# A Quantitative Theory of Time-Consistent Unemployment Insurance *

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Abstract

During recessions, the U.S. government substantially increases the duration of unemployment insurance (UI) benefits through multiple extensions. Benefit extensions increase UI coverage and lead to higher average consumption, but the expectation of an extension may reduce unemployed worker's job search incentives and lead to higher future unemployment. We show that benefit extensions in recessions arise naturally when the government makes discretionary UI policies that respond to the underlying states of the economy. We do so by endogenizing a time-consistent no-commitment UI policy in a stochastic equilibrium search model. In recessions, as wages and search efficiency are lower, the costs of search disincentive and unemployment are relatively small, so the government has incentives to extend benefit duration. We quantitatively evaluate the benefit extensions implemented during the Great Recession. Switching to alternate commitment policies would reduce the unemployment by 1.6–2.5 percentage points with limited welfare gains.

Keywords: Time-consistent policy, Unemployment insurance, Search and matching
JEL classifications: E61, J64, J65, H21

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# 1 Introduction

The U.S. government has extended unemployment insurance (UI) benefits in response to higher unemployment since the 1950s. During the Great Recession, benefit durations were extended up to 99 weeks. On the one hand, these policies provide insurance coverage to more unemployed workers, increasing the average consumption. On the other hand, UI can create moral hazard problems by reducing workers' incentives to find jobs. In fact, a big debate in the literature is whether benefit extensions worsened unemployment during the past recession due to adverse incentives (e.g. Nakajima 2012, Hagedorn, Manovskii, and Mitman 2015, and Chodorow-Reich, Coglianese, and Karabarbounis forthcoming).

This paper takes a positive approach from the government's standpoint, and argues that benefit extensions can arise endogenously from the government's problem. Two ingredients are key to this result. First, low productivity and low search efficiency imply a low cost of unemployment during recessions. Second, a lack of commitment by the government to future policies means the UI policy is discretionary and chosen without regard to its ex ante effects. We characterize these effects in a time-consistent (Markov) equilibrium of the government's problem over the business cycle. We show that quantitatively the UI extensions chosen endogenously by the government in the model match the extensions during the Great Recession quite well. Counterfactually, if the government implemented commitment policies during the Great Recession, unemployment would be lower, but welfare gains are limited and vary depending on the timing and the type of policy implemented.

We analyze the government's choice of UI policy in a stochastic equilibrium search and matching model with rise-averse workers and endogenous search intensity. We use the Markov-perfect equilibrium to characterize the government's discretionary UI policy. Because in this equilibrium the government's policies only depend on current payoff-relevant states, the policies are time-consistent. Specifically, each period a utilitarian welfare-maximizing government chooses the UI benefit level and duration – modeled using the probability that the benefit continues – depending on the current levels of unemployment and aggregate productivity. Modeling the benefit duration using a probability rather than a fixed length keeps the government's problem tractable.[1] A key assumption is that once benefits expire, the unemployed worker does not regain eligibility before he finds a job. Under this assumption, benefit duration policy today, through changing the proportion of unemployed UI recipients, directly changes the measures of unemployment next period, and thus the future policies.

The private sector's decisions are modeled using a search-matching model with risk-averse workers, endogenous search intensity by the unemployed, and business cycles driven by shocks to the aggregate labor productivity. Unemployed workers search for jobs, while firms post vacancies. Both parties make decisions given the government's policy choices. Future government policies affect

[1] Similar ways of probabilistic modeling are used in the monetary policy literature ("Calvo fairy") and the sovereign debt literature (modeling long duration debt). One concern about using the probabilistic structure is all UI recipients are affected equally by changes in UI duration. In the discussion section we introduce a group of UI-recipients whose eligibility status is not affected by changes in UI duration policy. This alternate assumption reduces the government's incentives to extend UI during recessions.

worker's expected net value of employment, so the unemployed worker's search depends on the expectations about future government policies: generous future benefit policies (higher benefit level or longer benefit duration) reduce the unemployed UI recipient's incentive to search. Because duration policy directly changes the future states of the economy, it affects the worker's expectations about future policies. The government's policy decision takes into account these effects of expectations.

The main trade-off associated with the government's duration policy is between insurance and incentives. A longer duration increases the UI coverage today and raises the average consumption of unemployed workers. It also reduces the average job search through a larger share of unemployed UI recipients. And because UI recipients search less than non-recipients in the model, this change in the extensive margin raises future unemployment. In the equilibrium, workers expect longer future benefit duration when the unemployment is high. This expectation lowers search by the UI recipients, which also contributes to higher future unemployment.

In a recession, the government has incentives to extend benefit duration. A drop in labor productivity reduces the marginal return to production and the marginal gain from job creation. Lower productivity also reduces search efficiency as the labor market becomes less tight, and this makes policies that encourage search (i.e. shorter benefit duration) less effective and more costly. As a result, the government raises UI duration. As the unemployment rises, both the insurance and incentive effects are strong as more unemployed workers are potentially affected by a change in the duration policy. Quantitatively, the increase in the insurance effect outweighs the larger incentive effect, and benefit duration rises further.[2]

We apply the model to the U.S. economy between 2008 and 2013. We feed in exogenous job separation rates taken from the data and calibrate exogenous labor productivity. Overall, our model matches the variations in benefit durations very well, generating 90% of the overall benefit extension seen in the data.

The substantial UI extensions contribute to higher unemployment in the equilibrium. In comparison, if the government is able to implement optimal commitment (Ramsey) policy during the recession, unemployment would be up to 2.5 percentage points lower between 2008 and 2013. This is because with commitment, the government makes policies for all future dates at time 0. And at any time t the policy is chosen to reduce *ex ante* search disincentives, in addition to the trade-offs in the time-consistent government's problem. Quantitatively, in response to rising unemployment the gain from reducing ex ante search disincentive outweighs the cost of lower average consumption, and so the Ramsey government lowers UI duration. This policy attenuates high unemployment when productivity is low during recessions. Alternatively, if the government commits to keeping the UI policy unchanged at its pre-recession level ("acyclical policy"), unemployment would be up to 1.6

[2] The idea that the welfare gains and costs of UI vary over the business cycle is not new. For example, Krueger and Meyer (2002) argue that the efficiency loss from reduced search effort may be smaller during a recession than during a boom. More recently, Kroft and Notowidigdo (2016) empirically estimate the moral hazard cost and consumption smoothing gain of UI benefits, and find that the marginal welfare cost from generous benefits is procyclical and the marginal welfare gain is modest and varies positively with unemployment rate. While they focus on the changing moral hazard effect of UI benefits on individual workers, we investigate the optimal government's response to the changing efficiency loss.

percentage points lower. Despite the relatively large effects on unemployment, switching a commitment policy has small welfare effects. Switching to the Ramsey policy before or during the Great Recession would improve lifetime welfare by 0.11–0.14%, while switching to the acyclical policy improves welfare by at most 0.04%. A more practical alternate policy whereby the government keeps UI policy unchanged going forward would reduce welfare if implemented in the middle of the recession.

This paper is related to two strands of the literature. First, it contributes to the literature on optimal UI policy. A long tradition of literature have studied the optimal design of UI policy where UI creates search disincentive (see, for example, Hopenhayn and Nicolini 1997; Wang and Williamson 2002; Shimer and Werning 2008; Chetty 2008; Golosov, Maziero, and Menzio 2013). A relatively new topic studies the optimal response of UI to business cycles (see, for example, Mitman and Rabinovich 2015; Jung and Kuester 2015; Landais, Michaillat, and Saez 2018; McKay and Reis 2017). The existing literature has assumed government commitment to future UI policies. We complement this literature by relaxing the commitment assumption and characterizing the time-consistent UI policy. Because the government does not need commitment to implement the time-consistent policy, the policy studied here is arguably a more realistic counterpart to policy-making in reality. We use the concept of Markov-perfect equilibrium, which delivers quantitatively relevant predictions, thus giving us a framework to address quantitative questions.

Most closely related to our work are Mitman and Rabinovich (2015) and Jung and Kuester (2015). Also using a standard search model they find that UI policies, absent other labor market instruments, should be less generous during recessions. The difference in results, as we show, lies in the assumption about government commitment. Landais, Michaillat, and Saez (2018) use a sufficient statistics approach to decompose the changing gains and costs of UI over unemployment, and find it optimal to have more generous UI when unemployment is higher.[3] Our work also complements Birinci and See (2017) and Kekre (2017), who find in a search framework with incomplete markets (the former) or aggregate demand externality (the latter) more generous UI policies (with commitment) are welfare-improving in recessions.

Second, this paper is related to the vast literature on time-consistent policy. Time-consistent equilibrium has been widely used to study monetary policy and taxation (see, for example, Alesina and Tabellini 1990; Klein and Ríos-Rull 2003; Chari and Kehoe 2007; Battaglini and Coate 2008; Yared 2010; Song, Storesletten, and Zilibotti 2012; Bianchi and Mendoza (2018)). We apply the concept to UI policy. Following Klein, Krusell, and Ríos-Rull (2008) we use the Generalized Euler Equations to characterize the government's choices and compare to the full commitment Ramsey policy. The time-consistent equilibrium is especially applicable to UI choices during recessions when the political pressure is high to forgo prior commitment and implement discretionary policies.

The rest of the paper proceeds as follows. Section 2 describes the model environment and defines the private-sector equilibrium. Section 3 defines the Markov-perfect time-consistent equilibrium.

[3] Their key margin is an acyclical marginal cost of unemployment and a high marginal value of unemployment (low recruiting cost for firms) when unemployment is low. In our model recessions are driven by low productivity, which endogenously generates high unemployment. Because marginal cost of unemployment in terms of loss in production is low when productivity is lower, the government has incentives to extend UI in a recession.

We characterize the time-consistent benefit policy and compare it to the policy with government commitment to draw qualitative intuitions. Section 4 describes the parametrization strategy. Section 5 presents the quantitative equilibrium results. Section 6 focuses on UI extensions in recessions. Section 7 discusses alternative model specifications and assumptions. Section 8 concludes.

# 2 Model

In this section, we describe the model environment and characterize the equilibrium in the private sector given government policy. The model is based on a search-matching framework with aggregate productivity shocks.

## 2.1 Model environment

Time is discrete and infinite. The labor market in this model is populated by a continuum of workers and firms. The measure of workers is normalized to one. In any given period, a worker can be either employed or unemployed. Some unemployed workers receive UI benefits. Workers are risk averse and maximize expected lifetime utility given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, s_t), \quad U(c_t, s_t) = \log(c_t) - v(s_t)$$

where $\mathbb{E}_0$ is the period 0 expectation factor, and $\beta$ is the time discount factor. Period utility $U(\cdot,\cdot)$ comprises of utility from consumption $\log(c)$ and disutility from job search activity $v(s)$, where $v$ is a non-negative, strictly increasing, and convex function. Utility is increasing in $c$ and decreasing in $s$. The concavity introduced by a log function allows us to study the insurance motive of the government. Only unemployed workers choose positive search intensity; that is, there is no on-the-job search. Each period, an employed worker gets paid wages from production. Wage determination technology is specified later in this section. All unemployed workers derive utility from home production and leisure $h$. In addition, an unemployed worker on unemployment benefits ("UI recipient") receives UI benefits $b$ from the government each period. There is no private insurance market and workers cannot save or borrow.

Firms are risk neutral and maximize the expected discounted sum of profits, with the same discount factor $\beta$ as workers. A firm produces output if it hires a worker. In order to hire, the firm posts a vacancy by paying a flow cost $\kappa$.

Unemployed workers and vacancies form new matches. The number of matches formed increases in the number of vacancies and unemployment and when workers search harder. Let $I$ and $V$ denote the aggregate search by unemployed workers and the aggregate vacancy posting by firms, respectively. Then the number of new matches formed in a period is given by the matching function $M(I,V)$. The matching function exhibits constant returns to scale, is strictly increasing and strictly concave in both arguments and is bounded above by the maximum possible number of matches: $M(I,V) \leq \min\{I,V\}$. The job-finding probability per efficiency unit of search intensity ("search efficiency"), $f$, and the

job-filling probability per vacancy, $q$, are functions of the labor market tightness, $\theta = V/I$. More specifically,

$$\begin{aligned} f(\theta) &= \frac{M(I,V)}{I} = M(1,\theta) \\ q(\theta) &= \frac{M(I,V)}{V} = M\left(\frac{1}{\theta},1\right). \end{aligned}$$

Following the assumptions made on $M$, $f(\theta)$ is increasing in $\theta$ and $q(\theta)$ is decreasing in $\theta$. The job-finding probability for an unemployed worker searching with intensity $s$ is $sf(\theta)$. Existing matches are destroyed exogenously with job separation probability $\delta$.

Each period, a matched pair of a worker and a firm produces $z$, where $z$ is the aggregate labor productivity and is equal to $\bar{z}$ at the steady state.

## 2.2 Government policy

The government cannot borrow or save; instead, it balances the budget each period. The government finances the UI system through a lump sum tax, $\tau$, on all workers, both employed and unemployed. The government budget constraint is

$$\tau = u_{benefit} b. \tag{1}$$

where $u_{benefit}$ is the measure of UI recipients.

The government changes the generosity of the UI program by varying (1) the benefit level, $b \geq 0$, and (2) the benefit duration probability $d$, where with probability $d$ an unemployed worker eligible for benefits receives benefits today, and $1/(1-d)$ is the potential benefit duration. A higher probability $d$ increases the measure of UI recipients $u_{benefit}$, everything else equal.[4]

**Remark.** Two things are worth noting. First, using a probability $d$ captures the expected length of UI duration while keeping the model tractable. But it also means all UI recipients are affected equally by a change in UI extension policy, regardless of how long they have been collecting benefits. In Section 7.3 we allow a group of UI recipients to be unaffected by UI extensions ("the extension-neutral") as an alternate setup.

Second, a key assumption about duration probability is that the government excludes benefit-ineligible unemployed workers from receiving benefits: once an unemployed worker loses benefits, he has to find work first before becoming eligible for benefits again. The assumption is consistent with how UI policy normally works.[5]

[4] In specifying the decentralized labor market equilibrium we take the government policies as exogenous. In the next section, we allow the government to optimally choose these policies.

[5] During the most recent recession and especially during 2008-2009, some unemployed workers who had previously exhausted their benefits became eligible for new tiers of extensions. In other words, during this period the government does *not* exclude ineligible workers from collecting extended benefits. We interpret this non-exclusion as coming from convenience instead of optimality concerns: the government did not optimally *choose* to let these unemployed workers' benefits expire and then give them more tiers of extension. As such, our assumption about the duration probability is an abstraction of what happened during this period.

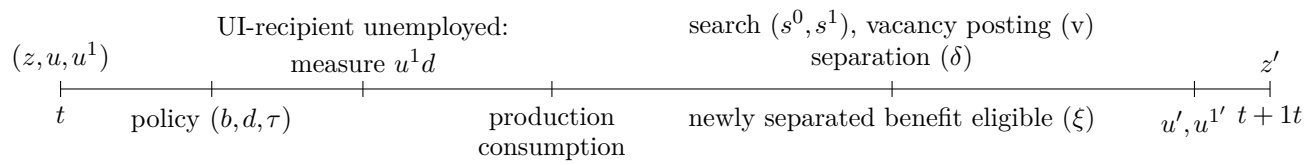



Figure 1: Timing of events.

## 2.3 Timing

The timing of events within a period is illustrated in Figure 1. The economy enters period $t$ with a measure of the total unemployed workers $u$ and a measure of the *benefit-eligible* unemployed workers $u^1$. The aggregate shock $z$ then realizes. $(z, u, u^1)$ are the aggregate states of the economy.

Once government policies $(b, d, \tau)$ for the period are announced, $u_{benefit} = u^1 d$ workers collect benefit (UI recipients). In other words, with probability $1 - d$, benefit-eligible unemployed workers lose benefit status in this period and will not be eligible until they find a job.

Employed workers produce $z$ and receive wages $w$. Unemployed workers get utility from non-market activity $h$ and, if collecting benefits, receive UI benefits $b$. All workers pay a lump sum tax $\tau$.

Finally, labor market turnovers (search, vacancy posting, and separation) take place. Given aggregate states and government policies for the period, unemployed workers choose how much to search: $s^1$ and $s^0$ for UI recipients and non-recipients, respectively. Firms post vacancies at cost $\kappa$. A fraction $\delta$ of the existing $1 - u$ matches are exogenously destroyed. With probability $\xi$ newly unemployed workers become benefit eligible for the next period.[6] The unemployment states of the next period $u', u^{1'}$ are determined, and in particular, benefit-eligible newly unemployed workers and UI recipients still unemployed at the end of the period make up next period's benefit-eligible unemployment $u^{1'}$. The matches formed today do not start producing until the next period.

The aggregate search is $I = u^1 d s^1 + (u - u^1 d) s^0$, aggregate vacancy posting is $V$, and market tightness is equal to $\theta = V/I$. More vacancies relative to unemployment and search mean the labor market is tight, whereas higher unemployment (or high search) relative to vacancy posting indicates a market is more slack. The laws of motion of unemployed workers are given by[7]

$$\text{total unemployment:} \quad u' = \underbrace{\delta(1-u)}_{\text{newly unemployed}} + \underbrace{(1 - f(\theta)s^0)(u - u^1 d) + (1 - f(\theta)s^1)u^1 d}_{\text{previously unemployed who do not find job}} \tag{2}$$

$$\text{benefit-eligible unemployed:} \quad u^{1'} = \underbrace{\delta(1-u)\xi}_{\text{benefit-eligible newly unemployed}} + \underbrace{(1 - f(\theta)s^1)u^1 d}_{\text{UI recipients who do not find job}} \tag{3}$$

[6] In reality, not all newly unemployed workers qualify for benefits. And among those who qualify a fraction choose not to collect. Following the literature on unemployment insurance, we model it using a probability for simplicity here.

[7] Equation (2) can be rewritten as $u' = u + \delta(1-u) - \bar{f}u$, where $\bar{f}$ is the average job finding rate defined as the product of search efficiency and the average search effort, i.e. $\bar{f} = f(\theta)\bar{s} \equiv f(\theta)\left([u^1 d s^1 + (u - u^1 d)s^0]/u\right)$.

Importantly, because consumption takes place before search, search activity today does not affect today's consumption. As such, changes in current period benefit level $b$ do not have a direct effect on search activity today. But because search today affects employment status and hence consumption tomorrow, the (anticipated) future benefit level $b'$ does affect search today.

**A note on notation:** In the rest of the section wherever appropriate we use $\mathcal{O}$ to summarize the states of the economy $(z, u, u^1)$. And denote by $g$ the government policy $(b, d, \tau)$.

## 2.4 Workers' value functions

A UI recipient consumes $h + b - \tau$ and chooses search intensity $s^1$; a non-recipient consumes $h - \tau$ and his search intensity is $s^0$. With probability $f(\theta)s,\ \ s \in \{s^0, s^1\}$, the unemployed worker finds a job and starts working the following period. Let $V^e(\mathcal{O}; g)$, $V^1(\mathcal{O}; g)$ and $V^0(\mathcal{O}; g)$ be the value of an employed worker, an unemployed UI recipient and non-recipient, respectively, given the aggregate states and government policy for the period.

The problem of an unemployed non-recipient (superscript 0 denotes no benefits) is

$$\begin{aligned} V^0(\mathcal{O}; g) \quad = \quad & \max_{s^0} \ \log(h - \tau) - v(s^0) + f(\theta)s^0\beta\mathbb{E}V^e(\mathcal{O}'; g') \\ & + (1 - f(\theta)s^0)\beta\mathbb{E}V^0(\mathcal{O}'; g'), \end{aligned} \tag{4}$$

and the problem of a UI recipient is

$$\begin{aligned} V^1(\mathcal{O}; g) \quad = \quad & \max_{s^1} \ \log(h + b - \tau) - v(s^1) + f(\theta)s^1\beta\mathbb{E}V^e(\mathcal{O}'; g') \\ & + (1 - f(\theta)s^1)\beta\mathbb{E}\left[(1 - d')V^0(\mathcal{O}'; g') + d'V^1(\mathcal{O}'; g')\right], \end{aligned} \tag{5}$$

UI duration policy tomorrow $d'$ affects the continuation value of the UI recipient: if he is still unemployed at the end of the period, with probability $d'$ he keeps benefits next period.

A worker entering a period employed produces and receives wage $w$ minus tax $\tau$. With probability $\delta$, he loses his job and becomes unemployed. A newly unemployed worker is benefit eligible next period with probability $\xi$.[8] The Bellman equation of an employed worker is then

$$\begin{aligned} V^e(\mathcal{O}; g) \quad = \quad & \log(w - \tau) + (1 - \delta)\beta\mathbb{E}V^e(\mathcal{O}'; g') \\ & + \underbrace{\delta(1 - \xi)\beta\mathbb{E}V^0(\mathcal{O}'; g')}_{\text{benefit ineligible newly unemployed}} + \underbrace{\delta\xi\beta\mathbb{E}\left[(1 - d')V^0(\mathcal{O}'; g') + d'V^1(\mathcal{O}'; g')\right]}_{\text{benefit eligible newly unemployed}}, \end{aligned} \tag{6}$$

**A note on notation:** For the rest of the paper, in order to avoid unnecessarily long equations we use $U^e$, $U^1$ and $U^0$ to denote the period u from consumption and disutility from search (if any) of workers, unemployed UI recipients and non-recipients: $U^e = \log(w - \tau)$, $U^1 = \log(h + b - \tau) - v(s^1)$, and $U^0 = \log(h - \tau) - v(s^0)$.

[8] There is no intra-temporal search, so a newly separated worker remains unemployed for at least one period.

