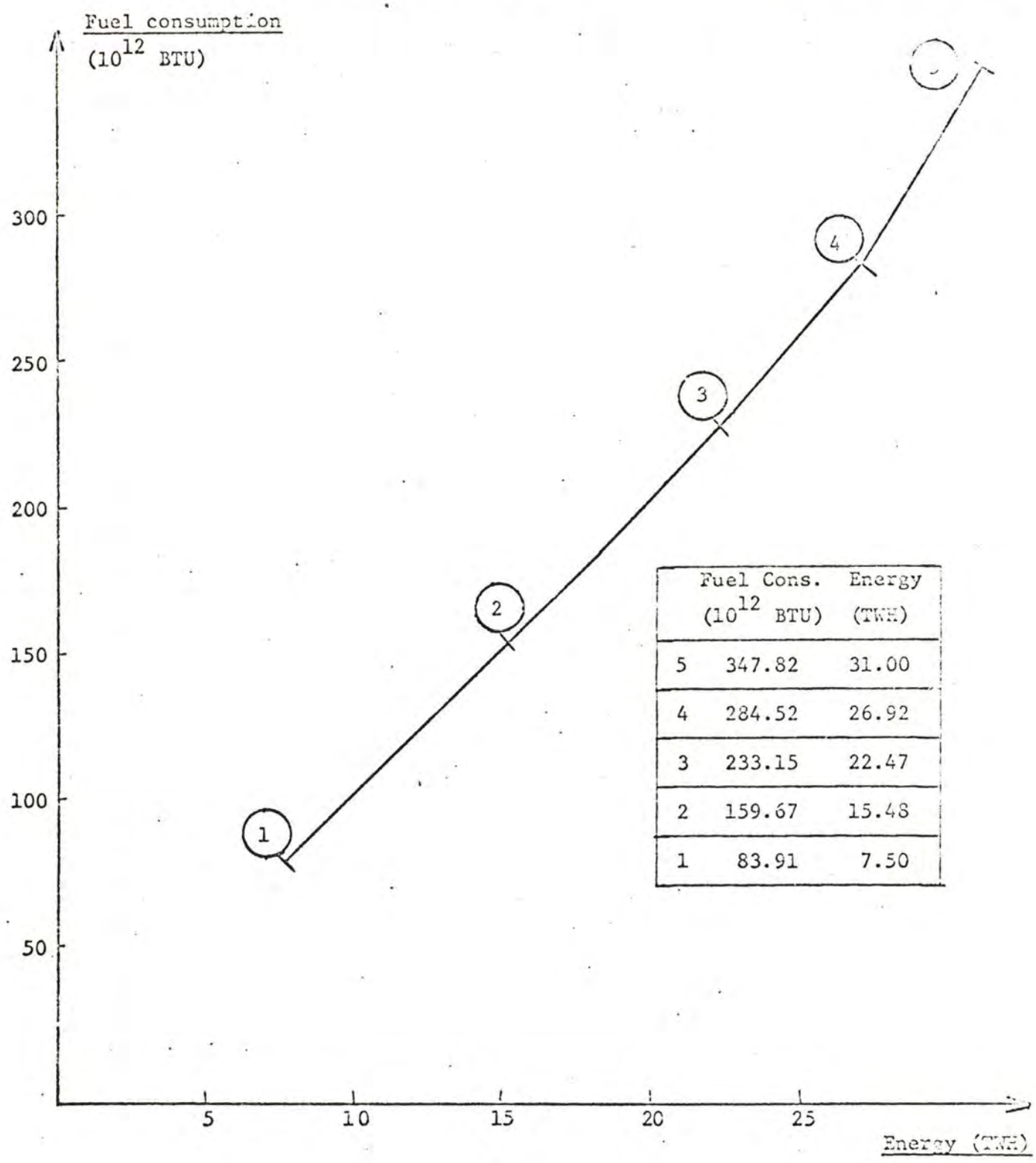
* The process of optimization, together with the convex shape of Figure 2, ensures that it will never be optimal to assign positive intensities to two non-adjacent interpolation variables. E.g., it will not be optimal to set INT 31 > 0 and simultaneously INT 51 > 0. A reduction in fuel inputs could then be achieved by increasing the intensity of INT 41, and reducing both INT 31 and INT 51.

Figure 2. Fuel consumption vs. energy supplied by fossil power stations operating in 1973



	Fuel Cons. (10^{12} BTU)	Energy (TWH)
5	347.82	31.00
4	284.52	26.92
3	233.15	22.47
2	159.67	15.48
1	83.91	7.50

Table 7. Costs in the electricity sector

		Generating Plants:					
	Unit size	Fossil 300 MW	Fossil 1000 MW	Nuclear 600 MW	Nuclear 1000 MW	Nuclear Breeder 1000 MW	Transmission + gas turbines for reserve capacity + overhead costs per 1000 MW of transmission capacity
I	Initial investment						
I.1	Total (M pesos)	426	1200	1374	2000	2000	1454
	annual change (%)	-0.5[a/]	-0.5[a/]	-1.0[b/]	-1.0[b/]	-2.0[c/]	0.0
I.2	Unit cost (M pesos/GW)	1426	1200	2290	2000	2000	1454
I.3	Domestic cost component (%)	65.0	50.0	25.0	25.0	25.0	70.0
	Annual change (%)	0.0	0.0	1.0[b/]	1.0[b/]	1.0[c/]	1.5[a/]
II	Unit fuel cost (M pesos/TWH)	35.55[d/]	35.33[d/]	16.25	15.75	8.37	
	Annual change (%)	0.0	0.0	-1.0	-1.0	-2.0	
III	Fixed operating costs (M pesos/year)	9.40	18.00	22.20	25.50	25.50	283.7
	Domestic cost component (%)	100	100	100	100	100	100
IV	First year available	1974	1981	1976	1981	1991	1974

Notes: [a/] From 1974

[b/] From 1976

[c/] From 1986

[d/] Valuing fossil fuel at 3.95 pesos/MBTU from 1981

Abbreviation: M = 10^6

Table 8. Optimal program for installation of generating capacity (unit: MW installed per year; 1GW = 1000 MW)

year	unit size	Fossil		Nuclear		Nuclear Breeder
		300 MW	1000 MW	600 MW	1000 MW	1000 MW
1974		676	n.a.	n.a.	n.a.	n.a.
1975		493	n.a.	n.a.	n.a.	n.a.
1976		50[a/]	n.a.	784	n.a.	n.a.
1977		26[a/]	n.a.	1018	n.a.	n.a.
1978		43[a/]	n.a.	241	n.a.	n.a.
1979		58[a/]	n.a.	1107	n.a.	n.a.
1980		64[a/]	n.a.	1331	n.a.	n.a.
1981 - 85		n.a.	0	n.a.	1786	n.a.
1986 - 90		n.a.	0	n.a.	2678	n.a.
1991-95		n.a.	1133	n.a.	n.a.	3037

Notes:

n.a. = not available

a/ Predetermined lower limit to ensure that fossil capacity will be installed in geographically isolated regions that are not interconnected with the major grids for the south - central and the northern systems.

These have been derived from the project characteristics given in Table 7, interpolating linearly for investment and operating costs within each of the three successive five-year intervals 1980-85, 1985-90, and 1990-95. With these numerical assumptions on costs, demands, operating modes, etc., the optimal installation program for generating capacity is given in Table 8. As is characteristic of linear programming, the solution tends to be of a bang-bang form: fossil units installed at their lower limit during 1976-80 and nuclear units for the balance of capacity requirements between 1976 and 1990. During the period in which breeders first become available (1991-95), it appears optimal to install a mix of fossil and breeder units. (If pumped-storage activities had been included explicitly within ENERGETICOS, it is unlikely that it would be optimal to install fossil units at the same time as breeders.)

During 1976-80, the period of immediate interest, note that the annual capacity increments would be of the order of one or two nuclear units, each of 600 MW size. Given that Mexico's consumption is not unified through a national transmission grid, this calculation understates the difficulties of an all-nuclear program. The transmission implications of such a program have yet to be worked out.