## 2.5 Firm's value function

An unmatched firm posts a vacancy to be matched with a worker and start production. A firm that posts a vacancy incurs a flow cost $\kappa$. With probability $q(\theta)$, a vacancy is filled and ready for production the following period. Let $J^u(\mathcal{O};g)$ and $J^e(\mathcal{O};g)$ be the values of an unmatched and a matched firm, respectively. A matched firm receives output net of wages $z-w$. With a probability $\delta$, a match is destroyed at the end of the period and the firm becomes vacant. The Bellman equation of a matched firm is

$$J^e(\mathcal{O};g) \quad = \quad z-w+(1-\delta)\beta\mathbb{E}J^e(\mathcal{O}';g')+\delta\beta\mathbb{E}J^u(\mathcal{O}';g'). \tag{7}$$

The Bellman equation of an unmatched firm is

$$J^u(\mathcal{O};g) \quad = \quad -\kappa+q(\theta)\beta\mathbb{E}J^e(\mathcal{O}';g')+(1-q(\theta))\beta\mathbb{E}J^u(\mathcal{O}';g'), \tag{8}$$

In the equilibrium and under free entry condition, the firm posts vacancies until

$$J^u(\mathcal{O};g)=0, \tag{9}$$

and free entry condition requires that the total expected value of profits to be equal to the flow cost of posting vacancy:

$$\frac{\kappa}{q(\theta)}=\beta\mathbb{E}J^e(\mathcal{O}';g'). \tag{10}$$

## 2.6 Wage determination

When a match is formed, the economic rent is shared between firm and worker. There are many alternative ways to specify how this rent is shared (e.g. Hagedorn and Manovskii 2008, Hall and Milgrom 2008). We use a convenient wage rule:[9]

$$w(z)=\bar{w}z^{\epsilon_w}, \quad \epsilon_w\in[0,1) \tag{11}$$

According to this wage rule, wages increase when labor productivity $z$ is higher, with an elasticity $\epsilon_w$ that is less than one. This way, workers and firms share the risk of fluctuating aggregate labor productivity. The choice of an exogenous wage rule simplifies the private-sector equilibrium and allows a more focused discussion of the government's problem.

**Remark.** Wages also potentially depend on UI policy. In the literature, there is a wide range of estimates for the size of this effect. For example, Hagedorn, Karahan, Manovskii, and Mitman (2013) find a large effect of UI extensions on the employment through the macro effects on wages, whereas Johnston and Mas (2016) find a small estimate.[10] The wage rule in (11) abstracts from the effect of UI on wages to keep the analysis more straightforward. Under this specification, vacancy

[9] Similar wage specifications are used in, for example, Landais, Michaillat, and Saez (2018) and McKay and Reis (2017).

[10] More recently, Jäger, Schoefer, Young, and Zweimüller (2018) exploits reforms in Austrian UI system to find that wages are insensitive to UI benefit levels with point estimates implying a wage response of less than \$0.01 to a \$1 increase in UI benefit.

mostly responds to changes in labor productivity and not with changes in UI policy. This is consistent with Marinescu (2017)'s finding that UI extension has little effect on job vacancies. In Section 7.1, we allow wages to additionally rise with UI duration in a similar way to McKay and Reis (2017)'s treatment of wage UI-dependency. This captures in a reduced-form way the effect of UI on the worker's outside option in a Nash bargaining process.

## 2.7 Equilibrium in the private sector

Again using $\mathcal{O}$ to summarize the states of the economy $(z, u, u^1)$. The equilibrium in the private sector given government's policy is defined as follows.

Definition 1. (Private-sector equilibrium) Given UI policy $g = (b, d, \tau)$, wage function (11) and initial states $\mathcal{O}^-$, an equilibrium consists of $\mathcal{O}$–measurable functions for worker's search intensities $s^0(\mathcal{O}; g)$ and $s^1(\mathcal{O}; g)$, market tightness $\theta(\mathcal{O}; g)$, total unemployment $u'(\mathcal{O}; g)$ and benefit-eligible unemployment $u^{1'}(\mathcal{O}; g)$, and value functions $V^e(\mathcal{O}; g)$, $V^0(\mathcal{O}; g)$, $V^1(\mathcal{O}; g)$, $J^e(\mathcal{O}; g)$ and $J^u(\mathcal{O}; g)$, such that for all $(\mathcal{O}; g)$:

- the value functions satisfy the worker's and firm's Bellman equations (4)-(8);
- the search intensities $s^0$ and $s^1$ solve the unemployed worker's maximization problems of (4) and (5), respectively;
- the market tightness $\theta$ is consistent with the free-entry condition (10);
- the measures of unemployment satisfy the laws of motion (2)-(3).

## 2.8 Characterization of private-sector optimality

The private-sector equilibrium can be characterized by three optimality conditions.[11] In what follows, primes denote variables of the following period, and subscripts denote derivatives.

**Worker's Search Incentive.** The optimal choice of search intensity $s^0$ and $s^1$ for the unemployed worker is characterized by

$$\text{non-UI recipients:} \quad \frac{v_s(s^0)}{f(\theta)} = \beta \mathbb{E}\left\{ U^{e'} - U^{0'} + [1 - f(\theta')s^{0'} - \delta(1-\xi)]\frac{v_s(s^{0'})}{f(\theta')} - \delta\xi \frac{v_s(s^{1'})}{f(\theta')} \right\} \tag{12}$$

$$\begin{aligned} \text{UI recipients:} \quad \frac{v_s(s^1)}{f(\theta)} &= \beta \mathbb{E}(1-d')\left\{ U^{e'} - U^{0'} + [1 - f(\theta')s^{0'} - \delta(1-\xi)]\frac{v_s(s^{0'})}{f(\theta')} - \delta\xi \frac{v_s(s^{1'})}{f(\theta')} \right\} \\ &+ \beta \mathbb{E} d'\left\{ U^{e'} - U^{1'} + [1 - f(\theta')s^{1'} - \delta\xi]\frac{v_s(s^{1'})}{f(\theta')} - \delta(1-\xi)\frac{v_s(s^{0'})}{f(\theta')} \right\} \end{aligned} \tag{13}$$

[11] To economize on notation, we suppress the dependence on aggregate states and government policy $(\mathcal{O}; g)$. It should be understood throughout that the equilibrium allocations are functions with arguments $(\mathcal{O}; g)$. Online Appendix B.1 contains a derivation of the optimality conditions.

The worker's optimality conditions state that the marginal cost (left-hand side) of higher job-finding probability equals the marginal value (right-hand side). The marginal cost is the marginal disutility from search weighted by the aggregate search efficiency. The marginal value is the expected value of the utility gain from employment next period *and* the benefit of economizing on future search cost. We can make two useful observations.

PROPOSITION 1. Unemployed UI recipients search less than non-recipients, $s^1 < s^0$, given $v_s(s) > 0$ and $v_{ss}(s) > 0$, and $0 < d' < 1$.

The UI recipient's marginal gain of higher job-finding probability (right-hand side of Equation 13) depends on the future UI duration policy $d'$. Because the first part is identical to the marginal gain of the non-recipients (equation 12) and is larger than the second part, the marginal gain from search is lower for the UI recipients, as long as the future duration probability ($d'$) is greater than 0. Given an increasing marginal search cost function, it then implies that the UI recipients search less. In other words, a non-zero probability of receiving benefits *tomorrow* creates a moral hazard problem today for the UI recipients.[12]

PROPOSITION 2. A higher future UI duration (larger $d'$) or a higher future benefit $b'$ reduces the search incentive for the UI recipients.

A larger $d'$ reduces the total marginal gain (right-hand side of Equation 13) and hence the search incentive of UI recipients. At the same time, the second part of the marginal gain is decreasing in $b'$ (inside $U^{1'}$), so the search incentive is lower when the future benefit is expected to be higher.

**Firm's Vacancy Posting Incentive.** From firm's free entry condition we get the optimality condition characterizing the labor market tightness:

$$\frac{\kappa}{q(\theta)} = \beta \mathbb{E}\left[z' - w(z') + (1-\delta)\frac{\kappa}{q(\theta')}\right], \tag{14}$$

The marginal cost (left-hand side) equals the marginal gain (right-hand side) of a filled vacancy. The marginal cost is the flow cost of posting a vacancy weighted by the probability of filling that vacancy. The marginal gain is the profit from a filled vacancy and the discounted future value of the match. Because a newly formed match does not become operational until the next period, the gain from production only has components from the next period. The current productivity level $z$ therefore does not have a direct impact on the firm's current hiring decision. Instead, due to persistence in the productivity process it affects the firm's expectation of future productivity and hence its current hiring decision.

In the baseline model, labor market tightness is not affected by the unemployment or UI policy because wages are not. In Section 7.1, we allow wages to depend on UI duration policy which

[12] In reality, non-UI recipients may be one of two types: either they did not qualify for (or choose not to take up) UI, or their benefits ran out. In either case there may be selection issue, that on average those with and without UI may be different people and as a result search differently. Such ex ante difference is not the mechanism behind Proposition 1, which highlights the difference between expected additional value of employment. The empirical estimates in the literature likely capture both the ex ante heterogeneity and the mechanism highlighted here, and it would be difficulty to disentangle the two.

endogenously depends on unemployment. Under this alternate assumption, labor market tightness responds to UI policy and unemployment.

## 3 Time-Consistent Equilibrium

The government is a utilitarian planner who maximizes the expected value of worker's utility. The government policy instruments include benefit level $b$, benefit duration probability $d$, and lump-sum tax $\tau$.

A potential time-inconsistency issue exists in the government's problem. The government at time $t$ has an incentive to promise less generous future UI policies (shorter duration or lower benefits) to encourage search and job posting, but after employment outcome of time $t$ realizes, the government has an incentive to be more generous to the unemployed workers for higher average consumption. Because of this time-inconsistency, policies such as Ramsey policy have to assume commitment by the government to future policies; otherwise the policy cannot be implemented by the government. We take a different approach and instead consider the time-consistent equilibrium policies that the government at time $t$ willingly follows. More specifically, we focus on the Markov-perfect equilibrium policy, which only depends on the payoff-relevant states — $(z, u, u^1)$ in our case.

Intuitively, one can think of the economy as having a different government each period. Each successive government chooses only the current policy, taking all future governments' policies as given. In other words, today's government cannot directly choose future policies. Instead, both today's government and the private sector form expectations about future government policy rules. Like Klein, Krusell, and Ríos-Rull (2008), we focus on equilibria where government policy depends *differentiably* on the aggregate states of the economy.

The timing of events is as illustrated in Figure 1. Because each worker and firm is infinitely small, they take future government policies as given. The equilibrium described above can be equivalently stated as an equilibrium where the government chooses policy and private-sector allocations together given the states of the economy. We express the lump-sum tax as a function of other policies using the government's budget constraint:

$$\tau = u^1 db. \tag{15}$$

The government assigns equal weight to each worker, and the *government period-welfare function* is equal to the average utility of all workers, given by[13]

$$R(z, u, u^1, b, d, s^0, s^1) = (1-u)U^e + u^1 dU^1 + (u - u^1 d)U^0 \tag{16}$$

where $U^e$, $U^0$ and $U^1$ again denote the period utility from consumption and disutility from search (if any) of workers, unemployed UI recipients and non-recipients, respectively.

[13] Because of firm's free entry condition, the total expected value of profits is equal to the flow cost of posting vacancy. In other words, firm's expected profit net of vacancy posting cost is zero. We follow the treatment in the literature (e.g. Mitman and Rabinovich 2015, Jung and Kuester 2015) and do not include firm's profit in the government's objective function.

A few redistributional (*insurance*) effects are obvious from the tax function and the period-welfare function. For example, an increase in the total unemployment ($u$) reduces the average utility by reducing the proportion of employed workers, ceteris paribus. A benefit extension (an increase in $d$) raises the average utility by increasing the proportion of UI recipients among unemployed workers; it raises taxes and shifts consumption from workers (lower marginal utility) and non-recipients (higher marginal utility) to UI recipients.

In the equilibrium, government chooses policies to maximize current and discounted future welfare, subject to the optimality conditions in the private sector. These optimality conditions capture the *incentive* effects of the policies.

Definition 2. (Markov-perfect equilibrium) A Markov-perfect equilibrium consists of a government's value function $G$, government policy rules $\Psi^b$ and $\Psi^d$, and private decision rules $\{S^0, S^1, \Theta, \Gamma, \Gamma^1\}$ such that for all aggregate states $\mathcal{O} \equiv (z, u, u^1)$, given wage rule (11), the policies $b = \Psi^b(\mathcal{O})$, $d = \Psi^d(\mathcal{O})$, $s^0 = S^0(\mathcal{O})$, $s^1 = S^1(\mathcal{O})$, $\theta = \Theta(\mathcal{O})$, $u' = \Gamma(\mathcal{O})$, and $u^{1'} = \Gamma^1(\mathcal{O})$ solve

$$\max_{b,d,s^0,s^1\theta,u',u^{1'}} \qquad R(\mathcal{O}, b, d, s^0, s^1) + \beta \mathbb{E} G(\mathcal{O}')$$

subject to

- The worker's laws of motion (2) and (3).
- The private-sector optimality conditions (12), (13) and (14).
- The government's budget constraint (15).
- The government value function satisfies the functional equation

$$G(\mathcal{O}) \quad \equiv \quad R(\mathcal{O}, \Psi^b(\mathcal{O}), \Psi^d(\mathcal{O}), S^0(\mathcal{O}), S^1(\mathcal{O})) + \beta \mathbb{E} G(z', \Gamma(\mathcal{O}), \Gamma^1(\mathcal{O})).$$

The equilibrium captures the effects of anticipated future UI policies ($\Psi^b(\mathcal{O}')$ and $\Psi^d(\mathcal{O}')$) on the search decisions of workers, and the government takes into account these effects when making policy decisions. In the equilibrium, both the government's and the private sector's problems are solved with a rational expectation on future policy rules, and all successive governments follow the same set of policy rules.

The first-order conditions of the government's problem can be found in Online Appendix B.2. The conditions contain derivatives of policy functions $\{\Psi^b, \Psi^d, S^0, S^1, \Theta\}$ with respect to the state variables $(u, u^1)$. These derivatives capture the effects of changing unemployment states on the future UI policy and the equilibrium labor market.

## 3.1 Generalized Euler Equation: Unpacking the government's problem

For a better understanding of the government's choices of UI policies, we combine government's first-order conditions into Generalized Euler Equations (GEEs) a la Klein, Krusell, and Ríos-Rull

(2008).[14] For expositional convenience, we use $MV(s^0;\mathcal{O}')$ and $MV(s^1;\mathcal{O}')$ to denote the *private* marginal values of search, which is the right-hand side of the unemployed workers' Euler equations (12)-(13). A change in this marginal value changes the private incentives to search. $\mu_0, \mu_1 > 0$ are the Lagrange multipliers on these Euler equations.[15]

Given the aggregate states of the economy and the private-sector optimality conditions, the duration probability $d$ can be characterized by the GEE[16]

$$
\begin{aligned}
0 \;=\; & \underbrace{u^1(U^1-U^0)}_{\text{insurance effect}>0} + \underbrace{\frac{\partial R}{\partial \tau}\frac{\partial \tau}{\partial d}}_{\text{tax effect}<0} + \underbrace{\lambda \frac{\partial u'}{\partial d}}_{\text{extensive margin search effect}<0} \\
& + \underbrace{\frac{\partial u^{1'}}{\partial d}}_{>0}\Bigg[\underbrace{\left(\mu_0 \frac{\partial MV(s^0;\mathcal{O}')}{\partial u^{1'}} + \mu_1 \frac{\partial MV(s^1;\mathcal{O}')}{\partial u^{1'}}\right)}_{\text{higher } u^{1'} \text{ on private choices today}} + \beta \mathbb{E} \underbrace{\left(\frac{\partial R'}{\partial u^{1'}} + \lambda' \frac{\partial u''}{\partial u^{1'}}\right)}_{\text{higher } u^{1'} \text{ on tomorrow's welfare}}\Bigg] \\
& + \beta \mathbb{E} \underbrace{\frac{\partial u^{1'}}{\partial d}}_{>0} \underbrace{\left.\frac{dd'}{du^{1'}}\right|_{\text{hold } u^{1''} \text{ constant}}}_{<0} \Bigg[\underbrace{u^{1'}(U^{1'}-U^{0'}) + \frac{\partial R'}{\partial \tau'}\frac{\partial \tau'}{\partial d'} + \lambda' \frac{\partial u''}{\partial d'}}_{\text{insurance, tax, and extensive margin search effects tomorrow}}\Bigg],
\end{aligned}
\tag{17}
$$

where $\lambda < 0$ is the shadow value of unemployment and is given by

$$
\begin{aligned}
\lambda \;=\; & \underbrace{\left(\mu_0 \frac{\partial MV(s^0;\mathcal{O}')}{\partial u'} + \mu_1 \frac{\partial MV(s^1;\mathcal{O}')}{\partial u'}\right)}_{\text{higher } u' \text{ on private choice today}} + \beta \mathbb{E} \underbrace{\left(\frac{\partial R'}{\partial u'} + \lambda' \frac{\partial u''}{\partial u'}\right)}_{\text{higher } u' \text{ on tomorrow's welfare}} \\
& + \beta \mathbb{E} \underbrace{\left.\frac{dd'}{du'}\right|_{\text{hold } u^{1''} \text{ constant}}}_{<0} \left[u^{1'}(U^{1'}-U^{0'}) + \frac{\partial R'}{\partial \tau'}\frac{\partial \tau'}{\partial d'} + \lambda' \frac{\partial u''}{\partial d'}\right],
\end{aligned}
\tag{18}
$$

and the UI benefit level $b$ is characterized by

$$
\underbrace{u^1 dU_c^1 + \frac{\partial R}{\partial \tau}\frac{\partial \tau}{\partial b}}_{\text{redistribution effect}} = 0. \tag{19}
$$

These equations summarize the marginal effects of a policy change from the Markov government's perspective. Equation (17) shows a benefit extension (through an increase in duration probability) has the following effects.

First, a **benefit extension** increases the share of unemployed workers who are UI recipients today. This raises the average utility of workers today as UI recipients have higher utility than non-recipients (first term of Equation 17). This is the *insurance* effect. This increased duration is financed through higher tax, which creates a negative tax effect on today's welfare (second term). Second, by increasing the share of UI recipients a benefit extension lowers average search, because UI recipients search less than non-recipients (Proposition 1). This leads to higher future unemployment, which has a

[14] Online Appendix B.2 contains the derivation of GEEs.

[15] Primes are used to denote future variables, e.g. $u'$ for total unemployment tomorrow and $U^{e'}$ for utility of workers tomorrow. Subscripts denote partial derivatives, e.g. $U_c^e = U_c(w-\tau)$

[16] $\frac{dd'}{du^{1'}}$ holding $u^{1''}$ constant $= -\frac{\partial u^{1''}}{\partial u^{1'}} / \frac{\partial u^{1''}}{\partial d'}$; and $\frac{dd'}{du'}$ holding $u^{1''}$ constant $= -\frac{\partial u^{1''}}{\partial u'} / \frac{\partial u^{1''}}{\partial d'}$.

shadow value $\lambda < 0$ on the total welfare. We call this the *extensive margin search* effect (third term: $\partial u'/\partial d = u^1 f(\theta)(s^0 - s^1)$).

The second line contains the effects of a benefit extension through raising the measure of benefit-eligible unemployment $u^{1'}$ ($\partial u^{1'}/\partial d > 0$). The anticipation of higher future unemployment alters private choices today. For example, $\partial MV(s^1;\mathcal{O}')/\partial u^{1'}$ is the effect of higher $u^{1'}$ on the marginal value of search by UI recipients. Because $u^{1'}$ is a state tomorrow, a change induces expected changes in tomorrow's UI policy and private choices, which in turn change the marginal value of search and the unemployed worker's search effort today. We call this the *intensive margin search* effect. A positive value means higher $u^{1'}$ raises the marginal value of search and hence increases today's search effort $s^1$. Here $\mu_1$ can be interpreted as the social value of this effect. In addition to changing private choices, higher $u^{1'}$ has direct welfare costs: on tomorrow's average utility ($\partial R'/\partial u^{1'}$) and future unemployment ($\partial u''/\partial u^{1'}$).

The third line contains the effects of an extension through changing tomorrow's duration. Holding $u^{1''}$ (eligible unemployment at the end of tomorrow) constant, an extension today reduces tomorrow's duration.[17] This shorter benefit duration tomorrow affects tomorrow's average utility and value of unemployment through the same channels as today (first line).

Equation (18) gives the expression for the shadow value of unemployment $u'$. This value is negative ($\lambda < 0$), implying that higher unemployment creates a negative net value on total welfare as expected. Unemployment affects total welfare through its effects on private choices: a positive $\partial MV(s^0;\mathcal{O}')/\partial u'$ or $\partial MV(s^1;\mathcal{O}')/\partial u'$ means a higher unemployment increases the marginal value of search. Quantitatively, in the equilibrium, this is indeed the case. At the same time, higher $u'$ leads to lower future average utility ($\partial R'/\partial u' = U^{0'} - U^{e'} < 0$) and higher future unemployment ($\partial u''/\partial u' > 0$). Lastly, because $u'$ is a state tomorrow, a higher value changes future UI duration policy, which has impact on future welfare (second line).[18]

Equations (17)-(18) also provide some insight for how benefit duration changes in recessions. **During recessions** lower productivity increases workers' marginal utility of consumption, so the tax effect is (negative and) larger. As a result, the marginal gain of UI extension from the combined insurance and tax effects is smaller. But this change is small.[19] At the same time, the marginal cost of UI extension is also smaller because the cost of unemployment is lower in recessions (smaller $-\lambda$). This is because lower wages mean the loss of average consumption from higher unemployment is small. Lower productivity reduces firm's job posting and leads to low search efficiency, which means the cost of reduced search activity is low (smaller $\partial u'/\partial d = u^1 f(\theta)(s^0 - s^1)$). As a result, the marginal cost of UI extension is lower. Because of a faster fall in the marginal cost than the marginal

[17] Note that holding $u^{1''}$ constant gives a partial equilibrium effect. In the full Markov equilibrium, benefit extension today raises $u^{1'}$ which raises benefit duration tomorrow.

[18] Holding $u^{1''}$ constant, a higher $u'$ reduces future UI duration in the partial equilibrium. In the full Markov equilibrium, a higher $u'$ induces benefit extension tomorrow.

[19] To see why the change in the combined insurance and tax effect is small, suppose workers are risk neutral, then the combined effect reduces to $u^1[v(s^0) - v(s^1)]$, as the consumption part of the insurance effect cancels out with the tax effect. A change in productivity has very small effects on this term. With risk averse workers, the consumption part of the insurance and tax effects do not cancel out, but the net effect is small.

gain of UI extension, as productivity drops the government can "afford" longer UI extensions.

Another feature of recessions is rising unemployment (both total and benefit-eligible unemployment). In response to rising unemployment, both the insurance-tax effect and the extensive margin effect are larger. Intuitively, at higher unemployment, the gain from an extension is larger because a benefit extension creates more UI recipients and thus increases the average consumption more strongly; and the cost is also larger because an extension lowers search of more unemployed workers. But because the shadow cost of unemployment $-\lambda$ also becomes smaller when the unemployment is higher, it mitigates the rise in the marginal cost. On net, when the unemployment rises, the gain of benefit extensions rises faster than the cost, and the government has incentives to extend benefits.

Turning to **benefit level**. Equation (19) summarizes the welfare effects of changing benefit level.[20] The effects here are simple: A redistributional effect increases consumption of the unemployed UI recipients, at the expense of higher taxes. Notice that unlike the effects of a benefit extension (Equation 17), changing benefit level here does not impact future welfare or change private choices. This is because a change in benefit level does not directly affect future unemployment ($u'$, $u^{1'}$).[21] In a recession, government has incentives to lower benefit level because wages are low due to lower productivity, which raise the marginal cost of higher taxes ($\partial R/\partial \tau$ part of Equation 19). At the same time, higher total unemployment increases the measure of unemployed non-UI recipients, so the government has incentives to lower benefit level to equalize consumption.[22]

Given the equations that characterize the UI policies, the Markov-perfect equilibrium is characterized by a system of *functional* equations (2), (3), (12)–(14), and (17)–(19). An analytical characterization of the Markov-perfect equilibrium is not possible; instead, we solve for the equilibrium numerically by approximating the government policy rules and the private-sector decision rules using the Chebyshev collocation method. The solution is a set of policy functions over the state space $\mathcal{O}$.[23]

## 3.2 A comparison with Ramsey commitment policy

Although the focus of this paper is on the time-consistent UI policy, it is natural to wonder how the time-*in*consistent (Ramsey) policy differs.[24] The Ramsey government makes contingent policies for all future periods at time zero and is assumed to stick to these pre-determined policies. If at any

[20] Equation 19 can be simplified into $(1-u)[U_c^1 - U_c^e] - (u - u^1 d)[U_c^0 - U_c^1] = 0$.

[21] Even though a change in current benefit level does not affect search incentives, higher expected *future* benefits does lower incentives to search today (Proposition 2).

[22] Higher unemployment increases the measure of non-UI recipients and reduces the measure of employed workers. The former has higher marginal utility of consumption, so the effect is stronger.

[23] The GEE contains derivatives of policy rules, which make solving the Markov equilibrium different from solving a standard growth model or the optimal policy problem of a government with commitment.

[24] Mitman and Rabinovich (2015) and Jung and Kuester (2015) among others look at the Ramsey UI policy over the business cycle. Mitman and Rabinovich (2015) do so in a setup similar to ours with differences in timing, tax structure, and government's budget constraint. Jung and Kuester (2015) consider other policy tools alongside UI. Neither papers study the time-consistent UI policy, which is the focus of this paper. With this comparison between the Ramsey and the time-consistent equilibria, we shed light on the differences between the two policies, which we show quantitatively in Section 5.2.

time t>0 the government is given the choice to break from the pre-determined policy rules, it would find it optimal to do so, because the welfare from time-t onward is maximized by deviating from the pre-determined plan. In other words, policy commitment is needed to implement the Ramsey policy. This incentive to deviate from the original plan also makes the Ramsey policy time inconsistent.

Definition 3. (Ramsey policy) Given initial measures of unemployed population $(u_0, u_0^1)$ and aggregate labor productivity $z_0$, the Ramsey government policy consists of a sequence of benefit level and duration probability $\{b_t, d_t\}_{t=0}^{\infty}$ that solves

$$\max_{\{b_t, d_t, s_t^0, s_t^1, \theta_t, u_{t+1}, u_{t+1}^1\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t R(z_t, u_t, u_t^1, b_t, d_t, s_t^0, s_t^1)$$

over the set of all policies that satisfy the worker's flow equations (2)-(3), the private-sector optimality conditions (12)-(14), and the government's budget constraint (15) for all time $t$ and aggregate shock $\{z_t\}_{t=0}^{\infty}$.

To highlight the differences between the Ramsey commitment policy and the time-consistent Markov policy, we compare the Ramsey optimality conditions with the GEEs of the Markov government (Equations 17-19). Define the Lagrange multipliers on the time-$t$ unemployed worker's optimality conditions (12)-(13) as $\beta^t \tilde{\mu}_{0,t}$ and $\beta^t \tilde{\mu}_{1,t}$. For easy comparison, we write Ramsey conditions recursively, using "–" to indicate variables of the previous period.[25]

The optimal Ramsey duration probability $d$ can be characterized by

$$\begin{aligned} 0 \;=\; & u^1(U^1 - U^0) + \frac{\partial R}{\partial \tau}\frac{\partial \tau}{\partial d} + \tilde{\lambda}\frac{\partial u'}{\partial d} + \underbrace{\frac{1}{\beta}\left[\tilde{\mu}_0^{-}\frac{\partial MV(s^{0-})}{\partial d} + \tilde{\mu}_1^{-}\frac{\partial MV(s^{1-})}{\partial d}\right]}_{\text{effect on private choices previous period}} \\ & + \frac{\partial u^{1'}}{\partial d}\left\{\tilde{\mu_0}\frac{MV(s^0)}{\partial u^{1'}} + \tilde{\mu_1}\frac{\partial MV(s^1)}{\partial u^{1'}} + \beta\mathbb{E}\left[\frac{\partial R'}{\partial u^{1'}} + \tilde{\lambda}'\frac{\partial u''}{\partial u^{1'}}\right]\right\} \\ & + \beta\mathbb{E}\frac{\partial u^{1'}}{\partial d}\left.\frac{dd'}{du^{1'}}\right|_{u^{1''}\text{constant}}\left\{u^{1'}[U^{1'} - U^{0'}] + \frac{\partial R'}{\partial \tau'}\frac{\partial \tau'}{\partial d'} + \tilde{\lambda}'\frac{\partial u''}{\partial d'} + \underbrace{\frac{1}{\beta}\left[\tilde{\mu}_0\frac{\partial MV(s^0)}{\partial d'} + \tilde{\mu}_1\frac{\partial MV(s^1)}{\partial d'}\right]}_{\text{effect of } d' \text{ on private choices today}}\right\}, \end{aligned} \tag{20}$$

where the shadow value of unemployment $\tilde{\lambda} < 0$ is given by

$$\begin{aligned} \tilde{\lambda} \;=\; & \beta\mathbb{E}\left\{\frac{\partial R'}{\partial u'} + \tilde{\lambda}'\frac{\partial u''}{\partial u'}\right\} \\ & + \beta\mathbb{E}\left.\frac{dd'}{du'}\right|_{u^{1''}\text{ constant}}\left\{u^{1'}[U^{1'} - U^{0'}] + \frac{\partial R'}{\partial \tau'}\frac{\partial \tau'}{\partial d'} + \tilde{\lambda}'\frac{\partial u''}{\partial d'} + \underbrace{\frac{1}{\beta}\left[\tilde{\mu}_0\frac{\partial MV(s^0)}{\partial d'} + \tilde{\mu}_1\frac{\partial MV(s^1)}{\partial d'}\right]}_{\text{effect of } d' \text{ on private choices today}}\right\}, \end{aligned} \tag{21}$$

and the optimal UI benefit level $b$ is characterized by

$$\left(u^1 dU_c^1 + \frac{\partial R}{\partial \tau}\frac{\partial \tau}{\partial b}\right) + \underbrace{\frac{1}{\beta}\left[\tilde{\mu}_0^{-}\frac{\partial MV(s^{0-})}{\partial b} + \tilde{\mu}_1^{-}\frac{\partial MV(s^{1-})}{\partial b}\right]}_{\text{effect on private choice previous period}} = 0. \tag{22}$$

[25] Online Appendix B.3 contains the full derivation of the Ramsey optimality conditions.

Because the Ramsey policies are chosen at time zero to maximize all future welfare, the policy at any time t internalizes the effects on time t-1. In other words, the Ramsey government is *backward-looking*, in addition to *forward-looking* as the Markov government. These backward effects are captured by the lagged multipliers: $\tilde{\mu}_0^-$ and $\tilde{\mu}_1^-$, which are absent from the Markov optimality conditions (17)-(19). Because of this backward-looking effect, the Ramsey government considers how a generous UI policy today reduces worker's search incentives yesterday, and as a result, the steady state Ramsey benefit level and duration are less generous than Markov. **During recessions** when the unemployment is high, the Ramsey government additionally realizes that benefit extensions discourage *more* unemployed workers from search (captured by $\tilde{\mu}_1^- \partial MV(s^{1-})/\partial d$ term in 20). This raises the marginal cost of an extension beyond the level considered by the Markov government. As such, compared to the Markov government, the Ramsey government has less incentive to extend (more incentive to *shorten*) benefit duration in a recession. The size of this difference is a quantitative question. For example, it depends on the elasticity of search with respect to UI duration changes for UI recipients, which we target in the quantitative analysis using empirical estimate from the literature.[26]

# 4 Parametrization

We describe our calibration strategy in this section. The model period is one month. We calibrate the parameters by matching moments of the Markov equilibrium to the empirical moments of the U.S. labor market between 2003.I and 2007.IV. We do this under the assumption that the government behaves as a benevolent utilitarian welfare-maximizer making discretionary policies and the equilibrium economy with such a government mirrors the U.S. economy. It turns out this assumption is not far from reality, as we show using the calibrated model, the Markov benefit extensions mimic the U.S. policies quite closely (Section 6).

The utility function is

$$U(c, s) = \log(c) - v(s),$$

where $v(\cdot)$ is the search cost function, which we specify following the literature:

$$v(s) = \alpha \frac{s^{1+\phi}}{1+\phi}.$$

For any $\alpha > 0$, $v_s(s) > 0$ and $v_{ss}(s) > 0$, and $v(0) = v_s(0) = 0$.

We adopt the matching function from Den Haan, Ramey, and Watson (2000), which is also used

[26] Another distinction from the Markov equilibrium is unlike in the Markov optimality conditions (17)-(18), the marginal values of search MV in the Ramsey government's optimality conditions do not have $\mathcal{O}'$. This is because the Ramsey government directly chooses future policies $d'$, $b'$ etc, whereas for the Markov government because of lack of commitment, the future policies are not directly chosen and are functions of future states $\mathcal{O}'$. Because of this difference, the Markov government's optimality conditions contain derivatives of policy functions with respect to endogenous states $(u', u^{1'})$ but not in the Ramsey government's problem.

in Hagedorn and Manovskii (2008) and Krusell, Mukoyama, and Sahin (2010) among others,

$$M(I,V) = \frac{V}{[1+(V/I)^{\chi}]^{1/\chi}}, \tag{23}$$

where $I = u^1 ds^1 + (u - u^1 d)s^0$ is the aggregate job search and $V$ is the aggregate vacancy posting in the economy. The market tightness $\theta$ is given by[27]

$$\theta = \frac{V}{I} \equiv \frac{V}{u^1 ds^1 + (u - u^1 d)s^0}. \tag{24}$$

This matching function guarantees that both the search efficiency (i.e. per-search unit job finding rate),

$$f(\theta) = \frac{\theta}{[1+\theta^{\chi}]^{1/\chi}},$$

and the job-filling rate,

$$q(\theta) = \frac{1}{[1+\theta^{\chi}]^{1/\chi}},$$

are strictly less than 1.

The externally determined parameters are summarized in Table 1. The discount factor $\beta$ is set at $0.99^{1/3}$, giving a quarterly discount factor of 0.99. The search cost curvature parameter $\phi$ is set to 1 following the average estimate in the literature.[28]

We calculate the average monthly job separation rate from the aggregate-level CPS data and obtain an average separation rate $\delta = 0.02$ for the period 2003.I-2007.IV.[29] We set the cost of vacancy creation $\kappa$ to be 58% of monthly labor productivity following Hagedorn and Manovskii (2008).

As in Shimer (2005), labor productivity $z$ is taken to be the average real output per employed person in the non-farm business sector. This measure is taken from the seasonally adjusted quarterly data constructed by the Bureau of Labor Statistics. We normalize the mean productivity to be $\bar{z} = 1$, and assume an AR(1) process for the shock to $z$:

$$\log z' = \rho \log z + \sigma_{\epsilon}\epsilon,$$

where $\rho \in [0,1)$, $\sigma_{\epsilon} > 0$, and $\epsilon$ are i.i.d. standard normal random variables. We target a quarterly autocorrelation of 0.762 and an unconditional standard deviation of 0.013 for the HP-filtered productivity process. At a monthly frequency this means setting $\rho = 0.9680$ and $\sigma_{\epsilon} = 0.006$.

Wages are determined according wage rule (11) and transformed with log below:

$$\log w = \log \bar{w} + \epsilon_w \log z.$$

[27] Note that here, as for any model with endogenous search (e.g. Mitman and Rabinovich 2015), the market tightness is a function of search $(s^1, s^0)$, in addition to vacancy (V) and unemployment ($u^1 d$ for measure of unemployed UI recipients, and $u - u^1 d$ for non-recipients). As such, the definition and calculation of market tightness differ from a model without *endogenous* search where tightness only depends on vacancy and total unemployment: $\theta = V/u$.

[28] Imposing $\phi$ equal to 1 gives a quadratic search cost function. This restriction is consistent with estimates by Yashiv (2000), Christensen et al. (2005), and Lise (2013), and calibration work of Nakajima (2012).

[29] Although some may argue that the U.S. economy during 2003.I-2007.IV is *above* the long-run trend, we believe it is an appropriate period to target for the labor market, especially because of the secular downward trend in job separation rate documented by, for example, Fujita (forthcoming). Given this trend, using the average job separation rate over a longer horizon would overestimate the recent steady-state numbers.

Table 1: Externally Determined Parameters

| Parameter | Description | Value |
|---|---|---|
| $\beta$ | Discount factor | $0.99^{1/3}$ |
| $\phi$ | Search cost curvature | 1 |
| $\delta$ | U.S. job separation rate | 0.02 |
| $\kappa$ | Vacancy posting cost | 0.58 |
| $\rho$ | Persistence of productivity | 0.968 |
| $\sigma_\epsilon$ | Standard deviation of innovation to productivity | 0.0060 |
| $\bar{w}$ | Steady state wage rate | 0.977 |
| $\epsilon_w$ | Elasticity of wage with respect to productivity | 0.45 |

Note: Calibration targets are monthly statistics of the U.S. economy.

Following the estimate of Hagedorn and Manovskii (2008) we set the elasticity of wages with respect to productivity $\epsilon_w = 0.45$. The steady state wage is set $\bar{w} = 0.977$.[30]

We jointly calibrate four parameters using four moments.[31] The four parameters are (1) the value of nonmarket activity $h$, (2) the level parameter of search cost $\alpha$, (3) the matching function parameter $\chi$, and (4) the probability that a newly unemployed worker is benefit-eligible $\xi$. The four target moments are: (1) the UI replacement ratio,[32] (2) the average job finding rate,[33] (3) the effect of a one-week benefit extension on the unemployment duration of UI recipients,[34] and (4) the proportion of UI recipients among unemployed workers. Intuitively, the UI replacement ratio pins down the value of nonmarket activity $h$, because benefit level is endogenously chosen and is a function of $h$; average job finding rate and the micro-effect of benefit extension together pin down the matching parameter $\chi$ and disutility of search $\alpha$; and the proportion of covered unemployment pins down the probability of gaining UI status $\xi$.

We target the UI replacement ratio at 40%, based on the replacement ratio reported on the US Department of Labor (DOL) website for post-2000. The average job finding rate during 2003.I-2007.IV is 0.40. Using DOL reported population of weekly continuing UI claims and unemployment popula-

[30] We set $\bar{w} = 0.977$ to guarantee that given $\kappa$ and $\epsilon_w$ values, firms still have enough expected profits to post vacancy at the lowest productivity level. With a higher $\bar{w}$, firm's expected profit becomes too low in recessions, and as a result the job filling rate $q$ implied by the firm's free entry condition (Equation 14) is greater than one. This is not a problem in a competitive equilibrium given government policies, because the matching function (23) guarantees $q$ is bounded by 1. But an implied $q > 1$ creates potential issues in solving the government's problem.

[31] We use a derivative-free algorithm for least-squares minimization to perform joint calibration. See Zhang, Conn, and Scheinberg (2010) for details.

[32] Unlike Shimer (2005) and Hagedorn and Manovskii (2008), the benefit level in our model is endogenously chosen by the government and is a function of nonmarket activity in the steady state. This is why we can target replacement ratio to pin down the value of nonmarket activity.

[33] The average job finding rate in the model $\bar{s}f(\theta)$ is calculated using the *average* search $\bar{s} = I/u \equiv [u^1 ds^1 + (u - u^1 d)s^0]/u$.

[34] The effect of 1-week extension is computed as the coefficient of the OLS regression with the unemployment duration of UI recipients $1/(f(\theta)s^1)$ as the dependent variable and UI duration (in weeks) as the independent variable. Consistent with what the estimates in the literature try to measure, we use the partial (micro) elasticity by holding fixed the economy-wide search efficiency $f(\theta)$ (macro effect) and allow individual search effort $s^1$ to change in response to benefit extensions.

Table 2: Estimated effect of UI benefit extension on unemployment duration (micro-elasticity)

| Estimate<br>ΔUI dur→Δunemp dur | Source | Estimation methodology |
|---|---|---|
| 1 week→0.16 weeks | Moffitt (1985) | Cross-state and time differences in UI duration |
| 1 week→0.16-0.20 weeks | Katz and Meyer (1990) | Simulated UI extension from 26 to 39 weeks |
| 13 weeks→1 week | Card and Levine (2000) | 13 week extension in NJ in 1996 (non-recession) |
| 10 weeks →1.5 weeks | Valletta (2014) | Discrete hazard analysis; follow Rothstein (2011) |
| 1 month→10 days | Johnston and Mas (2016) | UI duration cut in MS in 2011; regression discontinuity |

Note: Estimates cited are the increase in individual (those who collect benefits) unemployment duration to a given increase in UI duration (e.g. 1 week, 13 weeks, 1 month).

tion we obtain an average proportion of UI-covered unemployment of 0.45. We take the response of unemployment duration to benefit extension from micro-estimates in the literature summarized in Table 2. The estimated effect ranges from an average increase of 0.08 weeks (Card and Levine 2000) to more than 0.3 weeks (Johnston and Mas 2016) in response to a one-week benefit extension. We take a median value of 0.16 as the calibration target.

Table 3: Internally Calibrated Parameters: Markov Economy

| Parameter | Description | Value |
|---|---|---|
| $h$ | Value of nonmarket activity | 0.59 |
| $\chi$ | Matching parameter | 4.52 |
| $\alpha$ | Disutility of search | 3.2 |
| $\xi$ | Prob. newly unemployed is benefit eligible | 0.49 |
| **Target Moments** | **Data** | **Model** |
| UI replacement ratio | 40% | 38% |
| Average job finding rate $\bar{s}f(\theta)$ | 0.40 | 0.42 |
| % UI recipients among unemployed $u^1d/u$ | 0.45 | 0.45 |
| Δunemp duration/ΔUI duration | 0.16 | 0.152 |
| **Key Non-Target Moments** | **Data** | **Model** |
| Average UI Duration | 26 | 27 |
| Std dev of unemployment $u$ | 0.123 | 0.106 |
| Std dev of vacancy posting $v$ | 0.142 | 0.142 |
| Std dev of v-u ratio | 0.257 | 0.217 |

Note: See text for how model-generated moments are calculated. Seasonally adjusted unemployment series, $u$, is constructed by the BLS from the CPS. Vacancy-posting, $v$, is Barnichon (2010)'s spliced series of seasonally adjusted help-wanted advertising index constructed by the Conference Board and the job-posting data from the JOLTS. Both $u$ and $v$ are quarterly averages of monthly series. All second-order moments are reported in logs as deviations from an HP trend with smoothing parameter 1,600.

Table 3 reports these internally calibrated parameters and calibration targets. The calibrated model delivers an untargeted benefit duration of 27 weeks, compared to the benefit duration of 26 weeks in the U.S. during normal times. The last section of Table 3 compares some key second-order

moments in the model and in the data.[35] Two model and calibration features help generate second-order moments that are reasonably close to the data. First, we set the elasticity of wages with respect to productivity to the empirical estimate. The relatively low wage elasticity (0.45) makes firm's profit and vacancy posting responsive to business cycle shocks and helps generate enough volatility in unemployment. Second, in the equilibrium, unemployed workers anticipate higher benefit durations when productivity is low, which lowers search even more and leads to larger cyclical responses in search and hence unemployment.

# 5 Time-Consistent UI Policy

In this section, we present the equilibrium Markov UI policy and highlight the differences between Ramsey commitment policy and Markov policy.

## 5.1 Markov equilibrium UI policy

The discussion of Section 3.1 provides some intuition for how Markov UI policies change in response to changes in unemployment and productivity. Here we show the Markov equilibrium UI policy functions based on the calibrated model. Because the policy functions are multi-dimensional, we show plots in two ways. Figure 2 takes a slice of the policy function along one dimension at a time: Panel A plots the UI policy rules over total unemployment, and Panel B plots over productivity. In each plot, the solid line represents the policy rule, and the dashed line marks the steady-state unemployment or productivity. Figure 3 looks at the UI policy functions in two dimensions at a time: Panel A holds productivity at steady state and looks at policy changes over unemployment states ($u$ and $u^1$), while Panel B holds benefit-eligible unemployment ($u^1$) at steady state and looks at UI policy over total unemployment and productivity.

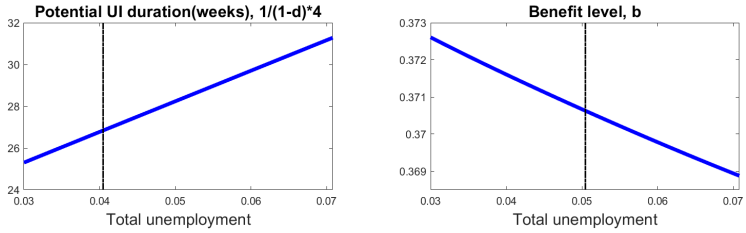
Consistent with the intuition of Section 3.1, UI duration is longer when unemployment is higher or when productivity is lower. UI benefit level is higher when productivity is higher or when total unemployment is lower. Overall, the equilibrium UI duration is countercyclical, whereas the benefit level is slightly procyclical.