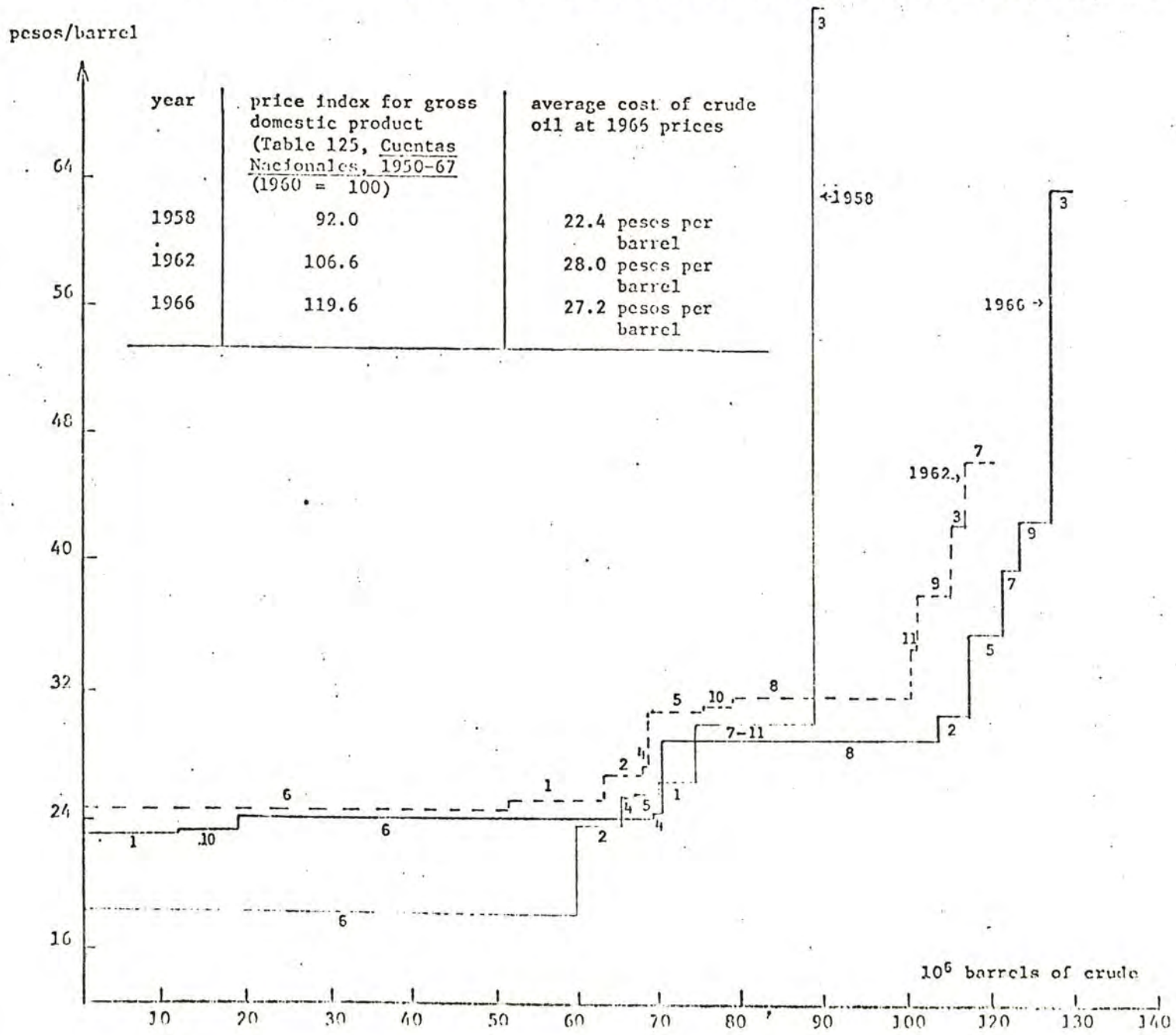
6. Petroleum and gas industry

ENERGETICOS is stated as though the objective were to satisfy the domestic demands for petroleum products at minimum cost, exporting and importing whichever items are most advantageous. It is not stipulated that absolute priority be given to self-sufficiency, nor to employment within the petroleum and gas industry. Rather, we explore the tradeoffs between total costs and foreign exchange costs - allowing for a 15% rate of protection to domestic costs against imports.

In order to concentrate on the problem of intersectoral linkage, we have focussed on oil and gas production and on refining - excluding the transportation and marketing aspects. Within ENERGETICOS, the costs of domestic crude oil and gas production constitute the most problematical set of parameters. It is not at all clear what have been the past costs of domestic production - let alone what is appropriate in an extrapolation for a decade into the future.