**A comparative static analysis of benefit extension.** In this exercise we decompose government's optimality condition (17) into marginal gain and marginal cost of extending UI duration.[36] Figure 4 shows how the gain and cost change over productivity and unemployment, holding UI policy unchanged. This exercise provides a visual illustration of the discussion in Section 3.1 and a middle step between the government's optimality condition and the duration probability policy function.

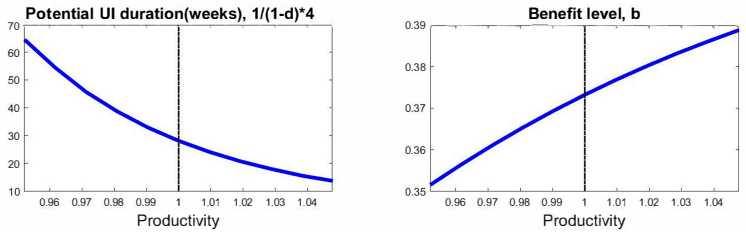
[35] Here we use the long-run average job separation rate when calculating the model generated second-order moments. In Section 6, we focus on the Great Recession and use the realized path of job separation rates to generate period-to-period movements.

[36] Marginal gain comprises today's and tomorrow's net value from the insurance and tax effects. Marginal cost consists of today's and tomorrow's extensive margin search effects of benefit extension, as well as the effect of extension through raising benefit-eligible unemployment (second line of 17).

Panel A: Over total unemployment, u



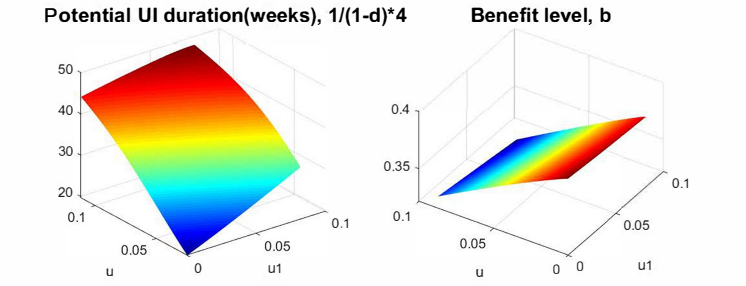

Panel B: Over productivity, z




Markov policy  Markov steady state

Figure 2: Markov equilibrium UI policy functions over total unemployment and productivity

Panel A: Over total and benefit-eligible unemployment




Panel B: Over total unemployment and productivity

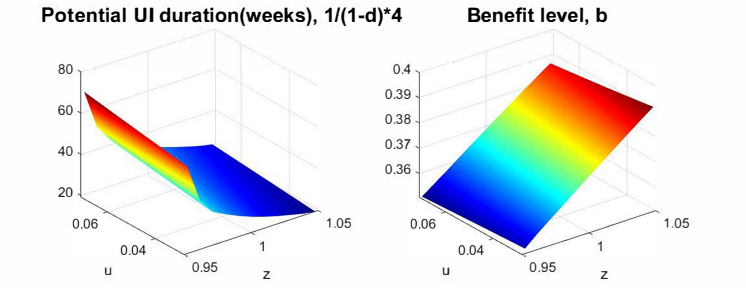



Figure 3: Markov equilibrium UI policy functions in 3-D

Consistent with the discussion of Section 3.1, as productivity falls (left panel), the marginal gain from benefit extensions is slightly lower, while the marginal cost falls much more. The marginal cost falls faster in response to a productivity drop than its rise when productivity rises. This curvature

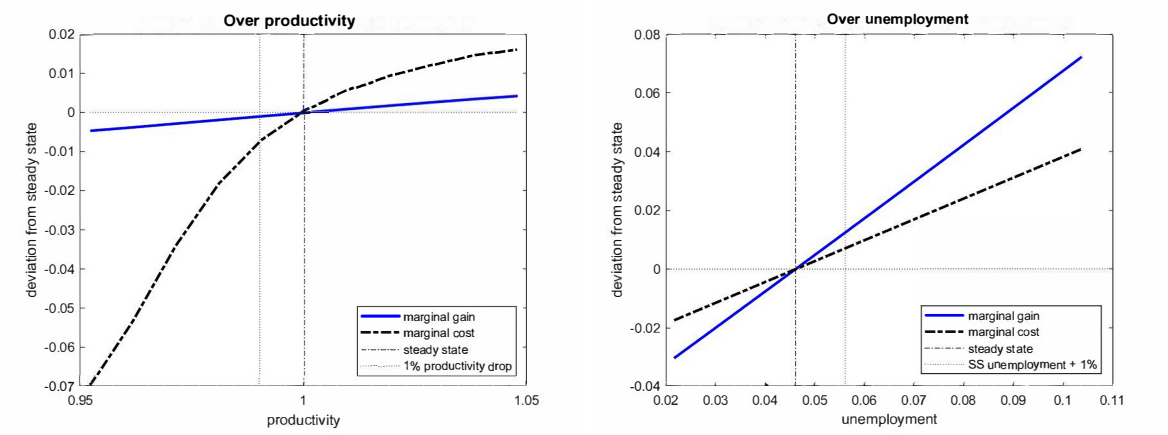



Figure 4: Marginal gain and marginal cost of extension

comes from changing sensitivity of unemployment to UI extension: As firms post more vacancies in response to rising productivity, search efficiency rises and unemployment becomes more sensitive to UI extension; the speed of increase slows down as more vacancy posting reduces each vacancy's filling rate, thus causing a congestion and slowing down the increase in the response of unemployment to UI extensions. This curvature implies the government has stronger incentives to increase UI duration in recessions when productivity is below steady state, and less incentive to reduce duration when productivity is above steady state.

At higher unemployment levels (right panel), both the marginal gain and marginal cost of benefit extensions are higher. But the rise in marginal cost is offset by a declining shadow cost of unemployment. So on net, the increase in marginal gain is larger than the rise in marginal cost. Overall, UI duration increases in response to lower productivity or higher unemployment.

## 5.2 A comparison with Ramsey UI policy

As highlighted in the comparison of Section 3.2, because of commitment to future policies, the Ramsey government internalizes the effect of current policy on past incentives. In particular, the Ramsey government realizes that an expectation of generous benefits (higher benefit level or long benefit duration) at time t creates disincentives for UI recipients to search at time t-1. As a result, the steady state Ramsey benefits are less generous than the Markov benefits.

Table 4 compares the steady states of the Ramsey vs Markov economies. The Ramsey steady state UI duration is shorter (15.8 weeks vs 27 weeks) because the Ramsey government takes into account that a long duration creates ex ante search disincentives. A shorter duration reduces search disincentive, so search by UI recipients $s^1$ is significantly higher, and proportion of UI recipients is much lower in the Ramsey economy. As a result, unemployment is lower (3.99% vs 4.55%).

Figure 5 plots the Ramsey UI policy rules along two dimension: total unemployment (Panel A) and productivity (Panel B), holding other states at steady state. Similar to the Markov UI policies, the Ramsey UI duration is longer when productivity is lower, and the benefit level is higher when

Table 4: Equilibrium Comparison: Markov versus Ramsey Policy

| Statistics | Mean | | Standard deviation | |
|---|---|---|---|---|
| | Markov | Ramsey | Markov | Ramsey |
| Benefit level, $b$ | 0.371 | 0.370 | 0.006 | 0.017 |
| Duration(weeks) | 27 | 15.8 | 5.69 | 0.082 |
| Benefit search, $s^1$ | 0.29 | 0.38 | 0.051 | 0.009 |
| No-benefit search, $s^0$ | 0.55 | 0.54 | 0.015 | 0.014 |
| Vacancy posting, $v$ | 0.032 | 0.033 | 0.142 | 0.125 |
| Proportion UI recipients, $u^1 d/u^*$ | 0.45 | 0.33 | 0.032 | 0.056 |
| Average job finding rate, $\bar{s}f(\theta)^*$ | 0.42 | 0.48 | 0.095 | 0.050 |
| Unemployment, $u(\%)$ | 4.55 | 3.99 | 0.106 | 0.052 |

Note: Statistics of Markov and Ramsey equilibrium are computed using the same exogenous parameters in Table 1 and internally calibrated parameters in Table 3. Means are reported in levels, standard deviations are reported in logs as quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600. The means of the statistics marked with * are calibration targets.

unemployment is lower or when productivity is higher. The key difference from the Markov policy is in the response of UI duration to unemployment. Panel A shows the Ramsey government reduces UI duration when unemployment is higher, whereas the Markov government increases UI duration as shown in the previous section. As pointed out in Section 3.2, this difference is because the ex ante disincentive effect of benefit extension is larger when the unemployment is higher. And because the Ramsey government internalizes this ex ante effect, it has extra incentives to lower benefit duration (or less incentive to raise duration) when the unemployment is higher.[37]

The last two columns of Table 4 compares the volatility of key variables in the Markov equilibrium vs the Ramsey economy. In comparison, the Ramsey UI duration is much less volatile. This is because the Ramsey UI duration is longer when productivity is higher or unemployment is lower, and productivity and unemployment are (closely) negatively correlated, so the size of overall change in UI duration is muted. Notice that because the Ramsey policy is designed to offset the ex ante search disincentives, search, job finding rate and unemployment are also less volatile compared to the Markov economy.

**Response to a one-time productivity shock.** We next look at the dynamic responses in both UI policy and the labor market to a 1% drop in labor productivity. Because the Markov and Ramsey steady states are quite different, we plot the deviations of variables from their respective steady states in Figure 6. This exercise highlights the *qualitative* differences between the two economies. Later when we use the model to look at the Great Recession in Section 6 we look at both the quantitative and qualitative differences.

[37] The properties of UI duration policy over unemployment and productivity shown here are qualitatively consistent with the Ramsey UI policies of Mitman and Rabinovich (2015). Benefit level here, however, increases when productivity is higher, whereas it falls in their Ramsey policy rule. This difference is driven by different assumptions on government budget. While we assume balanced government budget each period, Mitman and Rabinovich (2015) allows unbalanced government budget. When productivity is low, the burden of taxation is heavier with a balanced budget, and the government has more incentive to reduce benefit level.

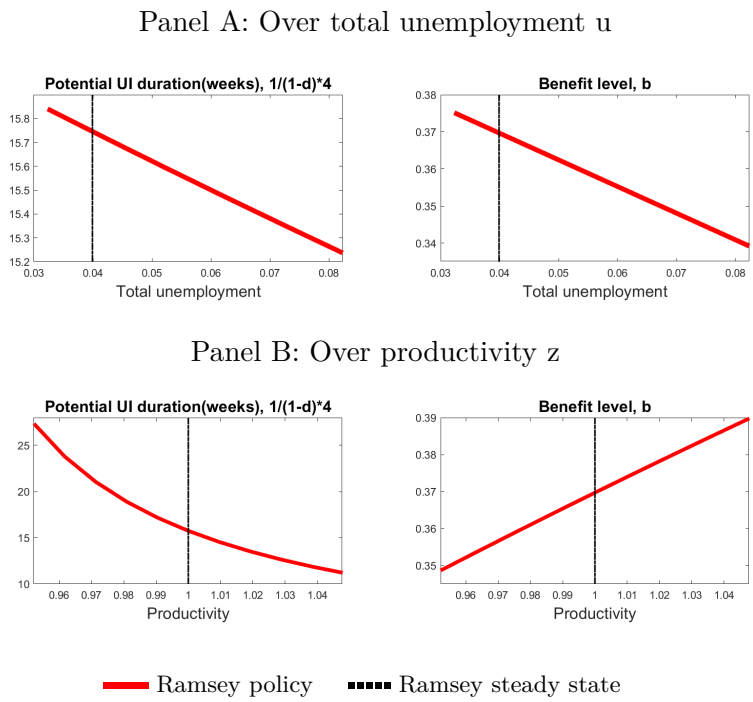



Figure 5: Ramsey UI policy functions over total unemployment and productivity

The responses of UI policies in both economies are consistent with the policy functions. Both the Markov and Ramsey governments increase UI duration and reduce benefit level in response to the initial drop in productivity. As the productivity recovers and the unemployment rises, the Markov UI duration falls gradually back to steady state, because the effects of rising unemployment and productivity work in opposite directions. The Ramsey government, in contrast, lowers UI duration drastically below steady state to reduce the increasing ex ante search disincentives as unemployment rises.

Because of dynamically different UI duration policies, responses in the economy are also quite different. On the intensive margin, search of UI recipients $s^1$ falls substantially in the first period in the Markov economy, a combined effect of lower productivity and longer UI duration. In contrast, in the Ramsey economy the fall is much smaller because of an anticipated reduction in UI duration below steady state. On the extensive margin, the higher UI duration in the Markov economy raises the proportion of UI recipients above steady state. In the Ramsey economy, this proportion falls below steady state level as the government cuts UI duration. Both the intensive and extensive margin effects contribute to the different responses in average search: in the Markov economy average search falls below steady state and recovers slowly, whereas in the Ramsey economy the initial fall is recovered much faster and average search briefly rises above steady state as the proportion of UI recipients falls. Unemployment rises in both economies, driven by lower search efficiency – a result of the negative productivity shock. But because of the different responses in search, unemployment rises much more in the Markov economy and recovery is also much slower.

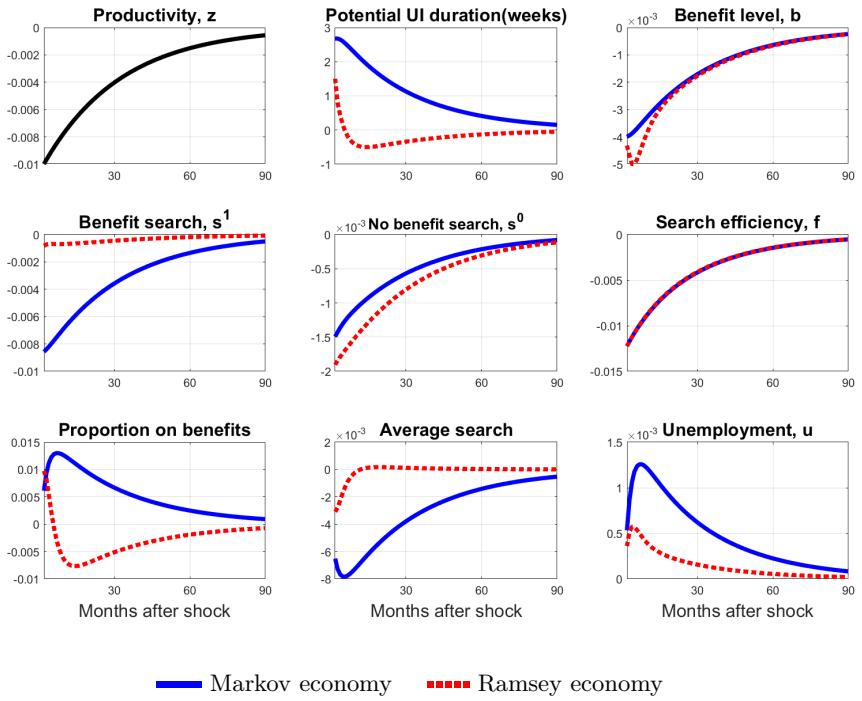



Figure 6: Responses (deviations from steady state) to a 1% drop in productivity.

# 6 UI Duration Extensions in Recessions

In this section we use the time-consistent (Markov) equilibrium to look at UI extensions during recessions in the U.S. Using calibrated shock series, the time-consistent UI policy can generate extensions close to those seen in the US. For comparisons, we look at two counterfactual policies: 1) the Ramsey optimal commitment policy, and 2) an acyclical policy where the government keeps UI policy unchanged.

## 6.1 Empirical evidence of UI benefit extensions in recessions

**UI duration and unemployment.** We first document the variations in UI duration during each recession since the 1970s. Figure 7 plots the variations in unemployment and UI duration during all five recession episodes.[38] The shaded regions mark the National Bureau of Economic Research (NBER) official recession dates. For each recession episode, the dotted red line (right axis) plots the unemployment rate, and the solid blue line (left axis) plots the maximum UI duration in weeks. The timing and the size of changes in the UI duration follow the specifics of the federal unemployment compensation laws, which are available from the U.S. Department of Labor Employment and Training Administration (DOLETA) website. Two things are worth noting. First, during all recession

[38] The recession from January to July 1980 was both shorter and milder than the other recessions. In addition, it was followed immediately by the much longer recession from July 1981 to November 1982. We therefore leave out the former recession period.

episodes, the UI duration reached its highest level around the time the unemployment peaked. Second, comparing across recessions, the recession with higher unemployment is in general associated with higher maximum UI durations, except for the 1980s recession.

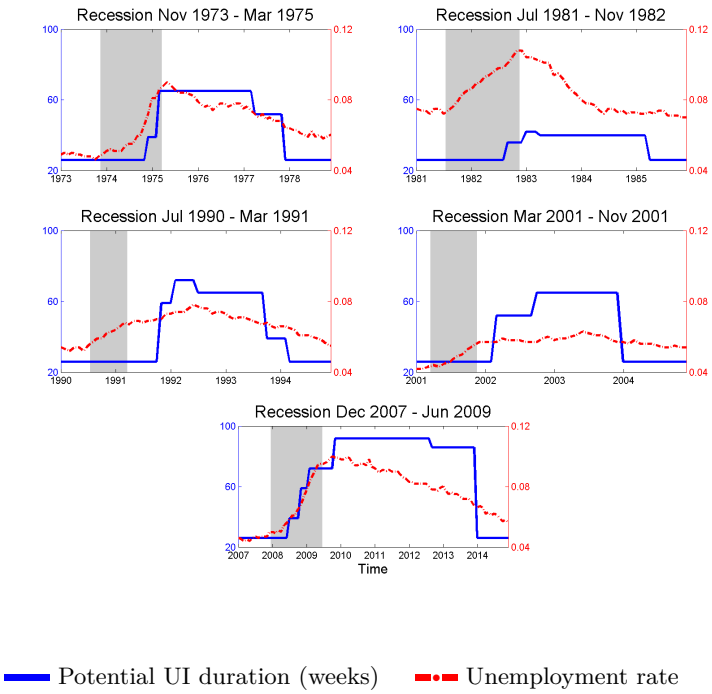
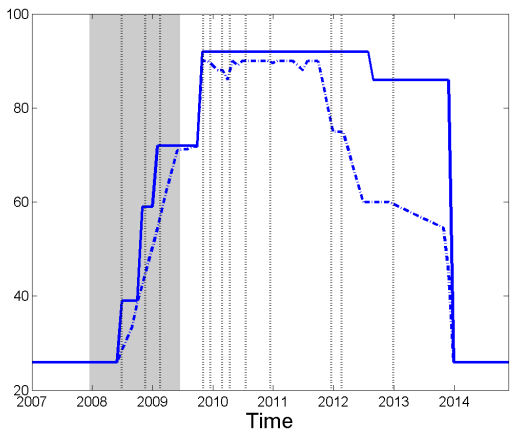



Figure 7: Empirical changes in unemployment (right axis) and UI duration (left axis) during recessions since the 1970s.

**Frequency of UI duration change.** Because more detailed data are available for the Great Recession, we document the frequency of legislation on UI policy during and following this recession in Figure 8. The vertical dotted lines indicate the timings of UI-related legislation. Each legislation specifies how and when UI duration policy will be changed (e.g. extended to 70 weeks). These legislations are summarized in Online Appendix A. Notice that the frequency of legislation increased substantially from the mid-2008, especially from the late 2009, to 2011. This suggests that UI extensions during recessions are discretionary policy as opposed to pre-determined policy rules.[39]

**Weighted UI duration.** Because the state-level implementations of UI benefit extension are conditional on the state's economic conditions, especially on the state's insured unemployment rate (IUR) and total unemployment rate (TUR), we use the two statistics to compute whether the state was eligible for longer durations in the month that a UI-related legislation was passed during the Great Recession. We then create a weighted measure of the potential UI durations across states using the number of total insured unemployed workers in each state as weights. Online Appendix A provides more details on the construction of this weighted measure. Figure 8 plots the weighted UI duration

[39] In contrast to the discretionary nature of extensions during recessions, another benefit extensions program, the " Extended Benefits" (EB) program can be understood as a commitment extensions program. The EB program is an automatic benefit extensions program triggered any time a state's unemployment rate exceeds 6.5% or 8%. It can be triggered regardless of the national economic conditions (recession or not).




Data: Maximum UI duration (weeks) Data: Weighted UI duration (weeks)
Data: UI legislation dates

Figure 8: Empirical changes in UI duration and timing of UI-related legislation during the Great Recession.

(dashed blue line). For the quantitative analysis, we use this weighted average series as the empirical counterpart for a more accurate description of the implemented UI duration policy.

## 6.2 The Great Recession

We next put the model in an environment similar to the U.S. economy during December 2007-December 2013.

**Shock and model fit.** We use an exogenous series of job separation rates over this period calculated from the aggregate-level CPS data.[40] For the productivity series, it is well known that labor productivity is hard to measure. For this reason we specify a path for productivity to match the model generated unemployment to data during this period.[41] It turns out a piecewise linear productivity path consisting of a drop, a flattening out and a slower recovery path generates a good fit for unemployment. In Section 7.4 we discuss an alternative way to pin down labor productivity.

As shown in Figure 9, given the shocks the model-generated unemployment (solid blud line) matches data (dashed black line) by construction. The UI duration endogenously chosen by the

[40] Note that the model is solved and calibrated with only the productivity shock $z$ and constant separation rate. During recessions separation rates fluctuate quite a bit within a relatively short period of time. So when we use the model to look at recessions, we use time-varying separation rates. We can do this in the model because unlike the productivity shock $z$ which realizes at the beginning of the period, job separation happens at the end of the period (see Figure 1). As such, it does not need to be a state of the economy. The underlying assumption here is that the private sector (and the government) do not internalize the time-varying separation rates and expect separation rate to return to its steady state level in the next period. In Section 7.2 we look at the case where the agents form expectations about future job separation rates.

[41] In the literature, there are many ways to measure labor productivity (see e.g. McGrattan and Prescott 2010, 2014 for a review). The present paper does not take a stand about the true level or change in labor productivity. Our model also does not have anything to say about the slow recovery following the Great Recession (see instead, e.g. Stock and Watson 2012, Shimer 2012, and Heathcote and Perri 2015).

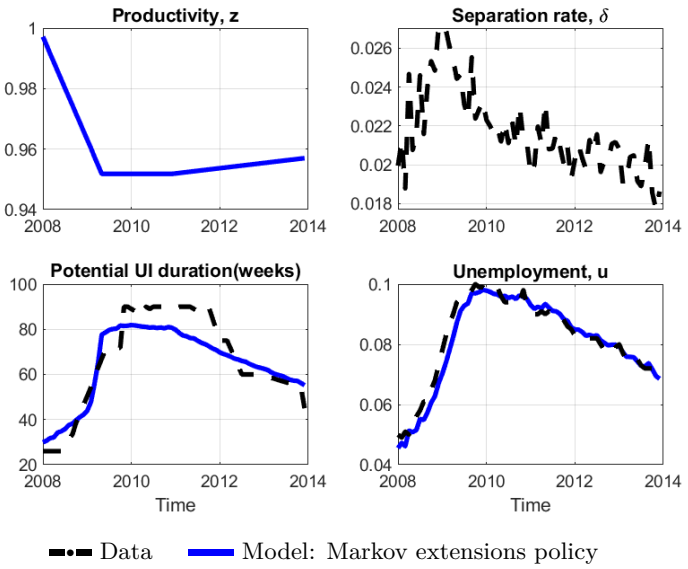



Figure 9: Exogenous shock processes, UI duration and unemployment in the Great Recession.

Markov government matches the general shape of its data counterpart. UI duration in the model rises from steady state to 80 weeks in mid-2009, compared to the data counterpart of 90 weeks. There is some mismatch in the timings of the rise and fall of UI extension between model and data. This may be attributed to political frictions which make it harder to change the policy direction in reality.

**Comparison of UI policy regimes.** We compare the Markov extension policy to two alternate UI policy schemes: the acyclical policy and the Ramsey policy in Figure 10. The question we ask here is what are the effects of *switching to* the alternate policy scheme before the start of the recession. With the acyclical policy, the government commits to not changing the UI policy. With the Ramsey policy the government makes all future policies in the period before recession and has commitment to carry out these policies at later dates. Note that while the Markov and Ramsey policies are equilibrium outcomes, the acyclical policy is not.

In all three economies lower productivity leads to lower wages and lower search efficiency $f(\theta)$. Because wages only depend on productivity, changes in wages and search efficiency are the same across the three economies.

With the acyclical policy (dashed light blue line), search falls in a recession because the anticipation of lower wages reduce the value of employment and lower search efficiency reduces the return to search.[42] The proportion of UI recipients among all unemployed workers also falls, even without any change in UI duration. This is because a lower search efficiency increases total unemployment more than benefit-eligible unemployment, and as a result, the number of UI recipients falls relative

[42] Lower search in recessions is a feature of the standard search model with endogenous search effort. The empirical findings on the cyclicality of search effort are mixed. Shimer (2004) and Mukoyama, Patterson, and Şahin (2018), for example, find countercyclical search effort, while DeLoach and Kurt (2013) find evidence of procyclical search effort. Gomme and Lkhagvasuren (2015) controls for the heterogeneity in the unemployed worker's past wages and hours, and find evidence that search is procyclical, consistent with the prediction of the structural search literature.

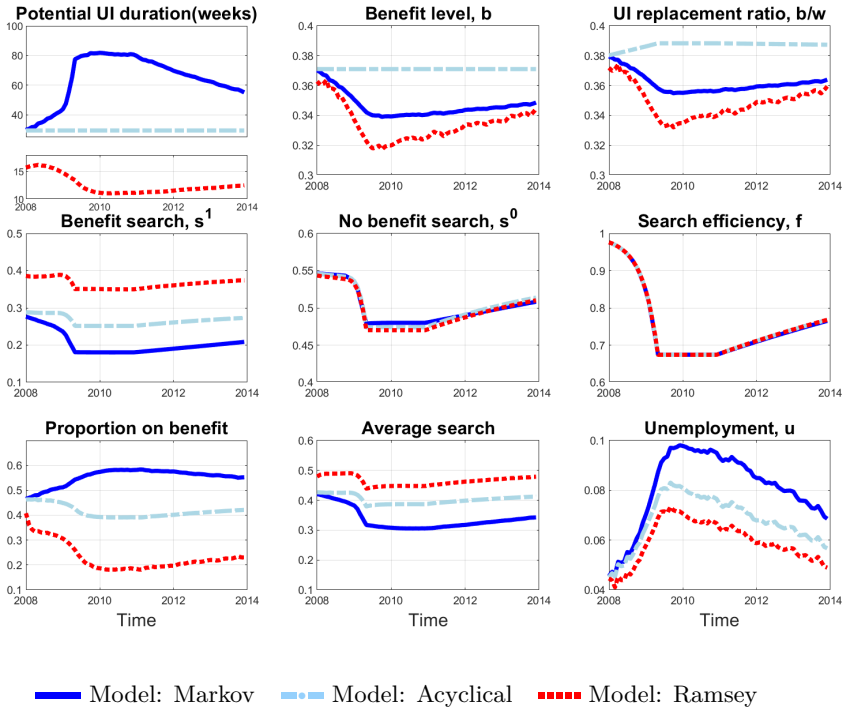



Figure 10: UI policies in the Great Recession: Markov vs Acyclical vs Ramsey policy.

to total unemployment.[43] This fall in UI-recipiency rate is consistent with data pattern: in the early 2008 before benefit extensions kicked in, the proportion of unemployed population receiving benefits fell to below 40%. Lower individual search leads to lower average search, and combined with higher separation rates and lower search efficiency lead to rising unemployment.[44]

The Markov government (solid blue line) extends UI duration and lowers benefit level during the recession.[45] Both are the results of low productivity and rising unemployment and are consistent with the intuition discussed earlier. With Markov policy, the expectation of UI extensions in recessions creates search disincentives for UI recipients, and this effect outweighs the anticipated reduction in benefit level. As a result, their search $s^1$ falls more substantially than under the acyclical policy (intensive margin effect of extension). Because of UI extensions, the proportion of UI recipients rose to 60% (extensive margin effect of extension), which is consistent with the UI-recipiency rate in the data during late 2009-early 2010. Because of both lower search and rising UI-recipiency rate, average search falls more than under the acyclical policy. As a result, unemployment increases by an additional 2 percentage points compared to the economy with the acyclical policy.

[43] To see why, from the flow equations (2)-(3), $-\partial u'/\partial f(\theta) = s^0(u - u^1 d) + s^1 u^1 d$ is larger than $-\partial u^{1'}/\partial f(\theta) = s^1 u^1 d$.

[44] Recall that with endogenous search, unemployment $= \delta/(\delta +$ average job finding rate$)$ and average job finding rate $= f(\theta)\bar{s}$ where $f(\theta)$ is the search efficiency, $\bar{s}$ is average search = UI-recipiency rate $\times s^1$ + (1-UI-recipiency rate) $\times s^0$.

[45] The falling benefit level in a recession may appear counter-factual. In the U.S. benefit level is determined by a proportion of worker's pre-unemployment wage. At the start of a recession, given relatively high pre-recession wages, average benefit is high. But as the recession drags on and average wages fall, benefit also falls.

Turning to the Ramsey policy (dotted red line), the government increases UI duration slightly from 15.7 to 16.2 weeks at the start of the recession in response to the negative productivity shock and while unemployment is still low.[46] As unemployment rises the Ramsey government reduces UI duration to 11 weeks to create search incentives for the rising group of unemployed workers. Benefit level follows a similar path as duration: it is raised slightly in the first period and then reduced to below its steady state level. The anticipation of shorter benefit duration and lower benefit level partially offsets the search disincentives due to lower productivity. As a result, UI recipients reduce search less than in the other two economies (intensive margin). Shorter UI duration also reduces the proportion of UI recipients more than under the acyclical policy (extensive margin). As a result of both the intensive and extensive margin effects, average search falls less, and the unemployment increases less (to 7.3%) compared to both the Markov (9.8%) and acyclical policies (8.2%).

**Welfare analysis.** To evaluate the welfare gain/loss of switching to alternate policies, we compute the consumption equivalent variation based on the utilitarian welfare function (16). To be more specific, we compute the percent change in lifetime consumption – of all workers, employed and unemployed – that would make welfare under the Markov policy equivalent to switching to the alternate policy. This welfare calculation includes the transition path. The results are summarized in Table 5.

Table 5: Welfare comparisons in the Great Recession: Ramsey and Acyclical relative to Markov

| Timing of policy switch | | Economic condition | | Welfare gains(%) switching to | | |
|---|---|---|---|---|---|---|
| #Month | Time | Unemp(%) | $\Delta z$ | Ramsey | Acyclical pre-rec policy | Acyclical time-t policy |
| 1 | 2007M12 | 4.55 | 0 | +0.11 | +0.012 | +0.012 |
| 2 | 2008M1 | 4.59 | -0.0031 | +0.11 | +0.013 | +0.011 |
| 10 | 2008M9 | 5.50 | -0.0279 | +0.12 | +0.025 | −0.026 |
| 19 | 2009M6 | 8.78 | -0.0566 | +0.13 | +0.036 | −0.19 |
| 28 | 2010M3 | 9.76 | -0.0566 | +0.14 | +0.039 | −0.19 |
| 40 | 2011M3 | 9.18 | -0.0552 | +0.14 | +0.039 | −0.18 |
| 70 | 2013M9 | 7.33 | -0.0412 | +0.13 | +0.032 | −0.13 |

Note: Statistics of Markov and Ramsey equilibrium are computed using the same exogenous parameters in Table 1 and internally calibrated parameters in Table 3. Welfare measure computes expected discounted average utility over 500 months and 500 simulations. Acyclical policy regime commits to keeping either Markov steady state UI policy or time-t policy unchanged going forward.

We first compute the welfare gain of switching to the alternate policy in Dec. 2007 before the recession. Given the states of the economy, if the government implements the Ramsey policy, welfare would be 0.11% higher, compared to a 0.012% welfare gain if the government commits to not changing UI policy for all future dates.[47]

Next we compute the welfare gains in subsequent periods during the recession (with productivity drop, i.e. $\Delta z<0$). In this exercise, the government switches from the Markov policy to the alternate

[46] Notice that Ramsey duration is lower than Markov duration even in the first period (15.7 vs 27 weeks). This is because given the same initial states of the economy (unemployment, productivity), the Ramsey government chooses lower duration than in the Markov government, precisely because of the ex ante search effect discussed in Section 3.2.

[47] Both alternate policies require some kind of commitment to be credible. Otherwise the private sector would *expect* the government to deviate from plan and implement the Markov policy.

policy at time t and keeps using the alternate policy from then on. We consider two scenarios for the acyclical policy: one where the government switches to the pre-recession UI policy (same as period 1), while in the other the government keeps policies unchanged at the *time-t levels*. We think this distinction is relevant because it is likely difficult and politically unpopular to switch back to the pre-recession policy halfway through a recession, and easier for the government to politically justify keeping policy unchanged going forward.

Unsurprisingly, switching to the Ramsey policy any time on the transition path improves welfare, by 0.11-0.14%. The gain is larger when the switch happens in a worse economic environment, e.g. in March 2010, compared to the beginning of the recession. Switching to an acyclical policy at the pre-recession level has smaller welfare gains, ranging from 0.013% at the start of the recession to 0.039% at the peak. Finally, switching to an acyclical policy at the time-t level reduces welfare at any time after the initial periods. This is because the duration policy is the most generous when the economic condition is the worst, and an acyclical policy at this high duration level raises unemployment for all future dates regardless of productivity levels.

## 6.3 Other recessions

Benefit extensions are not unique to the Great Recession. As we show in the empirical section, comparing across recession episodes since the 1970s, those with higher unemployment were associated with, in general, higher benefit durations.[48] In this section, we test whether our model delivers this characteristic. We recalibrate the model parameters to the pre-recession economy for each recession episode. Table 6 summarizes the pre-recession labor market statistics for each episode.

Table 6: Calibration Targets for Other Recessions

| | | Pre-recession labor market statistics | |
|---|---|---|---|
| Recession | Pre-recession period | Separation | Job finding |
| Nov 1973-Mar 1975 | 1973.I-1973.III | 0.026 | 0.51 |
| Jul 1981-Nov 1982 | 1980.II-1981.I | 0.033 | 0.41 |
| Jul 1990-Mar 1991 | 1988.I-1990.II | 0.027 | 0.47 |
| Mar 2001-Nov 2001 | 1999.I-2000.IV | 0.020 | 0.496 |

As with the Great Recession, we use the path of job separation rates from data, and target the observed unemployment path to recover the path of productivity for each recession. Figure 11 shows the model-generated UI duration (solid blue lines), unweighted UI duration from the data (dashed blue lines), and the model-generated unemployment (broken red lines, right axis) for each recession documented in the empirical analysis.

Three observations are worth noting. First, the model generates benefit extensions that match the

[48] Even though benefit extension programs existed since the 1950s, it was not until the 1973-1975 recession that the federal government started actively adjusting extensions. Such active adjustment corresponds to our interpretation of a discretionary policy. We thus focus on recessions since the mid-1970s.

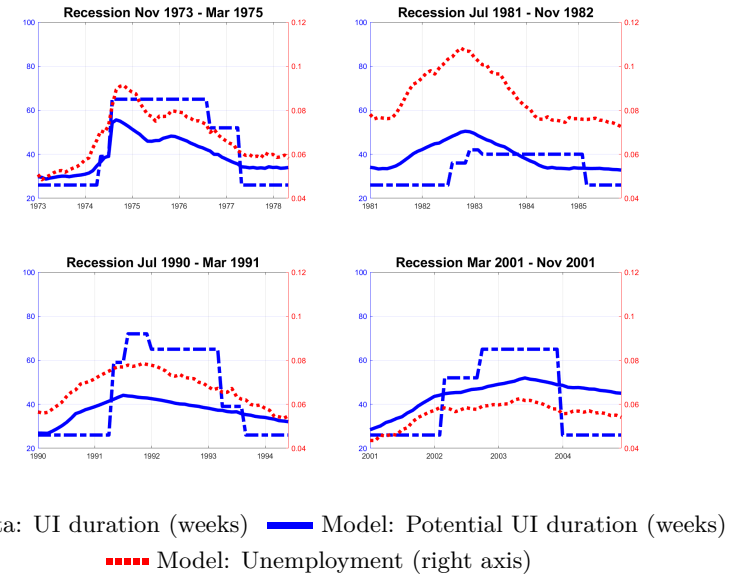



Figure 11: UI duration and unemployment in other recessions: Model versus data

rise and fall in the data counterparts (solid blue line vs. dashed blue line) in each recession. Second, consistent with patterns documented in the data, during all four recessions, the model-generated UI duration reached its highest level around the time the unemployment peaked (broken red line). Lastly, recessions where the unemployment was higher also had, in general, higher UI duration (solid blue line), except in the 1980s recession.

# 7 Robustness and Discussion

This section discusses how alternate model assumptions and specifications affect the analysis. We focus on the analysis of the Great Recession and compare the alternatives to the baseline Markov equilibrium in Section 6.2. The first three exercises explore alternate model assumptions, which we compare to the baseline model under the same calibrated parameters (i.e. we do not re-calibrate) to illustrate the effects of the alternate assumption.

## 7.1 UI-dependent wages

In the baseline model wages only respond to changes in productivity. Here we explore a case where wages additionally respond to changes in UI duration. We do so by assuming that wages increase when UI duration is increased from an anchor level, which we specify as the baseline steady state UI duration.

Because wages respond positively to UI duration, a longer duration increases wages of employed workers and raises the average utility (captured by an extra term in the government's optimality condition with respect to $d$). This is an additional marginal gain. But at the same time, higher wages reduce firm's profit margin and hence vacancy posting and search efficiency, and as a result

unemployed worker's search effort is lower, which raises the steady state unemployment. This is an additional marginal cost.[49]

We specify that a one-week increase in benefit duration from the steady state raises wage by 0.02%. This translates into an approximately 1% increase in wages when benefit duration is increased by 50 weeks.[50] At steady state, the additional marginal gain outweighs the additional marginal cost, and duration is longer than in the baseline (28.6 vs 27 weeks).[51]

In recessions, UI duration extensions attenuate the fall in wages. Figure 12 compares the baseline Markov equilibrium with the case here. When wages increase in response to benefit extensions (dashed purple line), wages fall less than in the baseline (solid blue line), with a difference of up to 1.1%. Because of higher wages, firm's profit margin is squeezed more, so market tightness and search efficiency fall more than in the baseline case (i.e. more slack). From the government's perspective, lower productivity increases the worker's marginal utility of consumption, which raises the value of higher wages, compared to steady state. This wage effect gives the government an additional incentive to extend UI duration. At the same time, because search efficiency is even lower than in the baseline, the cost of reduced search is lower, which gives the government another reason to extend UI duration beyond the baseline level. In this economy, UI is extended up to 90 weeks, compared to 80 weeks in the baseline. Relative to the baseline, lower search efficiency and longer expected duration both reduce search incentive, while higher wages increase it. On net, unemployment is about the same as in the baseline.

## 7.2 Cyclical job separation risk

In the baseline model agents only form expectations about future labor productivity but not about job separation rate. In this section, we allow agents to form expectations about future job separation rate as well. Since job separation rate closely tracks productivity and to avoid introducing additional state variables, we estimate an AR(1) process for separation rate that depends on the contemporaneous labor productivity

$$\delta(z) = \bar{\delta} + I_\delta(z - \bar{z}),$$

where $\bar{\delta}$ is the steady-state job separation rate, and $I_\delta < 0$ is the rate of change of the separation rate with respect to the aggregate productivity.

[49] A similar additional marginal cost is present in McKay and Reis (2017)'s setup with a positive UI-wage elasticity. Because of this additional marginal cost, their model generates *lower* steady state UI benefit compared to without UI-wage elasticity.

[50] Hagedorn, Karahan, Manovskii, and Mitman (2013) find UI extension has a significant effect on the wages of job stayers, translating into a 0.33% higher wages in a county with 70 weeks of benefits compared to a county with 50 weeks of benefits (compared to an implied value of 0.4% in our model.) By using a sample of job stayers, their estimate indicates the effect of extension on wages works through increases in worker's outside option. Nekoei and Weber (2017) use regression discontinuity design in Austria to find a 0.5% increase in average wages in response to a nine-week UI extension. They find evidence that most of the increase is the result of unemployed workers finding better matches rather than the effect of UI on worker's outside value in the wage bargaining process.

[51] Steady state duration under other response factor values: with a response factor of 0.01% , steady state duration is 28.4 weeks; a factor of 0.05% gives a steady state duration of 31.6 weeks; and a factor of 0.1% gives 35.4 weeks.

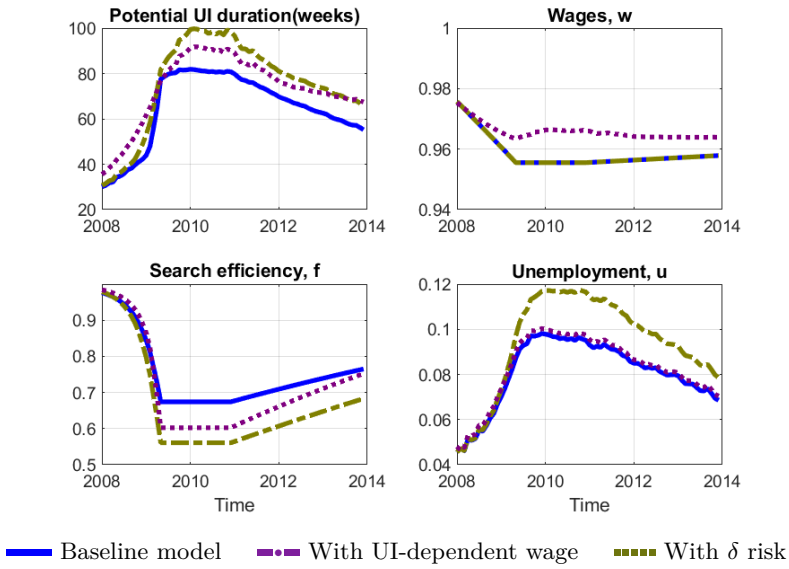



Figure 12: Alternate model specifications: UI-dependent wage and cyclical separation risk.

Given this process, agents form expectations about the end-of-period separation rate based on the realization of labor productivity at the beginning of the period. This specification has the interpretation that job separation rate increases when productivity and hence profits are low.[52] We estimate this process using job separation and labor productivity data over 1951.I-2007.IV and obtain $I_\delta = -0.15$.

Countercyclical separation rate risk reinforces the effect of the productivity shock. When productivity is low, firms expect higher separation rates, which reduce the expected value of a filled position. As a result, vacancy posting and market tightness fall more than in the baseline without separation risk. Accordingly, search efficiency falls more, as shown in Figure 12 (dotted dark green line). Lower search efficiency reduces search more and contributes to even higher unemployment in recessions. At the same time, lower search efficiency means the value of search is reduced more compared to the baseline, and so the government has stronger incentives to increase UI duration in recessions, which also contribute to higher unemployment.

## 7.3 Extension-neutral UI recipients

In the baseline model, an expected future UI extension reduces the search incentive for the UI recipients. The assumption is that all UI recipients are affected equally by a change in the UI policy. This assumption keeps the model tractable. But one may worry that it exaggerates the effects of UI extensions, as in practice not all UI recipients are affected equally. For example, someone collecting the 1st week of benefits is not *immediately* affected by a benefit extension from 40 to 50 weeks, whereas someone in her 39th week is affected right away.

[52] When labor productivity is low, wages are low as wages are also a function of productivity. Because the elasticity of wages with respect to productivity is less than 1, lower productivity means lower profit, or $z - w$ in the model.