Some clues on the future may be obtained from Figure 3, a cumulative distribution of crude oil costs by fields in 1958, 1962 and 1966. The historical figures suggest: (a) that there was a significant increase in the domestic cost of crude oil between 1958 and 1962 but not between 1962 and 1966 (after deflation by the gross domestic product price index); (b) that there is a significant difference between the costs of production in individual fields; (c) that the average field has costs that are competitive with an import price of 27.5 pesos per barrel = U.S. $2.20 per barrel; and (d) that there has consistently been a small volume of production from marginal fields with costs that lie well above the import price.

Figure 3. Pemex Accounting Department estimates of crude oil costs by fields - 1958, 1962, 1966



year	price index for gross domestic product (Table 125, Cuentas Nacionales, 1950-67 (1960 = 100)	average cost of crude oil at 1966 prices
1958	92.0	22.4 pesos per barrel
1962	106.6	28.0 pesos per barrel
1966	119.6	27.2 pesos per barrel

The classical academic impulse is to conclude that the high-cost fields ought to be shut down, that imported crude replace the domestic output, and that the necessary foreign exchange be earned through shifting the displaced petroleum workers into producing, say, fruits and vegetables for export. A more pragmatic reaction would be to question whether the historical accounting estimates bear any relation to the economic theorist's definition of opportunity costs. Underlying Figure 3 are arbitrary allocations between two joint products - crude oil and associated gas. There are quality differences between types of crude. There is the possibility that low-cost fields have not been exploited to their capacity in the past. There are arbitrary allocations of exploration and drilling costs, arbitrary estimates of proved reserves, and arbitrary allowances for the return on capital. Given this view of the data, it is hopeless to calculate an optimal make-or-buy choice for crude oil. For the decision-maker who is subject to political pressure from petroleum workers, the line of least resistance is therefore to adopt the criterion of self-sufficiency in raw materials - regardless of costs. (Consideration of this type are not unknown in the U.K. coal mines nor in the U.S. petroleum industry.)

Rather than abandon cost criteria completely, it seems preferable to make some assumptions on raw material costs and then to test how these assumptions influence the optimal choice of processes elsewhere within the system. For the decade of the 1970's, it can be supposed that the import price will remain constant at 27.5 pesos per barrel for crude oil and at 25.0 for the fuel oil equivalent of dry gas.

(See Table 9*). Given the uncertainties in cost extrapolations - and given the inevitable political pressures against imports - we have attempted to err in the direction of underestimating future domestic costs. For this reason, it is supposed that 85% of the future domestic supplies will be available at a cost substantially below that of imports, and that an additional 10% of the domestic supplies will be competitive if foreign exchange is valued at the conventional rate of 12.5 pesos per U.S. dollar. For the remaining 5% of potential oil and gas supplies, the cost parameters have been adjusted so that domestic material will cost more than imports even after allowance is made for a 15% rate of protection against imports. Typically, unless there is a premium value assigned to foreign exchange, it then turns out to be optimal to exploit just 95% of the potentially available domestic supplies, and to import the balance of the raw material requirements for the petroleum refining industry.

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* For many of the estimates that appear in Tables 9-12, we are indebted to Ing. Victor L. Gamez and Carlos Velasco, Instituto Mexicano del Petróleo.

Table 9. Projections of domestic availability and cost of oil, condensates and dry gas

	Availability (unit: 10^6 barrels)								Cost coefficients (row CSCORt) (unit: pesos per barrel) a/				
	'74	'75	'76	'77	'78	'79	'80	1974-80	% of domestic supplies available at stated cost	85%	10%	5%	import price
time period t	0	1	2	3	4	5	6	%/ year growth rate					
Crude oil	172.2	176.1	191.4	209.3	228.3	234.5	255.0	6.8%		24.0	27.0	35.0	27.5
Condensates	31.9	34.5	37.2	39.6	42.2	44.9	47.5	6.8		c/	c/	c/	-
Dry gas b/	118.0	131.3	144.1	157.3	171.4	188.4	204.2	9.6		20.0	24.5	31.3	25.0 d/

Note:

a/ Foreign exchange component (row CSMCORt) is taken at 10% of domestic costs and 100% of import price.

b/ Fuel oil equivalent of dry gas at 6.321 10^6 BTU/barrel of fuel oil.

c/ Since the quantity of condensates is projected exogenously, no cost coefficients need to be estimated for this item.

d/ Import price of fuel oil.

Table 10. Matrix of coefficients for petroleum refining

Row (for identification and units of measurement, see Tables 1 and 2) / Activity		Initial (1973) refining capacity	New refining capacity [b]			Outputs per million barrels of domestic condensates	Industrial fuel equivalents			
			conventional	visbreaking	maximum hydrogen processing					
		PECP73[a]	PECt	PEVt	PEHt	PEDCONt [c]	PEQGSCt	PEQGSRt	PEQGPLt	PEQCOMt
cost inputs	CSCORt		-6.2	-10.8	-16.4					
	CSMCORt		- .62	- 1.64	- 1.08					
physical	CMBTUt						6.321	6.321	4.173	6.321
inputs (-)	PEGSCt			- .0181	- .0328		-1.0			
and	PECRDt	-170.0	-1.0000	-1.0000	-1.0000					
outputs (+);	PEGSRt	5.9	.0240	.0080	.0289			-1.0		
unit:	PEGPLt	12.5	.0280	.0602	.0836	.5005			-1.0	
10^6 barrels,	PEGSLt	71.5	.2580	.4876	.5859	.3900				
except for	PEKDt	52.6	.2770	.2546	.2863	.1095				
row CMBTUt	PECOMt	35.0	.2870	.2218	.1282					-1.0

Notes:

[a] Initial capacity activity is operated at level of 1.0 during all time periods.

[b] All three processing sequences are based upon an integrated new refinery with 115 TB/D of primary distillation capacity, 50 TB/D of vacuum distillation capacity, and a 10 TB/D gasoline reformer. Other capacities would be as follows (in TB/D = thousand barrels per stream day);

	conventional	visbreaking	maximum hydrogen processing
H-oil	0	0	25
Isomax	0	30	30
Visbreaking	10	25	0
Catalytic cracking	27	0	0
Hydrogen (10^6 scf/day)	0	50	100

[c] Domestic condensates activity is operated at predetermined level during all time periods. See Table 9 for this availability level.

Table 11. Marginal costs and optimal material balances, 1977 - petroleum industry

(omitting inputs and outputs where optimal activity level is zero)

(unit: millions of barrels/year - except for industrial fuel)

Description of activity / Activity identification / Item	Refining capacity			Domestic gas a/		Domestic condensates	Domestic crude b/		Fuel oil equivalents			Imports			Intersectoral flows		Final demand	Marginal cost of supplying item in 1977 (pesos/barrel-except for industrial fuel)
	PECP73	PEC3	PEV3	PEDGSC3	PEDGSCA3	PEDCOND3	PEDCRD3	PEDCRDA3	PEQCRS3	PEQCRR3	PEQCRM3	PEMCRD3	PEMCGL3	PEMCPK3	PELRQF3	PESRQF3		
CMBTU3 c/									905.4	39.6	223.8				-231.7	-138.7	798.4	3.93
PEGSC3			-.4	133.7	9.9				-143.2								0	24.87
PECR03	-170.0	-7.7	-22.7				177.9	20.9				1.6					0	31.63
PECSR4	5.9	.2	.2							- 6.3							0	24.87
PECD13	12.5	.2	1.4			19.8							6.1				40.0	33.12
PEGSL3	71.5	2.0	11.1			15.4											100.0	44.06
PECD3	52.6	2.1	5.8			4.3								.5			65.3	54.40
PECC13	35.0	3.0	5.0								- 35.4						7.6	24.87

Notes:

a/ Activity PEDGSC3 denotes domestic gas obtainable at cost of 20.0 pesos/barrel; PEDGSCA3 denotes domestic gas obtainable at 24.5 pesos/barrel.

b/ Activity PEDCRD3 denotes domestic crude obtainable at cost of 22.0 pesos/barrel; PEDCRDA3 denotes domestic crude obtainable at 27.0 pesos/barrel.

c/ Unit: 10^{12} BTU/year.

d/ Includes asphalts only.