Without modeling duration with full discreteness, we introduce a third type of workers — the extension-neutral UI recipients. This group of workers collect benefits but are not affected by changes in UI duration. To avoid introducing additional endogenous states, we use a stochastic process to assign UI recipients to this extension-neutral group. Specifically, each period a proportion $\gamma$ of UI recipients for sure have benefits the next period if they stay unemployed. We use $\gamma = 0.05$ or 0.1 for this exercise.[53]

Table 7: Key second moments in model with extension-neutral UI recipients

| Key Second Moments | Baseline Model | With extension-neutral type | |
|---|---|---|---|
| | | $\gamma = 0.05$ | $\gamma = 0.1$ |
| Std dev of duration | 5.69 | 5.24 | 4.43 |
| Std dev of unemployment $u$ | 0.106 | 0.102 | 0.095 |
| Std dev of Vacancy posting $v$ | 0.142 | 0.138 | 0.133 |
| Elasticity unemp wrt UI duration | 0.017 | 0.018 | 0.021 |
| $\Delta$unemp duration/$\Delta$UI duration | | | |
| UI recipient | 0.152 | 0.170 | 0.185 |
| extension-neutral UI recipient | – | 0.130 | 0.137 |

Note: Standard deviations are reported in logs as quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600. Elasticity is calculated using log deviations from HP-filtered trends. Responses of unemployment duration to a 1-week increase in UI duration are reported in levels.

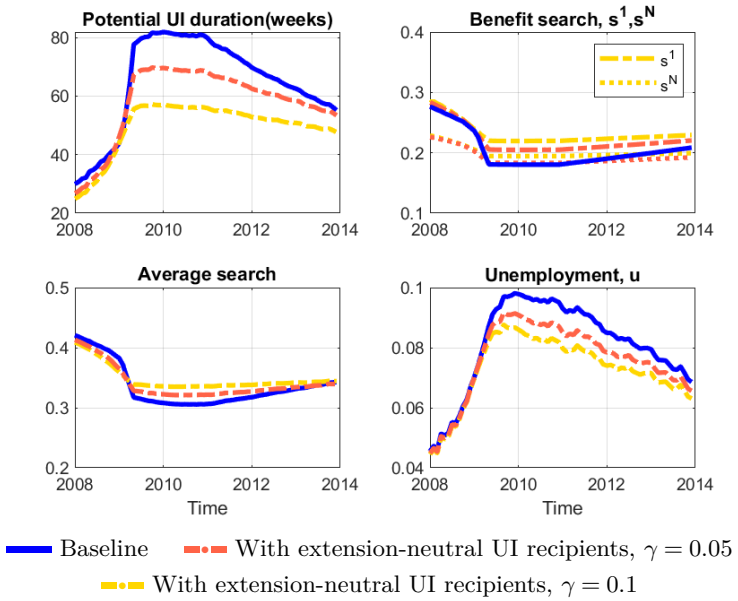



Figure 13: Alternate model specification: adding extension-neutral UI recipients.

Because the extension-neutral UI recipients are not immediately affected by UI duration changes,

[53] Online Appendix C provides details of the modified model, including the workers' and government's optimality conditions.

their search is less responsive to changes in UI duration. Table 7 shows the unemployment duration of these workers increases by less in response to an increase in UI duration, compared to other UI recipients. At the same time, introducing the extension-neutral type makes other UI recipients more sensitive to UI policy: in response to 1 week increase in UI duration, recipient's unemployment duration on average increases by 0.17 weeks with $\gamma = 0.05$ and 0.185 weeks when $\gamma = 0.1$, compared to 0.152 weeks in the baseline. Overall, unemployment is more responsive to UI duration changes compared to the baseline. As a result, the marginal cost of increasing UI duration is larger, and the steady state UI duration is shorter relative to the baseline: 24.8 weeks when we set $\gamma = 0.05$ and 23.6 weeks when $\gamma = 0.1$.

Figure 13 compares the modified model with the baseline during the Great Recession. Introducing the extension-neutral workers reduces UI extensions, more so with a larger $\gamma$: benefit duration is extended to 57 weeks when $\gamma = 0.1$ (dashed yellow line) and 70 weeks with $\gamma = 0.05$ (dashed orange line), compared to 80 weeks in the baseline. Because of shorter benefit extensions (and despite the larger response to UI duration changes), search by UI recipients $s^1$ falls less compared to the baseline. The figure shows search by the extension-neutral UI recipients $s^N$ (dotted yellow and orange lines) alongside $s^1$: because the extension-neutral type for sure have benefits the next period they have less incentive to search; their search also drops less in response to rising UI duration, because they are less responsive to benefit extensions. Overall, the average search falls less and the unemployment increases less compared to the baseline: 9.1% and 8.7% with $\gamma = 0.05$ and 0.1, respectively.[54]

## 7.4 Alternative calibration of productivity path in Great Recession

In the baseline analysis we calibrate the labor productivity path to match the model-generated unemployment to the data during 2008-2013. We then evaluate the model-generated (Markov) UI extensions against the extensions in the U.S. In this section we alternatively calibrate the productivity to match UI duration and compare the Markov equilibrium unemployment rate with U.S. unemployment.

Figure 14 shows that with this calibration strategy (dotted blue line), the model generates a good fit for unemployment. Equilibrium unemployment peaks around the same level as the baseline calibration (solid blue line) and data (dashed black line), although the peak happens later and stays high for longer.

## 7.5 Discussion of other model assumptions

Here we briefly discuss a few other model assumptions. The discussion focuses on how alternative assumptions could affect our results or how to reconcile assumption with reality.

**Household borrowing and saving.** Our model abstracts from household borrowing and saving decisions. While these decisions are potentially important for the effects of UI policy, we need to keep

[54] Note that once productivity is re-calibrated such that unemployment is matched to the data, UI duration will also be higher and closer to data.

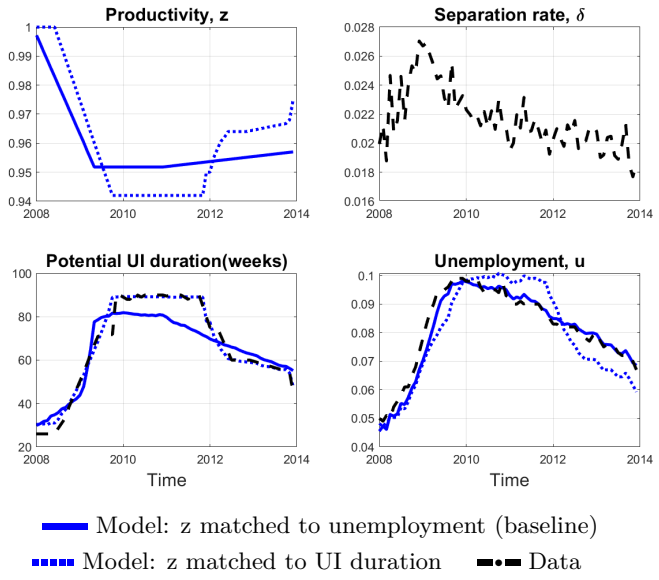



Figure 14: Comparison of alternative calibration of productivity path.

the model tractable in order to properly model the government's problem. In fact, recent works in the literature suggest that incorporating incomplete markets strengthens the argument for more generous UI during recessions. Birinci and See (2017) incorporate incomplete markets into a direct search model, and find that more generous UI during recessions reduces household's precautionary savings motive and is welfare improving. Introducing nominal rigidities into an environment with incomplete markets and search frictions, Kekre (2017) finds the aggregate demand externality created by more generous UI is large enough to offset any moral hazard problem during the Great Recession.

**Who pays taxes.** In our model UI is financed through a lump-sum tax on all workers both employed and unemployed. In the U.S. UI is funded through a payroll tax on firms, part of which may pass to workers through wages. While the tax system in the model differs from the U.S., it minimizes the distortion from rising taxation in a recession. In a model where firms pay taxes, higher unemployment or UI extensions increase tax rate in a recession, which further reduces firm's vacancy posting beyond the effect of lower productivity. Alternatively, if only employed workers pay taxes, UI extensions would lower the value of jobs relative to unemployment through higher taxes and create additional disincentive to find jobs in recessions.

**A theory of UI policy during normal times?** While our model generates benefit extensions during recessions, it does not produce the lack of policy movements during non-recessionary times (e.g. post-2014). In fact, our model predicts when productivity rises and unemployment falls, the government has incentives to reduce benefit duration to below the steady state level. But to the extent that during normal times, productivity and unemployment do not change drastically, the Markov UI policy also does not change much. In addition, the response of the duration policy to productivity is very nonlinear: it increases faster in response to falling productivity than it falls in response to rising productivity (as shown in Section 5.1). As a result, the government's incentive to reduce UI duration during non-recessionary times is small relative to the incentives to extend in a recession.

Finally, it is politically costly – both in terms of time and legislative process – to change UI duration. So given relatively small changes in productivity and unemployment during non-recessionary times, the gain from changing may be outweighed by the cost.

## 8 Conclusion

This paper studies UI extensions during recessions from the government's perspective by modeling the problem of a government without commitment to future policies. Because low productivity reduces the social values of search and employment, the discretionary UI policy chosen by a Markov government extends UI durations in a recession. Calibrated to the US economy, the time-consistent UI extensions are quantitatively consistent with the extensions during the Great Recession. Compared to either the optimal commitment (Ramsey) policy or an acyclical policy, the time-consistent UI extension raises the unemployment by an additional 2.5% and 1.6%, respectively. Switching to the optimal commitment policy improves welfare, whereas switching to an acyclical policy has either minimal welfare gain or welfare loss, depending on the implementation and the timing of the switch.

We consider this paper an application of the time-consistent Markov equilibrium analysis (a la Klein, Krusell, and Ríos-Rull 2008) to a search and matching environment. This framework is especially applicable to government policies during recessions such as UI extensions, because this is likely when the political pressure is high to forgo prior commitment and implement discretionary policies. We focus only on UI policy in our analysis, because UI extension has been and likely will be a popular and contentious labor market policy implemented during recessions. But changes in other labor market policies (e.g. hiring subsidies) may create interesting interactions with UI policy, so considering these other policies in combination with UI policy in a time-consistent setting will provide a more comprehensive policy recommendation.

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# Online Appendix for
# "A Quantitative Theory of Time-Consistent Unemployment Insurance"

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# A Unemployment Insurance Benefit Extensions in the Great Recession

The U.S. government has extended unemployment insurance (UI) benefit duration during recessions since the 1950s. During normal non-recessionary times, an eligible unemployed worker may receive UI benefits for up to 26 weeks in most states under the regular UI program. During economic downturns, automatic benefits extensions are triggered under the Extended Benefits (EB) program, whereby additional 13 or 20 weeks are added on top of the usual 26 weeks of UI benefits. The number of additional weeks depends on the state's insured unemployment rate (IUR) or the total unemployment rate (TUR). In addition, during each recession since the 1970s, the federal government has financed extra benefit extensions depending on the economy. The Emergency Unemployment Compensation (EUC08) were launched in 2008 in response to high unemployment. The program extended benefits in waves based on evaluations of unemployment and the underlying economic situation. Four waves called "Tiers" were implemented progressively between Nov 2008 and the end of 2013. Tiers I was effective without any conditions on the states' unemployment level. Tiers II, III and IV required certain condition on the states' IUR and/or TUR to take place.

We use the series of the IUR and TUR for 51 U.S. states to compute if the state was eligible for the EB and the EUC08 programs during any month between 2008 and 2013. This gives us the maximum duration of UI benefits for each state over time. We follow the methodology in Albertini and Poirier (2015) and weight these series to build an aggregate indicator. We use the number of total insured unemployed workers in the state divided by the total insured unemployed workers in the U.S. as weights. Statistics on insured unemployment population come from the U.S. Department of Labor Employment and Training Administration (DOLETA). Table 8 reports a timeline for policy changes and unweighted expected maximum duration under the EUC08 and EB programs.

Table 8: Federally-Funded Unemployment Insurance Extensions 2008-2013

| Start date | Program extension of EUC08 | End date | Additional Weeks |
|---|---|---|---|
| Jul 2008 | 13 weeks for all states | Nov 2008 | 13 |
| Nov 2008 | Tier I: 20 weeks for all states | Mar 2009 | 33 |
| | Tier II: 13 weeks for states with TUR $\geq 6\%$ | | |
| Mar 2009 | keep exisiting structure | Nov 2009 | 33 |
| Nov 2009 | Tier I - 20 weeks for all states | Dec 2009 | 53 |
| | Tier II: 14 weeks for all states | | |
| | Tier III: 13 weeks if states TUR $\geq 6\%$ | | |
| | Tier IV: 6 weeks if states TUR $\geq 8.5\%$ | | |
| Dec 2009 | keep exisiting structure | Aug 2010 | 53 |
| Mar 2010 | keep exisiting structure | Sep 2010 | 53 |
| Apr 2010 | keep exisiting structure | Nov 2010 | 53 |
| Jul 2010 | keep exisiting structure | May 2011 | 53 |
| Dec 2010 | keep exisiting structure | Jun 2012 | 53 |
| Dec 2011 | keep exisiting structure | Aug 2012 | 53 |
| Feb 2012 | Tier I: 20 weeks for all states | May 2012 | 53 |
| | Tier II: 14 weeks for all states | | |
| | Tier III: 13 weeks if states TUR $\geq 6\%$ | | |
| | Tier IV: 6 weeks if states TUR $\geq 8.5\%$ | | |
| | (16 weeks if no active EB and TUR $\geq 8.5\%$) | | |
| Jun 2012 | Tier I: 20 weeks for all states | Sep 2012 | 53 |
| | Tier II: 14 weeks if states TUR $\geq 6\%$ | | |
| | Tier III: 13 weeks if states TUR $\geq 7\%$ | | |
| | Tier IV: 6 weeks if states TUR $\geq 9\%$ | | |
| Sep 2012 | Tier I: 14 weeks for all states | Dec 2012 | 47 |
| | Tier II: 14 weeks if states TUR $\geq 6\%$ | | |
| | Tier III: 9 weeks if states TUR $\geq 7\%$ | | |
| | Tier IV: 10 weeks if states TUR $\geq 9\%$ | | |
| Jan 2013 | keep exisiting structure | Dec 2013 | 47 |

| Start date | Program extension of EB | End date | Additional Weeks |
|---|---|---|---|
| Feb 2009 | 6.5% 13 week IUR and IUR $\geq 110\%$ of prior 3 years | Dec 2013 | 13 |
| | 8% 13 week IUR and IUR $\geq 110\%$ of prior 3 years | | 26 |

Source: DOLETA, Whittaker and Isaacs (2013), Albertini and Poirier (2015)

# B Derivations and Proofs

## B.1 Derivation of private sector optimality conditions

Throughout this section, we drop the dependence of functions on productivity shock $z$ to economize on notation.

- Solving problem of unemployed person without benefit by taking derivative with respect to $s^0$

$$\frac{v_s(s^0)}{f(\theta)} = \beta[V^{e'} - V^{0'}] \tag{25}$$

Solving problem of unemployed person with benefit by taking derivative with respect to $s^1$

$$\frac{v_s(s^1)}{f(\theta)} = \beta[V^{e'} - (1-d')V^{0'} - d'V^{1'}] \tag{26}$$

Using worker's bellman equations

$$\begin{aligned} & V^e - V^0 \\ = \; & U^e(w(z,d)-\tau) - U^0(h-\tau,s^0) + \beta[1-f(\theta)s^0 - \delta(1-\xi)][V^{e'} - V^{0'}] - \beta\delta\xi[V^{e'} - (1-d')V^{0'} - d'V^{1'}] \qquad (27) \\ & V^e - (1-d)V^0 - dV^1 \\ = \; & (1-d)\Big\{U^e(w(z,d)-\tau) - U^0(h-\tau,s^0) + \beta[1-f(\theta)s^0 - \delta(1-\xi)][V^{e'} - V^{0'}] - \beta\delta\xi[V^{e'} - (1-d')V^{0'} - d'V^{1'}]\Big\} \\ & +d\Big\{U^e(w(z,d)-\tau) - U^1(h+b-\tau,s^1) + \beta[1-f(\theta)s^1 - \delta\xi][V^{e'} - (1-d')V^{0'} - d'V^{1'}] - \beta\delta(1-\xi)[V^{e'} - V^{0'}]\Big\} \qquad (28) \end{aligned}$$

Combine (27) with (25) and (26)

$$V^e - V^0 = U^e(w(z,d)-\tau) - U^0(h-\tau,s^0) + [1-f(\theta)s^0 - \delta(1-\xi)]\frac{v_s(s^0)}{f(\theta)} - \delta\xi\frac{v_s(s^1)}{f(\theta)}$$

Update one period, and substitute into (25)

$$\frac{v_s(s^0)}{f(\theta)} = \beta\left\{U^e(w(z',d')-\tau') - U^0(h-\tau',s^{0'}) + [1-f(\theta')s^{0'} - \delta(1-\xi)]\frac{v_s(s^{0'})}{f(\theta')} - \delta\xi\frac{v_s(s^{1'})}{f(\theta')}\right\}$$

Combine (28) with (25) and (26)

$$\begin{aligned} & V^e - (1-d)V^0 - dV^1 \\ = \; & (1-d)\left\{U^e(w(z,d)-\tau) - U^0(h-\tau,s^0) + [1-f(\theta)s^0 - \delta(1-\xi)]\frac{v_s(s^0)}{f(\theta)} - \delta\xi\frac{v_s(s^1)}{f(\theta)}\right\} \\ & +d\left\{U^e(w(z,d)-\tau) - U^1(h+b-\tau,s^1) + [1-f(\theta)s^1 - \delta\xi]\frac{v_s(s^1)}{f(\theta)} - \delta(1-\xi)\frac{v_s(s^0)}{f(\theta)}\right\} \end{aligned}$$

Update one period, and substitute into (26)

$$\begin{aligned} \frac{v_s(s^1)}{f(\theta)} \; = \; & \beta(1-d')\left\{U^e(w(z',d')-\tau') - U^0(h-\tau',s^{0'}) + [1-f(\theta')s^{0'} - \delta(1-\xi)]\frac{v_s(s^{0'})}{f(\theta')} - \delta\xi\frac{v_s(s^{1'})}{f(\theta')}\right\} \\ & +\beta d'\left\{U^e(w(z',d')-\tau') - U^1(h+b'-\tau',s^{1'}) + [1-f(\theta')s^{1'} - \delta\xi]\frac{v_s(s^{1'})}{f(\theta')} - \delta(1-\xi)\frac{v_s(s^{0'})}{f(\theta')}\right\} \end{aligned}$$

- From unmatched firm's value function, assuming free entry, i.e. $J^u(u,u^1) = 0$

$$\frac{\kappa}{q(\theta)} = \beta J^e(u',u^{1'})$$

Then firm's value function can be rewritten as

$$J^e(u, u^1) = z - w(z, d) + (1 - \delta)\frac{\kappa}{q(\theta)}$$

Update one period

$$J^e(u', u^{1'}) = z' - w(z', d') + (1 - \delta)\frac{\kappa}{q(\theta')}$$

Substitute into the first equation

$$\frac{\kappa}{q(\theta)} = \beta\left[z' - w(z', d') + (1 - \delta)\frac{\kappa}{q(\theta')}\right]$$

## B.2 Derivation of the Markov GEEs (17)-(19)

Throughout this section, we drop the dependence of functions on productivity shock $z$ to economize on notation. Denote the period welfare function by $R(z,u,u^1,b,d,s^0,s^1)$. Denote the workers' flow equations by $f_1(u,u^1,d,s^0,s^1,\theta,u')=0$ and $f_2(u,u^1,d,s^1,\theta,u^{1'})=0$. Denote the private sector's optimality conditions by $\eta_1(s^0,\theta,u',u^{1'})=0$, $\eta_2(s^1,\theta,u',u^{1'})=0$ and $\eta_3(\theta,u',u^{1'})=0$. Let $\lambda$, $\lambda_1$, $\mu_0$, $\mu_1$, $\gamma$ be the Lagrange multipliers on workers' flow equations and private sector's optimality conditions respectively.

1. Take derivatives of the government's problem with respect to $b$, $d$, $s^0$, $s^1$, $\theta$, $u'$ and $u^{1'}$

$$\begin{aligned}
b: &\quad R_b=0\\
d: &\quad \lambda f_{1d}+\lambda_1 f_{2d}-R_d=0\\
s^0: &\quad \lambda f_{1s^0}+\mu_0\eta_{1s^0}-R_{s^0}=0\\
s^1: &\quad \lambda f_{1s^1}+\lambda_1 f_{2s^1}+\mu_1\eta_{2s^1}-R_{s^1}=0\\
\theta: &\quad \lambda f_{1\theta}+\lambda_1 f_{2\theta}+\mu_0\eta_{1\theta}+\mu_1\eta_{2\theta}+\gamma\eta_{3\theta}=0\\
u': &\quad \lambda f_{1u'}+\mu_0\eta_{1u'}+\mu_1\eta_{2u'}+\gamma\eta_{3u'}=\beta G'_u\\
u^{1'}: &\quad \lambda_1 f_{2u^{1'}}+\mu_0\eta_{1u^{1'}}+\mu_1\eta_{2u^{1'}}+\gamma\eta_{3u^{1'}}=\beta G'_{u^1} \qquad \text{(FOC)}
\end{aligned}$$

   where primes denote next period, and subscripts are derivatives.