Table 12. Product demands; optimal imports and exports

(unit: 10^6 barrels/year - except for industrial fuel)

		export/ import price (pesos/ barrel)	year	'74	'75	'76	'77	'78	'79	'80	1974-80 %/ year growth rate of final demand	Identification of activity	Identification of item
			period t	0	1	2	3	4	5	6			
Industrial fuel [a/]	final demand			616.5	672.0	732.5	798.4	870.3	948.6	1034.0	9.0%		
	intersectoral flows, electricity			250.0	285.2	240.9	231.7	231.6	215.3	197.3		ELRQFt	CNBTUt
	intersectoral flows, steel [b/]			117.2	124.2	130.9	138.7	147.1	156.4	166.5		SDRQFt	
Liquefied petroleum gas	final demand	-		33.8	35.9	38.0	40.0	42.0	44.0	45.8	5.2		PECPLt
	imports	28.8		(5.3)	(6.0)	(6.0)	(6.1)	(6.0)	(5.8)	(5.5)		PEMCPLt	
gasoline	final demand	-		80.0	86.3	93.0	100.0	107.4	115.1	123.1	7.4		
	exports	31.5		3.9	0	0	0	0	0	0		PEXGSLt	PECSLt
	imports	47.3		0	0	0	0	0	0	0		PEMGSLt	
kerosine and diesel oil	final demand	-		55.1	58.4	61.8	65.3	68.9	73.4	76.7	5.7		
	exports	31.5		1.0	0	0	0	0	0	0		PEXKDt	PEKDt
	imports	47.3		0	(.6)	(.4)	(.5)	0	(.7)	(.1)		PEMKDt	
residual fuel oil	final demand [c/]	-		5.9	6.5	7.0	7.6	8.2	8.9	9.6	8.5		
	exports	21.2		0	0	0	0	0	0	1.3		PEXCOMt	PECOMt
	imports	25.0		(8.5)	(9.8)	0	0	0	0	0		PEMCOMt	
crude oil	imports	27.5		(6.4)	(7.9)	(6.1)	(1.6)	0	(5.8)	(.6)		PEMCRDt	PECRDt

Notes:

a/ Unit: 10^{12} BTU/year,

b/ Total industrial fuel requirements for steel industry includes rolling stage. This excludes coke requirements, and refers only to fuel oil plus natural gas.

c/ Includes asphalts only.

For alternative refining processes, it is straightforward to estimate physical inputs and outputs. (See Table 10.) Each of the three alternative activities for new refining capacity is viewed as though it were vertically integrated from crude oil through refined products, after netting out intermediate materials such as vacuum gas oil. If crude oil were the only cost element and if the refining industry's objective were to maximize the output of gasoline (the highest-priced product) it would be straightforward to conclude that maximum hydrogen processing (process PEHt) is always optimal. Here, with a more complex objective function - satisfying demands at minimum costs - the optimal process choice is not maximum hydrogen processing. The process choice is sensitive to the refining cost coefficients shown in rows CSCORt and CSMCORt.*

Table 11 provides the marginal costs and optimal material balances for a representative year, 1977. For each year within the planning horizon of 1974-80, similar costs and material balances are available. Typically, as shown in Table 12, the optimal solution entails small-scale imports of crude oil and liquefied petroleum gas, and virtual self-sufficiency in other items. With other numerical assumptions (e.g., a slower rate of introduction of nuclear power), the solution will change. In particular, note that the optimality of the solution depends upon

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* The numerical values adopted for these cost coefficients are based upon the rather shaky estimate that annual refinery costs-including capital charges, labor, overheads and refinery inputs other than oil and gas - will be 40% of the initial investment costs for an integrated plant.

the import and export prices assumed for the various raw materials and products. To the extent that we have overstated the transport and marketing cost differentials between the import and the export price of each petroleum product, the results have been biased in favor of self-sufficiency.

7. Sensitivity analysis - discount rate and foreign exchange scarcity premium

Up to this point, all results have been based upon those project evaluation criteria that we believe to be typical within the energy sector. It has been supposed that the public enterprises within this sector evaluate alternative projects as though capital were available at a 10% annual discount rate, as though foreign exchange were valued at the conventional rate of 12.5 pesos per U.S. dollar, and as though there were a 15% protection rate for domestic as against imported items.*

With respect to import substitution choices, a 15% protection rate has effects that are identical to a 15% scarcity premium imputed to foreign exchange. With respect to export promotion, however, the two criteria differ. E.g., to stipulate a zero scarcity premium on foreign exchange and a 15% protection rate implies that the energy sector would prefer to save $1.00 of foreign exchange through import substitution rather than earn up to $1.15 of foreign exchange through exports. A domestic protection rate, then, is to be interpreted as a price that Mexico is willing to pay in order to insulate her economy from fluctuations transmitted through international trade.

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*Throughout the energy sector, domestic inputs are given preference over imported ones. For heavy electrical equipment financed by World Bank loans, the standard current practice for bid evaluation is a 15% preference rate. Outside the electrical sector, the protection criteria are seldom stated in explicit form.

An alternative interpretation of the 15% preference rate is the following: Even if absolute preference is to be given to domestic raw materials such as oil and gas, the consumers of these items should evaluate their own process alternatives as though fuel were not priced much above the international level. If marginal domestic cost pricing is applied in this "second-best" situation, there will be an uneconomic stimulus to economize on oil and gas, e.g., to speed up the introduction of nuclear power into the electricity industry and of hydrogen processing into petroleum refineries.

Table 13 summarizes how the optimal solution would shift under alternative assumptions with respect to the foreign exchange premium and the discount rate - holding the protection rate constant at 15% of import costs. Let p denote the scarcity premium imputed to foreign exchange. Then recalling the definition given in Table 2, the cost minimand represents the discounted sum of one-period costs. The costs incurred in period t would be evaluated as follows:

costs at conventional prices + .15 (import costs) + p (import costs - export earnings)

$$CSCOR_t + .15\ (CSMCOR_t) + p\ (CSMCOR_t - CSXCOR_t)$$

As p ranges between 0 and 60% (i.e., as the value of foreign exchange ranges between the conventional rate of 12.5 and the

shadow price of 20.0 pesos per U.S. dollar*), note that the discounted import costs less export earnings would decrease, and that total costs evaluated at conventional prices would rise by a comparatively small amount.

Underlying the aggregative results, there are detailed import-export choices - which in turn depend upon the selection of production processes. For simplicity in presentation, Table 13 summarizes the results for the single representative year, 1977. Within the steel industry, the optimal choice would remain process #5 (blast furnace + LD converter) - regardless of whether the discount rate is 10% or 20% per year and regardless of whether $p = 0$ or $p = 60\%$.

Within the electric power and the petroleum sectors, the process choices are sensitive both to the discount rate and the foreign exchange premium. Some of these choices are self-evident, e.g., the shift to high-cost domestic crude as the foreign exchange scarcity premium increases. Not all the results are so clearcut. E.g., the higher the discount rate, the greater becomes the capital cost of the domestic fossil fuel supplied by the petroleum to the electricity industry. Nonetheless, in changing the annual discount rate from 10 to 20%, the greater becomes the optimal amount of nuclear electric generating capacity. The high capital cost component of nuclear capacity would more than offset the rise in fossil fuel costs.

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* Given the prevailing levels of effective tariffs on imports of consumers' durables, an efficiency price of 20.0 pesos is not altogether unrealistic. See, e.g., Bueno (1969, Table 7).

In a subsequent memorandum, we hope to show how these process analysis results may be linked to the dynamic multi-sector model DINAMICO. Tentatively, it appears that the most appropriate device for linkage will be through the six alternative time-phased vectors of investment and foreign exchange expenditures that are shown in Table 14.

Table 13. Effects of foreign exchange premium and of discount rate for petroleum, electricity and steel industries