2. Take derivative of the Bellman equation with respect to $u$ and $u^1$, respectively

$$\begin{aligned}
G_u &= R_u+R_b\Psi^b_u+R_d\Psi^d_u+R_{s^0}S^0_u+R_{s^1}S^1_u+\beta G'_u\Gamma_u+\beta G'_{u^1}\Gamma^1_u \qquad \text{(ENV1)}\\
G_{u^1} &= R_{u^1}+R_b\Psi^b_{u^1}+R_d\Psi^d_{u^1}+R_{s^0}S^0_{u^1}+R_{s^1}S^1_{u^1}+\beta G'_u\Gamma_{u^1}+\beta G'_{u^1}\Gamma^1_{u^1} \qquad \text{(ENV2)}
\end{aligned}$$

   substitute the last two FOCs into ENV1 and ENV2 to eliminate $\beta G'_u$ and $\beta G'_{u^1}$

$$\begin{aligned}
G_u &= R_u+R_b\Psi^b_u+R_d\Psi^d_u+R_{s^0}S^0_u+R_{s^1}S^1_u\\
&\quad +\Gamma_u\left\{\lambda f_{1u'}+\mu_0\eta_{1u'}+\mu_1\eta_{2u'}+\gamma\eta_{3u'}\right\}+\Gamma^1_u\left\{\lambda_1 f_{2u^{1'}}+\mu_0\eta_{1u^{1'}}+\mu_1\eta_{2u^{1'}}+\gamma\eta_{3u^{1'}}\right\} \quad (29)\\
G_{u^1} &= R_{u^1}+R_b\Psi^b_{u^1}+R_d\Psi^d_{u^1}+R_{s^0}S^0_{u^1}+R_{s^1}S^1_{u^1}\\
&\quad +\Gamma_{u^1}\left\{\lambda f_{1u'}+\mu_0\eta_{1u'}+\mu_1\eta_{2u'}+\gamma\eta_{3u'}\right\}+\Gamma^1_{u^1}\left\{\lambda_1 f_{2u^{1'}}+\mu_0\eta_{1u^{1'}}+\mu_1\eta_{2u^{1'}}+\gamma\eta_{3u^{1'}}\right\} \quad (30)
\end{aligned}$$

3. Differentiate private-sector optimality conditions ($\eta_1$, $\eta_2$, $\eta_3$) and worker's flow equations ($f_1$ and $f_2$) with respect to $u$

$$\begin{aligned}
\eta_{1s^0}S^0_u+\eta_{1\theta}\Theta_u+\eta_{1u'}\Gamma_u+\eta_{1u^{1'}}\Gamma^1_u &= 0 \quad (31)\\
\eta_{2s^1}S^1_u+\eta_{2\theta}\Theta_u+\eta_{2u'}\Gamma_u+\eta_{2u^{1'}}\Gamma^1_u &= 0 \quad (32)\\
\eta_{3\theta}\Theta_u+\eta_{3u'}\Gamma_u+\eta_{3u^{1'}}\Gamma^1_u &= 0 \quad (33)\\
f_{1u}+f_{1d}\Psi^d_u+f_{1s^0}S^0_u+f_{1s^1}S^1_u+f_{1\theta}\Theta_u+f_{1u'}\Gamma_u &= 0 \quad (34)\\
f_{2u}+f_{2d}\Psi^d_u+f_{2s^1}S^1_u+f_{2\theta}\Theta_u+f_{2u^{1'}}\Gamma^1_u &= 0 \quad (35)
\end{aligned}$$

4. Substitute (31)-(35) and the FOCs into (29)

$$G_u = R_u-\lambda f_{1u}-\lambda_1 f_{2u} \quad (36)$$

   Similarly, differentiate private-sector optimality conditions and the worker's flow equations with respect to $u^1$, and substitute into (30)

$$G_{u^1} = R_{u^1}-\lambda f_{1u^1}-\lambda_1 f_{2u^1} \quad (37)$$

5. Update (36)-(37) and substitute into the last two FOCs, respectively

$$\lambda f_{1u'} + \mu_0 \eta_{1u'} + \mu_1 \eta_{2u'} + \gamma \eta_{3u'} = \beta \left[ R'_u - \lambda' f'_{1u} - \lambda'_1 f'_{2u} \right] \tag{38}$$

$$\lambda_1 f_{2u^{1\prime}} + \mu_0 \eta_{1u^{1\prime}} + \mu_1 \eta_{2u^{1\prime}} + \gamma \eta_{3u^{1\prime}} = \beta \left[ R'_{u^1} - \lambda' f'_{1u^1} - \lambda'_1 f'_{2u^1} \right] \tag{39}$$

6. Substituting (39) into the FOC for $d$ and (38), the duration policy $d$ is characterized by

$$\begin{aligned} 0 = \; & R_d - \lambda f_{1d} \\ & - \frac{f_{2d}}{f_{2u^{1\prime}}} \left[ -\mu_0 \eta_{1u^{1\prime}} - \mu_1 \eta_{2u^{1\prime}} - \gamma \eta_{3u^{1\prime}} + \beta \left( R'_{u^1} - \lambda' f'_{1u^1} \right) \right] \\ & - \beta \frac{f_{2d}}{f_{2u^{1\prime}}} \left( - \frac{f'_{2u^1}}{f'_{2d}} \right) \left( R'_d - \lambda' f'_{1d} \right) \end{aligned}$$

where the multiplier $\lambda$ is characterized by

$$\begin{aligned} \lambda f_{1u'} = \; & -\mu_0 \eta_{1u'} - \mu_1 \eta_{2u'} - \gamma \eta_{3u'} + \beta \left( R'_u - \lambda' f'_{1u} \right) \\ & + \beta \left( - \frac{f'_{2u}}{f'_{2d}} \right) \left( R'_d - \lambda' f'_{1d} \right) \end{aligned}$$

The benefit policy $b$ is characterized by the FOC for $b$

$$R_b = 0$$

These three equations together characterize the Markov government's UI policy choices (GEEs).

7. For easy interpretation, rewrite the above three equations using the the auxiliary functions (specified later). The duration policy $d$ is characterized by

$$\begin{aligned} 0 = \; & u^1 \left( U^1 - U^0 \right) + \frac{\partial R}{\partial \tau} \frac{\partial \tau}{\partial d} + \lambda \frac{\partial u'}{\partial d} \\ & + \frac{\partial u^{1\prime}}{\partial d} \left[ -\mu_0 \eta_{1u^{1\prime}} - \mu_1 \eta_{2u^{1\prime}} - \gamma \eta_{3u^{1\prime}} + \beta \left( \frac{\partial R'}{\partial u^{1\prime}} + \lambda' \frac{\partial u''}{\partial u^{1\prime}} \right) \right] \\ & + \beta \frac{\partial u^{1\prime}}{\partial d} \left. \frac{dd'}{du^{1\prime}} \right|_{u^{1\prime\prime} \text{constant}} \left[ u^{1\prime} \left( U^{1\prime} - U^{0\prime} \right) + \frac{\partial R'}{\partial \tau'} \frac{\partial \tau'}{\partial d'} + \lambda' \frac{\partial u''}{\partial d'} \right], \end{aligned} \tag{40}$$

where the multiplier $\lambda$ is characterized by

$$\begin{aligned} \lambda = \; & -\mu_0 \eta_{1u'} - \mu_1 \eta_{2u'} - \gamma \eta_{3u'} + \beta \left( \frac{\partial R'}{\partial u'} + \lambda' \frac{\partial u''}{\partial u'} \right) \\ & + \beta \left. \frac{dd'}{du'} \right|_{u^{1\prime\prime} \text{constant}} \left[ u^{1\prime} \left( U^{1\prime} - U^{0\prime} \right) + \frac{\partial R'}{\partial \tau'} \frac{\partial \tau'}{\partial d'} + \lambda' \frac{\partial u''}{\partial d'} \right], \end{aligned} \tag{41}$$

and the benefit policy $b$ is characterized by

$$u^1 dU^1_c + \frac{\partial R}{\partial \tau} \frac{\partial \tau}{\partial b} = 0. \tag{42}$$

Note that $-\eta_{1u'}$ is the derivative of the marginal value of search for non-recipients with respect to state $u'$, i.e. $\frac{\partial MV(s^0;\mathcal{O})}{\partial u'}$. Similarly, $-\eta_{1u^{1\prime}} = \frac{\partial MV(s^0;\mathcal{O})}{\partial u^{1\prime}}$, $-\eta_{2u'} = \frac{\partial MV(s^1;\mathcal{O})}{\partial u'}$, and $-\eta_{2u^{1\prime}} = \frac{\partial MV(s^1;\mathcal{O})}{\partial u^{1\prime}}$. Also in the baseline model, the equilibrium $\theta$ does not depend on unemployment, so $\eta_{3u'} = 0$ and $\eta_{3u^{1\prime}} = 0$. So Equations (40)-(42) become Equations (17)-(19).

**Auxiliary functions.** Denote $U^e = U(w(z) - \tau(u^1, b, d))$, $U^0 = U(h - \tau(u^1, b, d)) - v(s^0)$, $U^1 = U(h + b - \tau(u^1, b, d)) - v(s^1)$. Let $U^e_c$, $U^0_c$, $U^1_c$ be their respective derivatives with respect to consumption, and $U^0_s$, $U^1_s$ be their respective derivatives with respect to search. The derivatives of the auxiliary functions $R(z, u, u^1, b, d, s^0, s^1)$,

$f_1(u,u^1,d,s^0,s^1,\theta,u')$, $f_2(u,u^1,d,s^1,\theta,u^{1'})$, $\eta_1(s^0,\theta,u',u^{1'})$, $\eta_2(s^1,\theta,u',u^{1'})$ and $\eta_3(\theta,u',u^{1'})$ are

$$
\begin{aligned}
R_u &\equiv \frac{\partial R}{\partial u} = -U^e + U^0 \\
R_{u^1} &\equiv \frac{\partial R}{\partial u^1} = d\left(U^1 - U^0\right) + \frac{\partial R}{\partial \tau}\frac{\partial \tau}{\partial u^1} \\
R_b &\equiv \frac{\partial R}{\partial b} = u^1 d U_c^1 + \frac{\partial R}{\partial \tau}\frac{\partial \tau}{\partial b} \\
R_d &\equiv \frac{\partial R}{\partial d} = u^1\left(U^1 - U^0\right) + \frac{\partial R}{\partial \tau}\frac{\partial \tau}{\partial d} \\
R_{s^0} &\equiv \frac{\partial R}{\partial s^0} = (u - u^1 d)U_s^0 \\
R_{s^1} &\equiv \frac{\partial R}{\partial s^1} = u^1 d U_s^1 \\
\frac{\partial R}{\partial \tau} &= -\left[(1-u)U_c^e + u^1 d U_c^1 + (u - u^1 d)U_c^0\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \tau}{\partial u^1} &= db \\
\frac{\partial \tau}{\partial b} &= u^1 d \\
\frac{\partial \tau}{\partial d} &= u^1 b
\end{aligned}
$$

$$
\begin{aligned}
f_{1u} &\equiv -\frac{\partial u'}{\partial u} = \delta - (1 - f(\theta)s^0) \\
f_{1u^1} &\equiv -\frac{\partial u'}{\partial u^1} = -f(\theta)(s^0 - s^1)d \\
f_{1d} &\equiv -\frac{\partial u'}{\partial d} = -f(\theta)(s^0 - s^1)u^1 \\
f_{1s^0} &\equiv -\frac{\partial u'}{\partial s^0} = f(\theta)(u - u^1 d) \\
f_{1s^1} &\equiv -\frac{\partial u'}{\partial s^1} = f(\theta)u^1 d \\
f_{1\theta} &\equiv -\frac{\partial u'}{\partial \theta} = f_\theta(\theta)\left[(u - u^1 d)s^0 + u^1 d s^1\right] \\
f_{1u'} &= 1 \\
f_{2u} &\equiv -\frac{\partial u^{1'}}{\partial u} = \delta\xi \\
f_{2u^1} &\equiv -\frac{\partial u^{1'}}{\partial u^1} = -(1 - f(\theta)s^1)d \\
f_{2d} &\equiv -\frac{\partial u^{1'}}{\partial d} = -(1 - f(\theta)s^1)u^1 \\
f_{2s^1} &\equiv -\frac{\partial u^{1'}}{\partial s^1} = f(\theta)u^1 d \\
f_{2\theta} &\equiv -\frac{\partial u^{1'}}{\partial \theta} = f_\theta(\theta)u^1 d s^1 \\
f_{2u^{1'}} &= 1
\end{aligned}
$$

$$
\begin{aligned}
\eta_{1s^0} &= \frac{v_{ss}(s^0)}{f(\theta)} \\
\eta_{1\theta} &= -\frac{v_s(s^0)f_\theta(\theta)}{f(\theta)^2} \\
\eta_{1u'} &= \beta\frac{\partial\tau'}{\partial b'}\left(U_c^{e'}-U_c^{0'}\right)\Psi_u^{b'}+\beta\frac{\partial\tau'}{\partial d'}\left(U_c^{e'}-U_c^{0'}\right)\Psi_u^{d'}\ldots \\
&\quad -\beta\left[1-f(\theta')s^{0'}-\delta(1-\xi)\right]\frac{v_{ss}(s^{0'})}{f(\theta')}S_u^{0'}+\beta\delta\xi\frac{v_{ss}(s^{1'})}{f(\theta')}S_u^{1'}+\beta\left[(1-\delta(1-\xi))v_s(s^{0'})-\delta\xi v_s(s^{1'})\right]\frac{f_\theta(\theta')}{f(\theta')^2}\Theta'_u \\
\eta_{1u^{1'}} &= \beta\frac{\partial\tau'}{\partial u^{1'}}\left(U_c^{e'}-U_c^{0'}\right)+\beta\frac{\partial\tau'}{\partial b'}\left(U_c^{e'}-U_c^{0'}\right)\Psi_{u^1}^{b'}+\beta\frac{\partial\tau'}{\partial d'}\left(U_c^{e'}-U_c^{0'}\right)\Psi_{u^1}^{d'}\ldots \\
&\quad -\beta\left[1-f(\theta')s^{0'}-\delta(1-\xi)\right]\frac{v_{ss}(s^{0'})}{f(\theta')}S_{u^1}^{0'}+\beta\delta\xi\frac{v_{ss}(s^{1'})}{f(\theta')}S_{u^1}^{1'}+\beta\left[(1-\delta(1-\xi))v_s(s^{0'})-\delta\xi v_s(s^{1'})\right]\frac{f_\theta(\theta')}{f(\theta')^2}\Theta'_{u^1}
\end{aligned}
$$

$$
\begin{aligned}
\eta_{2s^1} &= \frac{v_{ss}(s^1)}{f(\theta)} \\
\eta_{2\theta} &= -\frac{v_s(s^1)f_\theta(\theta)}{f(\theta)^2} \\
\eta_{2u'} &= \beta\left[(1-d')\frac{\partial\tau'}{\partial b'}\left(U_c^{e'}-U_c^{0'}\right)+d'\left[\frac{\partial\tau'}{\partial b'}U_c^{e'}+\left(1-\frac{\partial\tau'}{\partial b'}\right)U_c^{1'}\right]\right]\Psi_u^{b'}\ldots \\
&\quad +\beta\Bigg((1-d')\frac{\partial\tau'}{\partial d'}\left(U_c^{e'}-U_c^{0'}\right)+d'\frac{\partial\tau'}{\partial d'}\left(U_c^{e'}-U_c^{1'}\right)\ldots \\
&\qquad +\left[U^{1'}-U^{0'}+[1-f(\theta')s^{0'}]\frac{v_s(s^{0'})}{f(\theta')}-[1-f(\theta')s^{1'}]\frac{v_s(s^{1'})}{f(\theta')}\right]\Bigg)\Psi_u^{d'}\ldots \\
&\quad -\beta\left[(1-d')(1-f(\theta')s^{0'})-\delta(1-\xi)\right]\frac{v_{ss}(s^{0'})}{f(\theta')}S_u^{0'}-\beta\left[d'(1-f(\theta')s^{1'})-\delta\xi\right]\frac{v_{ss}(s^{1'})}{f(\theta')}S_u^{1'}\ldots \\
&\quad +\beta\left[v_s(s^{0'})\left[(1-d')-\delta(1-\xi)\right]+v_s(s^{1'})\left(d'-\delta\xi\right)\right]\frac{f_\theta(\theta')}{f(\theta')^2}\Theta'_u \\
\eta_{2u^{1'}} &= \beta(1-d')\frac{\partial\tau'}{\partial u^{1'}}\left(U_c^{e'}-U_c^{0'}\right)+\beta d'\frac{\partial\tau'}{\partial u^{1'}}\left(U_c^{e'}-U_c^{1'}\right)\ldots \\
&\quad +\beta\left[(1-d')\frac{\partial\tau'}{\partial b'}\left(U_c^{e'}-U_c^{0'}\right)+d'\left[\frac{\partial\tau'}{\partial b'}U_c^{e'}+\left(1-\frac{\partial\tau'}{\partial b'}\right)U_c^{1'}\right]\right]\Psi_{u^1}^{b'}\ldots \\
&\quad +\beta\Bigg((1-d')\frac{\partial\tau'}{\partial d'}\left(U_c^{e'}-U_c^{0'}\right)+d'\frac{\partial\tau'}{\partial d'}\left(U_c^{e'}-U_c^{1'}\right)\ldots \\
&\qquad +\left[U^{1'}-U^{0'}+[1-f(\theta')s^{0'}]\frac{v_s(s^{0'})}{f(\theta')}-[1-f(\theta')s^{1'}]\frac{v_s(s^{1'})}{f(\theta')}\right]\Bigg)\Psi_{u^1}^{d'}\ldots \\
&\quad -\beta\left[(1-d')(1-f(\theta')s^{0'})-\delta(1-\xi)\right]\frac{v_{ss}(s^{0'})}{f(\theta')}S_{u^1}^{0'}-\beta\left[d'(1-f(\theta')s^{1'})-\delta\xi\right]\frac{v_{ss}(s^{1'})}{f(\theta')}S_{u^1}^{1'}\ldots \\
&\quad +\beta\left[v_s(s^{0'})\left[(1-d')-\delta(1-\xi)\right]+v_s(s^{1'})\left(d'-\delta\xi\right)\right]\frac{f_\theta(\theta')}{f(\theta')^2}\Theta'_{u^1}
\end{aligned}
$$

$$
\begin{aligned}
\eta_{3\theta} &= -\frac{\kappa q_\theta(\theta)}{q(\theta)^2} \\
\eta_{3u'} &= \beta(1-\delta)\frac{\kappa q_\theta(\theta')}{q(\theta')^2}\Theta'_u \\
\eta_{3u^{1'}} &= \beta(1-\delta)\frac{\kappa q_\theta(\theta')}{q(\theta')^2}\Theta'_{u^1}
\end{aligned}
$$

## B.3 Derivation of the Ramsey problem's optimality conditions (20)-(22)

Throughout this section, we drop the dependence of functions on productivity shock $z_t$ to economize on notation, and use red to highlight differences from the Markov conditions.

Denote the period welfare function by $R(z_t, u_t, u_t^1, b_t, d_t, s_t^0, s_t^1)$. Denote the workers' flow equations by $\tilde{f}_1(u_t, u_t^1, d_t, s_t^0, s_t^1, \theta_t, u_{t+1}) = 0$ and $\tilde{f}_2(u_t, u_t^1, d_t, s_t^1, \theta_t, u_{t+1}^1) = 0$. Denote the private sector's optimality conditions by $\tilde{\eta}_1(s_t^0, \theta_t, u_{t+1}^1, b_{t+1}, d_{t+1}, s_{t+1}^0, s_{t+1}^1, \theta_{t+1}) = 0$, $\tilde{\eta}_2(s_t^1, \theta_t, u_{t+1}^1, b_{t+1}, d_{t+1}, s_{t+1}^0, s_{t+1}^1, \theta_{t+1}) = 0$ and $\tilde{\eta}_3(\theta_t, \theta_{t+1}) = 0$. Let $\beta^t \tilde{\lambda}_t$, $\beta^t \tilde{\lambda}_{1,t}$, $\beta^t \tilde{\mu}_{0,t}$, $\beta^t \tilde{\mu}_{1,t}$, $\beta^t \tilde{\gamma}_t$ be the Lagrange multipliers on workers' flow equations and private sector's optimality conditions respectively. Then the following FOCs characterize the Ramsey solution:

$$
\begin{aligned}
b_t: &\quad \tilde{\mu}_{0,t-1} \frac{\tilde{\eta}_{1b',t-1}}{\beta} + \tilde{\mu}_{1,t-1} \frac{\tilde{\eta}_{2b',t-1}}{\beta} - R_{b,t} = 0 \\
d_t: &\quad \tilde{\mu}_{0,t-1} \frac{\tilde{\eta}_{1d',t-1}}{\beta} + \tilde{\mu}_{1,t-1} \frac{\tilde{\eta}_{2d',t-1}}{\beta} + \tilde{\lambda}_t \tilde{f}_{1d,t} + \tilde{\lambda}_{1,t} \tilde{f}_{2d,t} - R_{d,t} = 0 \\
s_t^0: &\quad \tilde{\mu}_{0,t-1} \frac{\tilde{\eta}_{1s^{0\prime},t-1}}{\beta} + \tilde{\mu}_{1,t-1} \frac{\tilde{\eta}_{2s^{0\prime},t-1}}{\beta} + \tilde{\lambda}_t \tilde{f}_{1s^0,t} + \tilde{\mu}_{0,t} \tilde{\eta}_{1s^0,t} - R_{s^0,t} = 0 \\
s_t^1: &\quad \tilde{\mu}_{0,t-1} \frac{\tilde{\eta}_{1s^{1\prime},t-1}}{\beta} + \tilde{\mu}_{1,t-1} \frac{\tilde{\eta}_{2s^{1\prime},t-1}}{\beta} + \tilde{\lambda}_t \tilde{f}_{1s^1,t} + \tilde{\lambda}_{1,t} \tilde{f}_{2s^1,t} + \tilde{\mu}_{1,t} \tilde{\eta}_{2s^1,t} - R_{s^1,t} = 0 \\
\theta_t: &\quad \tilde{\mu}_{0,t-1} \frac{\tilde{\eta}_{1\theta',t-1}}{\beta} + \tilde{\mu}_{1,t-1} \frac{\tilde{\eta}_{2\theta',t-1}}{\beta} + \tilde{\gamma}_{t-1} \frac{\tilde{\eta}_{3\theta',t-1}}{\beta} + \tilde{\lambda}_t \tilde{f}_{1\theta,t} + \tilde{\lambda}_{1,t} \tilde{f}_{2\theta,t} + \tilde{\mu}_{0,t} \tilde{\eta}_{1\theta,t} + \tilde{\mu}_{1,t} \tilde{\eta}_{2\theta,t} + \tilde{\gamma}_t \tilde{\eta}_{3\theta,t} = 0 \\
u_{t+1}: &\quad \tilde{\lambda}_t \tilde{f}_{1u',t} = \beta \left\{ R_{u,t+1} - \tilde{\lambda}_{t+1} \tilde{f}_{1u,t+1} - \tilde{\lambda}_{1,t+1} \tilde{f}_{2u,t+1} \right\} \\
u_{t+1}^1: &\quad \tilde{\lambda}_{1,t} \tilde{f}_{2u^{1\prime},t} + \tilde{\mu}_{0,t} \tilde{\eta}_{1u^{1\prime},t} + \tilde{\mu}_{1,t} \tilde{\eta}_{2u^{1\prime},t} = \beta \left\{ R_{u^1,t+1} - \tilde{\lambda}_{t+1} \tilde{f}_{1u^1,t+1} - \tilde{\lambda}_{1,t+1} \tilde{f}_{2u^1,t+1} \right\} \qquad \text{(RAM)}
\end{aligned}
$$