		Unit of measurement	p = foreign exchange scarcity premium, % 0	10	20	30	40	50	60	
Total discounted costs at conventional prices + 15% protection on imports		10^9 pesos discounted to 1974 (year 0) at 10% per year a/	127.34	127.46	127.75	127.95	127.98	128.11	129.27	annual discount rate = 10%
Total discounted import costs less export earnings, at conventional prices		10^9 pesos discounted to 1974 (year 0) at 10% per year a/	23.67	21.91	20.06	19.29	19.22	18.90	16.70	
1977 process choices	Ingot output from 1974-77 capacity, steel process #5	10^6 ingot tons	2.9	2.9	2.9	2.9	2.9	2.9	2.9	
	Crude petroleum, domestic output, 1977	10^6 barrels	198.8	198.8	201.9	201.9	201.9	203.8	209.3	
	Crude petroleum, imports, 1977	10^6 barrels	1.6	3.0	0	0	0	0	0	
	Crude input to 1974-77 refining capacity (conventional)	10^6 barrels	7.7	10.7	10.9	10.9	10.9	14.9	23.4	
	Crude input to 1974-77 refining capacity (visbreaking)	10^6 barrels	22.7	21.1	21.0	21.0	21.0	18.9	15.9	
	Fossil electric generating capacity installed, 1974-77	MW	1,245.	1,869.	1,991.	2,483.	2,614.	2,621.	3,047.	
	Nuclear electric generating capacity installed, 1974-77	MW	1,802.	1,177.	1,145.	644.	433.	226.	0	
Total discounted costs at conventional prices + 15% protection on imports		10^9 pesos discounted to 1974 (year 0) at 20% per year a/	85.34	85.50	85.50	85.67	85.95	86.20	86.20	annual discount rate = 20%
Total discounted import costs less export earnings, at conventional prices		10^9 pesos discounted to 1974 (year 0) at 20% per year a/	18.35	13.35	13.35	12.60	11.89	11.30	11.30	
1977 process choices	Ingot output from 1974-77 capacity, steel process #5	10^6 ingot tons	2.9	2.9	2.9	2.9	2.9	2.9	2.9	
	Crude petroleum, domestic output, 1977	10^6 barrels	177.9	198.8	198.8	198.8	198.8	201.9	201.9	
	Crude petroleum, imports, 1977	10^6 barrels	23.9	3.0	3.1	3.1	3.1	0	0	
	Crude input to 1974-77 refining capacity (conventional)	10^6 barrels	10.7	10.7	10.9	10.9	10.9	10.9	10.9	
	Crude input to 1974-77 refining capacity (visbreaking)	10^6 barrels	21.1	21.1	21.0	21.0	21.0	21.0	21.0	
	Fossil electric generating capacity installed, 1974-77	MW	3046.	3046.	3046.	3046.	3046.	3046.	3046.	
	Nuclear electric generating capacity installed, 1974-77	MW	0	0	0	0	0	0	0	

Note:
a/ Discounted over 1974-89 horizon for petroleum and steel industries; over 1974-95 horizon for electricity industry.

(Corrected version of Table 14, memo 70-10)

Table 14. Macroeconomic implications of six alternative programs for petroleum, electricity and steel industries. (1970 prices)

Annual discount rate, %		10			20		
Foreign exchange scarcity premium, %		0	30	60	0	30	60
Investment expenditures	'74	3.39	3.94	4.08	3.09	3.39	3.94
	'75	2.83	2.99	2.87	2.03	2.83	2.99
for capacity	'76	5.25	4.56	4.54	4.18	4.59	4.54
	'77	6.01	5.62	5.61	5.25	5.39	5.22
first available	'78	3.36	3.20	3.09	3.25	3.33	3.30
	'79	6.28	6.13	5.95	5.45	5.63	5.97
in year (10^9 pesos)	'80	7.87	6.69	7.00	6.73	6.90	6.69
Total		34.99	33.13	33.14	29.98	32.06	32.65
Import costs	'74	1.12	.84	.78	2.00	1.32	1.05
	'75	1.45	1.09	1.09	2.40	1.68	1.33
less export	'76	1.48	1.16	1.15	2.58	1.79	1.47
	'77	1.57	1.39	1.23	2.86	2.00	1.76
earnings	'78	1.43	1.34	1.29	2.86	1.97	1.75
	'79	1.92	1.58	1.63	3.57	2.58	2.18
(10^9 pesos)	'80	1.87	1.89	1.82	3.94	2.87	2.59
Total		10.84	9.29	8.99	20.21	14.21	12.13
Export earnings	'74	.16	.16	.19	.16	.16	.16
	'75	0	0	.02	0	0	.01
(from petroleum	'76	0	0	0	0	0	0
	'77	0	0	.09	0	0	0
products only)	'78	0	.02	.06	0	0	.19
	'79	0	0	.03	0	0	0
(10^9 pesos)	'80	.03	.02	.11	0	0	.20
Total		.19	.20	.50	.16	.16	.56
Total discounted costs at conventional prices + 15% protection on imports (10^9 pesos discounted to 1974 at 10% per year) a/		127.34	127.95	129.27	128.62	132.43	132.68

Note: a/Discounted over 1974-80 horizon for petroleum and steel industries; over 1974-95 horizon for electricity industry

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