Substituting $\tilde{\lambda}_1$ from the last FOC into the FOCs for $d_t$ and $u_{t+1}$, the duration policy $d_t$ is characterized by

$$
\begin{aligned}
0 \;=\; & R_{d,t} - \tilde{\lambda}_t \tilde{f}_{1d,t} - \tilde{\mu}_{0,t-1} \frac{\tilde{\eta}_{1d',t-1}}{\beta} - \tilde{\mu}_{1,t-1} \frac{\tilde{\eta}_{2d',t-1}}{\beta} \\
& - \frac{\tilde{f}_{2d,t}}{\tilde{f}_{2u^{1\prime},t}} \Big[ - \tilde{\mu}_{0,t} \eta_{1u^{1\prime},t} - \tilde{\mu}_{1,t} \eta_{2u^{1\prime},t} + \beta \left( R_{u^1,t+1} - \tilde{\lambda}_{t+1} \tilde{f}_{1u^1,t+1} \right) \Big] \\
& - \beta \frac{\tilde{f}_{2d,t}}{\tilde{f}_{2u^{1\prime},t}} \left( - \frac{\tilde{f}_{2u^1,t+1}}{\tilde{f}_{2d,t+1}} \right) \left( R_{d,t+1} - \tilde{\lambda}_{t+1} \tilde{f}_{1d,t+1} - \tilde{\mu}_{0,t} \frac{\tilde{\eta}_{1d',t}}{\beta} - \tilde{\mu}_{1,t} \frac{\tilde{\eta}_{2d',t}}{\beta} \right)
\end{aligned}
$$

where the multiplier $\tilde{\lambda}_t$ is characterized by

$$
\begin{aligned}
\tilde{\lambda}_t \tilde{f}_{1u',t} \;=\; & \beta \left( R_{u,t+1} - \tilde{\lambda}_{t+1} \tilde{f}_{1u,t+1} \right) \\
& + \beta \left( - \frac{\tilde{f}_{2u,t+1}}{\tilde{f}_{2d,t+1}} \right) \left( R_{d,t+1} - \tilde{\lambda}_{t+1} \tilde{f}_{1d,t+1} - \tilde{\mu}_{0,t} \frac{\tilde{\eta}_{1d',t}}{\beta} - \tilde{\mu}_{1,t} \frac{\tilde{\eta}_{2d',t}}{\beta} \right)
\end{aligned}
$$

The benefit policy $b_t$ is characterized by the FOC for $b_t$

$$
R_{b,t} = \tilde{\mu}_{0,t-1} \frac{\tilde{\eta}_{1b',t-1}}{\beta} + \tilde{\mu}_{1,t-1} \frac{\tilde{\eta}_{2b',t-1}}{\beta}
$$

Rewrite the above three equations using the auxiliary functions (specified later) The duration policy $d_t$ is

characterized by

$$
\begin{aligned}
0 \quad = \quad & u_t^1\left(U_t^1 - U_t^0\right) + \frac{\partial R_t}{\partial \tau_t}\frac{\partial \tau_t}{\partial d_t} + \tilde{\lambda}_t \frac{\partial u_{t+1}}{\partial d_t} - \tilde{\mu}_{0,t-1}\frac{\tilde{\eta}_{1d',t-1}}{\beta} - \tilde{\mu}_{1,t-1}\frac{\tilde{\eta}_{2d',t-1}}{\beta} \\
& + \frac{\partial u_{t+1}^1}{\partial d_t}\left[-\mu\tilde{0}_{,t}\tilde{\eta}_{1u^{1'},t} - \mu\tilde{1}_{,t}\tilde{\eta}_{2u^{1'},t} + \beta\left(\frac{\partial R_{t+1}}{\partial u_{t+1}^1} + \tilde{\lambda}_{t+1}\frac{\partial u_{t+2}}{\partial u_{t+1}^1}\right)\right] \\
& + \beta\frac{\partial u_{t+1}^1}{\partial d_t}\left.\frac{dd_{t+1}}{du_{t+1}^1}\right|_{u_{t+2}^1\text{constant}}\left[u_{t+1}^1\left(U_{t+1}^1 - U_{t+1}^0\right) + \frac{\partial R_{t+1}}{\partial \tau_{t+1}}\frac{\partial \tau_{t+1}}{\partial d_{t+1}} + \tilde{\lambda}_{t+1}\frac{\partial u_{t+2}}{\partial d_{t+1}} - \tilde{\mu}_{0,t}\frac{\tilde{\eta}_{1d',t}}{\beta} - \tilde{\mu}_{1,t}\frac{\tilde{\eta}_{2d',t}}{\beta}\right] \qquad (43)
\end{aligned}
$$

where the multiplier $\tilde{\lambda}_t$ is characterized by

$$
\begin{aligned}
\tilde{\lambda}_t \quad = \quad & \beta\left(\frac{\partial R_{t+1}}{\partial u_{t+1}} + \tilde{\lambda}_{t+1}\frac{\partial u_{t+2}}{\partial u_{t+1}}\right) \\
& + \beta\left.\frac{dd_{t+1}}{du_{t+1}}\right|_{u_{t+2}^1\text{constant}}\left[u_{t+1}^1\left(U_{t+1}^1 - U_{t+1}^0\right) + \frac{\partial R_{t+1}}{\partial \tau_{t+1}}\frac{\partial \tau_{t+1}}{\partial d_{t+1}} + \tilde{\lambda}_{t+1}\frac{\partial u_{t+2}}{\partial d_{t+1}} - \tilde{\mu}_{0,t}\frac{\tilde{\eta}_{1d',t}}{\beta} - \tilde{\mu}_{1,t}\frac{\tilde{\eta}_{2d',t}}{\beta}\right] \qquad (44)
\end{aligned}
$$

and the benefit policy $b_t$ is characterized by

$$
\left(u_t^1 d_t U_{c,t}^1 + \frac{\partial R_t}{\partial \tau_t}\frac{\partial \tau_t}{\partial b_t}\right) = \tilde{\mu}_{0,t-1}\frac{\tilde{\eta}_{1b',t-1}}{\beta} + \tilde{\mu}_{1,t-1}\frac{\tilde{\eta}_{2b',t-1}}{\beta}. \qquad (45)
$$

Similar to the Markov optimality equations, $-\tilde{\eta}_{ix}$ is the derivative of the marginal value of search for unemployed worker $i$ with respect to variable $x$, i.e., $-\tilde{\eta}_{ix} = \frac{\partial MV(s^i)}{\partial x}$, $i = \{0,1\}$, and $x = \{b, d, u^{1'}\}$. So Equations (43)-(45) become Equations (20)-(22).

**Auxiliary functions.** Denote $U_t^e = U(w(z_t) - \tau(u_t^1, b_t, d_t))$, $U_t^0 = U(h - \tau(u_t^1, b_t, d_t)) - v(s_t^0)$, $U_t^1 = U(h + b_t - \tau(u_t^1, b_t, d_t)) - v(s_t^1)$. Let $U_{c,t}^e$, $U_{c,t}^0$, $U_{c,t}^1$ be their respective derivatives with respect to consumption, and $U_{s,t}^0$, $U_{s,t}^1$ be their respective derivatives with respect to search. The derivatives of the auxiliary functions $R(z_t, u_t, u_t^1, b_t, d_t, s_t^0, s_t^1)$, $\tilde{f}_1(u_t, u_t^1, d_t, s_t^0, s_t^1, \theta_t, u_{t+1})$, $\tilde{f}_2(u_t, u_t^1, d_t, s_t^1, \theta_t, u_{t+1}^1)$, $\tilde{\eta}_1(s_t^0, \theta_t, u_{t+1}^1, b_{t+1}, d_{t+1}, s_{t+1}^0, s_{t+1}^1, \theta_{t+1})$, $\tilde{\eta}_2(s_t^1, \theta_t, u_{t+1}^1, b_{t+1}, d_{t+1}, s_{t+1}^0, s_{t+1}^1, \theta_{t+1})$ and $\tilde{\eta}_3(\theta_t, \theta_{t+1})$ are

$$
\begin{aligned}
R_{u,t} \quad & \equiv \quad \frac{\partial R_t}{\partial u_t} = -U_t^e + U_t^0 \\
R_{u^1,t} \quad & \equiv \quad \frac{\partial R_t}{\partial u_t^1} = d_t\left(U_t^1 - U_t^0\right) + \frac{\partial R_t}{\partial \tau_t}\frac{\partial \tau_t}{\partial u_t^1} \\
R_{b,t} \quad & \equiv \quad \frac{\partial R_t}{\partial b_t} = u_t^1 d_t U_{c,t}^1 + \frac{\partial R_t}{\partial \tau_t}\frac{\partial \tau_t}{\partial b_t} \\
R_{d,t} \quad & \equiv \quad \frac{\partial R_t}{\partial d_t} = u_t^1\left(U_t^1 - U_t^0\right) + \frac{\partial R_t}{\partial \tau_t}\frac{\partial \tau_t}{\partial d_t} \\
R_{s^0,t} \quad & \equiv \quad \frac{\partial R_t}{\partial s_t^0} = (u_t - u_t^1 d_t)U_{s,t}^0 \\
R_{s^1,t} \quad & \equiv \quad \frac{\partial R_t}{\partial s_t^1} = u_t^1 d_t U_{s,t}^1 \\
\frac{\partial R_t}{\partial \tau_t} \quad & = \quad -\left[(1-u_t)U_{c,t}^e + u_t^1 d_t U_{c,t}^1 + (u_t - u_t^1 d_t)U_{c,t}^0\right] \\
\frac{\partial \tau_t}{\partial u_t^1} \quad & = \quad d_t b_t \\
\frac{\partial \tau_t}{\partial b_t} \quad & = \quad u_t^1 d_t \\
\frac{\partial \tau_t}{\partial d_t} \quad & = \quad u_t^1 b_t
\end{aligned}
$$

$$
\begin{aligned}
\tilde{f}_{1u,t} &\equiv -\frac{\partial u_{t+1}}{\partial u_t} = \delta - (1 - f(\theta_t)s_t^0) \\
\tilde{f}_{1u^1,t} &\equiv -\frac{\partial u_{t+1}}{\partial u_t^1} = -f(\theta_t)(s_t^0 - s_t^1)d_t \\
\tilde{f}_{1d,t} &\equiv -\frac{\partial u_{t+1}}{\partial d_t} = -f(\theta_t)(s_t^0 - s_t^1)u_t^1 \\
\tilde{f}_{1s^0,t} &\equiv -\frac{\partial u_{t+1}}{\partial s_t^0} = f(\theta_t)(u_t - u_t^1 d_t) \\
\tilde{f}_{1s^1,t} &\equiv -\frac{\partial u_{t+1}}{\partial s_t^1} = f(\theta_t)u_t^1 d_t \\
\tilde{f}_{1\theta,t} &\equiv -\frac{\partial u_{t+1}}{\partial \theta_t} = f_\theta(\theta_t)\left[(u_t - u_t^1 d_t)s_t^0 + u_t^1 d_t s_t^1\right] \\
\tilde{f}_{1u',t} &= 1 \\
\tilde{f}_{2u,t} &\equiv -\frac{\partial u_{t+1}^1}{\partial u_t} = \delta\xi \\
\tilde{f}_{2u^1,t} &\equiv -\frac{\partial u_{t+1}^1}{\partial u_t^1} = -(1 - f(\theta_t)s_t^1)d_t \\
\tilde{f}_{2d,t} &\equiv -\frac{\partial u_{t+1}^1}{\partial d_t} = -(1 - f(\theta_t)s_t^1)u_t^1 \\
\tilde{f}_{2s^1,t} &\equiv -\frac{\partial u_{t+1}^1}{\partial s_t^1} = f(\theta_t)u_t^1 d_t \\
\tilde{f}_{2\theta,t} &\equiv -\frac{\partial u_{t+1}^1}{\partial \theta_t} = f_\theta(\theta_t)u_t^1 d_t s_t^1 \\
\tilde{f}_{2u^1{}',t} &= 1
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\eta}_{1s^0,t} &= \frac{v_{ss}(s_t^0)}{f(\theta_t)} \\
\tilde{\eta}_{1\theta,t} &= -\frac{v_s(s_t^0)f_\theta(\theta_t)}{f(\theta_t)^2} \\
\tilde{\eta}_{1u^1{}',t} &= \beta\frac{\partial \tau_{t+1}}{\partial u_{t+1}^1}\left(U_{c,t+1}^e - U_{c,t+1}^0\right) \\
\tilde{\eta}_{1b',t} &= \beta\frac{\partial \tau_{t+1}}{\partial b_{t+1}}\left(U_{c,t+1}^e - U_{c,t+1}^0\right) \\
\tilde{\eta}_{1d',t} &= \beta\frac{\partial \tau_{t+1}}{\partial d_{t+1}}\left(U_{c,t+1}^e - U_{c,t+1}^0\right) \\
\tilde{\eta}_{1s^0{}',t} &= -\beta\left[1 - f(\theta_{t+1})s_{t+1}^0 - \delta(1-\xi)\right]\frac{v_{ss}(s_{t+1}^0)}{f(\theta_{t+1})} \\
\tilde{\eta}_{1s^1{}',t} &= \beta\delta\xi\frac{v_{ss}(s_{t+1}^1)}{f(\theta_{t+1})} \\
\tilde{\eta}_{1\theta',t} &= \beta\left[(1-\delta(1-\xi))v_s(s_{t+1}^0) - \delta\xi v_s(s_{t+1}^1)\right]\frac{f_\theta(\theta_{t+1})}{f(\theta_{t+1})^2}
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\eta}_{2s^1,t} &= \frac{v_{ss}(s_t^1)}{f(\theta_t)} \\
\tilde{\eta}_{2\theta,t} &= -\frac{v_s(s_t^1) f_\theta(\theta_t)}{f(\theta_t)^2} \\
\tilde{\eta}_{2u^{1\prime},t} &= \beta(1-d_{t+1})\frac{\partial \tau_{t+1}}{\partial u_{t+1}^1}\left(U_{c,t+1}^e - U_{c,t+1}^0\right) + \beta d_{t+1}\frac{\partial \tau_{t+1}}{\partial u_{t+1}^1}\left(U_{c,t+1}^e - U_{c,t+1}^1\right) \\
\tilde{\eta}_{2b',t} &= \beta(1-d_{t+1})\frac{\partial \tau_{t+1}}{\partial b_{t+1}}\left(U_{c,t+1}^e - U_{c,t+1}^0\right) + \beta d_{t+1}\left[\frac{\partial \tau_{t+1}}{\partial b_{t+1}} U_{c,t+1}^e + \left(1 - \frac{\partial \tau_{t+1}}{\partial b_{t+1}}\right) U_{c,t+1}^1\right] \\
\tilde{\eta}_{2d',t} &= \beta(1-d_{t+1})\frac{\partial \tau_{t+1}}{\partial d_{t+1}}\left(U_{c,t+1}^e - U_{c,t+1}^0\right) + \beta d_{t+1}\frac{\partial \tau_{t+1}}{\partial d_{t+1}}\left(U_{c,t+1}^e - U_{c,t+1}^1\right) \\
&\quad + \beta\left[U_{t+1}^1 - U_{t+1}^0 + [1 - f(\theta_{t+1}) s_{t+1}^0]\frac{v_s(s_{t+1}^0)}{f(\theta_{t+1})} - [1 - f(\theta_{t+1}) s_{t+1}^1]\frac{v_s(s_{t+1}^1)}{f(\theta_{t+1})}\right] \\
\tilde{\eta}_{2s^{0\prime},t} &= -\beta\left[(1-d_{t+1})(1 - f(\theta_{t+1}) s_{t+1}^0) - \delta(1-\xi)\right]\frac{v_{ss}(s_{t+1}^0)}{f(\theta_{t+1})} \\
\tilde{\eta}_{2s^{1\prime},t} &= -\beta\left[d_{t+1}(1 - f(\theta_{t+1}) s_{t+1}^1) - \delta\xi\right]\frac{v_{ss}(s_{t+1}^1)}{f(\theta_{t+1})} \\
\tilde{\eta}_{2\theta',t} &= \beta\left[v_s(s_{t+1}^0)[(1-d_{t+1}) - \delta(1-\xi)] + v_s(s_{t+1}^1)(d_{t+1} - \delta\xi)\right]\frac{f_\theta(\theta_{t+1})}{f(\theta_{t+1})^2} \\
\\
\tilde{\eta}_{3\theta,t} &= -\frac{\kappa q_\theta(\theta_t)}{q(\theta_t)^2} \\
\tilde{\eta}_{3\theta',t} &= \beta(1-\delta)\frac{\kappa q_\theta(\theta_{t+1})}{q(\theta_{t+1})^2}
\end{aligned}
$$

# C Model with Extension-Neutral UI-Recipients

In this extension to the baseline model, we assume each period a proportion $\gamma$ of UI recipients for sure have benefits the next period if they stay unemployed. Throughout this section, we drop the dependence of functions on state variables to economize on notation, and use red to highlight the difference from the baseline model. The results from this extension are discussed in Section 7.3.

The laws of motion of unemployed workers are now

$$\text{total unemployment:} \quad u' = \delta(1-u) + (1-f(\theta)s^0)(u-u^1d) + (1-f(\theta)s^1)u^1d(1-\gamma)\ldots + \underbrace{(1-f(\theta)s^N)u^1d\gamma}_{\text{extension-neutral UI recipients}}$$

$$\text{benefit-eligible unemployed:} \quad u^{1'} = \delta(1-u)\xi + (1-f(\theta)s^1)u^1d(1-\gamma) + \underbrace{(1-f(\theta)s^N)u^1d\gamma}_{\text{extension-neutral UI recipients}}$$

The unemployed worker's optimal search is characterized by

$$\text{Non-UI recipients:} \quad \frac{v_s(s^0)}{f(\theta)} = \beta\mathbb{E}[V^{e'} - V^{0'}]$$

$$\text{UI recipients:} \quad \frac{v_s(s^1)}{f(\theta)} = \beta\mathbb{E}[V^{e'} - (1-d')V^{0'} - (1-\gamma)d'V^{1'} - \gamma d'V^{N'}]$$

$$\text{Extension-neutral UI recipients:} \quad \frac{v_s(s^N)}{f(\theta)} = \beta\mathbb{E}[V^{e'} - V^{1'}]$$

where the value functions of workers are defined as

$$\text{Employed:} \quad V^e = \log(w-\tau) + (1-\delta)\beta\mathbb{E}V^{e'} + \delta(1-\xi)\beta\mathbb{E}V^{0'}\ldots + \delta\xi\beta\mathbb{E}\left[(1-d')V^{0'} + (1-\gamma)d'V^{1'} + \gamma d'V^{N'}\right]$$

$$\text{Non-recipients:} \quad V^0 = \max_{s^0} \ \log(h-\tau) - v(s^0) + f(\theta)s^0\beta\mathbb{E}V^{e'} + (1-f(\theta)s^0)\beta\mathbb{E}V^{0'}$$

$$\text{UI recipients:} \quad V^1 = \max_{s^1} \ \log(h+b-\tau) - v(s^1) + f(\theta)s^1\beta\mathbb{E}V^{e'}\ldots + (1-f(\theta)s^1)\beta\mathbb{E}\left[(1-d')V^{0'} + (1-\gamma)d'V^{1'} + \gamma d'V^{N'}\right]$$

$$\text{Extension-neutral UI recipients:} \quad V^N = \max_{s^N} \ \log(h+b-\tau) - v(s^N) + f(\theta)s^N\beta\mathbb{E}V^{e'} + (1-f(\theta)s^N)\beta\mathbb{E}V^{1'}$$

Government's period-welfare function is

$$R(z,u,u^1,b,d,s^0,s^1,s^N) = (1-u)U^e + (u-u^1d)U^0 + u^1(1-\gamma)dU^1 + u^1\gamma dU^N$$

The government's optimality conditions (GEEs) become

$$
\begin{aligned}
0 &= u^1[(1-\gamma)U^1 + \gamma U^N - U^0] + \frac{\partial R}{\partial \tau}\frac{\partial \tau}{\partial d} + \lambda \frac{\partial u'}{\partial d} \\
&\quad + \frac{\partial u^{1'}}{\partial d}\left[\left(\mu_0 \frac{\partial MV(s^0;\mathcal{O}')}{\partial u^{1'}} + \mu_1 \frac{\partial MV(s^1;\mathcal{O}')}{\partial u^{1'}} + \mu_N \frac{\partial MV(s^N;\mathcal{O}')}{\partial u^{1'}}\right) + \beta \mathbb{E}\left(\frac{\partial R'}{\partial u^{1'}} + \lambda' \frac{\partial u''}{\partial u^{1'}}\right)\right] \\
&\quad + \beta \mathbb{E} \frac{\partial u^{1'}}{\partial d} \left.\frac{dd'}{du^{1'}}\right|_{\text{hold } u^{1''} \text{ constant}} \left[u^{1'}((1-\gamma)U^{1'} + \gamma U^{N'} - U^{0'}) + \frac{\partial R'}{\partial \tau'}\frac{\partial \tau'}{\partial d'} + \lambda' \frac{\partial u''}{\partial d'}\right] \\
\lambda &= \left(\mu_0 \frac{\partial MV(s^0;\mathcal{O}')}{\partial u'} + \mu_1 \frac{\partial MV(s^1;\mathcal{O}')}{\partial u'} + \mu_N \frac{\partial MV(s^N;\mathcal{O}')}{\partial u'}\right) + \beta \mathbb{E}\left(\frac{\partial R'}{\partial u'} + \lambda' \frac{\partial u''}{\partial u'}\right) \\
&\quad + \beta \mathbb{E} \left.\frac{dd'}{du'}\right|_{\text{hold } u^{1''} \text{ constant}} \left[u^{1'}((1-\gamma)U^{1'} + \gamma U^{N'} - U^{0'}) + \frac{\partial R'}{\partial \tau'}\frac{\partial \tau'}{\partial d'} + \lambda' \frac{\partial u''}{\partial d'}\right] \\
0 &= u^1 d\left[(1-\gamma)U_c^1 + \gamma U_c^N\right] + \frac{\partial R}{\partial \tau}\frac{\partial \tau}{\partial b}
\end{aligned}
$$