Inequality as cholesterol:
Attempting to quantify inequality of opportunity

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The World Bank and IZA

Policy Research Talk
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Outline

1. Why measure inequality of opportunity? Motivation

2. What is (in)equality of opportunity? Definition and illustrations

3. Can inequality of opportunity be measured?
   a) A ‘canonical model’ and allocation rules
   b) One approach to measurement (among many)
   c) Empirical examples: “first generation”
   d) “Second generation” studies

4. Extensions
   a) Intergenerational mobility and economic growth
   b) Impact evaluation and algorithmic design

5. Conclusions
1a. Motivation: normative arguments

In pursuing social justice, what inequalities should you seek to eliminate?

• Is information on the distribution of outcomes enough?


1b. Motivation: evidence on preferences

1. It is now well-established that individuals value ‘fairness’, in the sense that many are prepared to give up private monetary gains to achieve what they perceive as a just allocation.
   – Offers made and rejected in ultimatum and dictator games.
   – Fehr and Gachter (2000); Fehr and Fischbacher (2003); Henrich et al. (2004)

2. There is also evidence that what is deemed to be a just allocation depends on how it is arrived at!
   – E.g. Cappelen, Sorensen and Tungodden (2010) asked Norwegian business students and alumni to propose a fair distribution of earnings from a typing exercise where “wages” ($p$) were randomly allocated, work duration ($q$) was chosen; and “productivity” ($a$) was measured. Respondents were grouped as follows:

<table>
<thead>
<tr>
<th>Preference groups</th>
<th>Responsibility sets</th>
<th>Frequency in sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strict egalitarians</td>
<td>$R^{SE} = \emptyset$</td>
<td>0.18</td>
</tr>
<tr>
<td>Choice egalitarians</td>
<td>$R^{CE} = {q}$</td>
<td>0.05</td>
</tr>
<tr>
<td>Meritocrats</td>
<td>$R^{M} = {q, a}$</td>
<td>0.47</td>
</tr>
<tr>
<td>Libertarians</td>
<td>$R^{L} = {q, a, p}$</td>
<td>0.30</td>
</tr>
</tbody>
</table>
“We know that equality of individual ability has never existed and never will, but we do insist that equality of opportunity still must be sought”
(Franklin D. Roosevelt, second inaugural address, 20 January 1937)

“The rise in inequality in the United States over the last three decades has reached the point that inequality in incomes is causing an unhealthy division in opportunities, and is a threat to our economic growth”
(Alan Krueger, Center for American Progress, 12 January 2012)

If these concepts matter for policymakers, can they be rigorously defined and measured?
2. What is equality of opportunity?
An “economic definition”

- Often expressed in terms of adherence to two central principles:
  - **Principle of compensation**: outcome differences due to factors beyond an individual’s responsibility (“circumstances”) are unfair, and are compensated.
  - **Principle of reward**: outcome differences reflecting differential reward to individual responsibility and effort are ethically legitimate, and are preserved.

- Pioneering economists who adopted and built on these ideas include:
  - Dirk van de Gaer (1993)
  - Marc Fleurbaey (1994, 2008)

- Building primarily on the Arneson / Cohen “control view” of equality of opportunity.

- Formally, equality of opportunity can be defined as a situation in which the outcome of interest ($x$) is distributed independently of (predetermined) circumstances ($C$) for which the individual ought not to be held responsible:

$$F(x|C) = F(x)$$
2. What is equality of opportunity?
   An “economic definition”

This approach “… performs for egalitarianism the considerable service of incorporating within it the most powerful idea in the arsenal of the anti-egalitarian right: the idea of choice and responsibility” (Cohen, 1989, p.993)
2. What is equality of opportunity? An “economic definition”

This approach “… performs for egalitarianism the considerable service of incorporating within it the most powerful idea in the arsenal of the anti-egalitarian right: the idea of choice and responsibility” (Cohen, 1989, p.993)

But it also expands the scope for normative judgement inherent in inequality analysis. Recall Atkinson (1970): “…this direct approach […] serves to emphasize that any measure of inequality involves judgements about social welfare” (p.257, emphasis in the original)

Beside normative views on how transfers in different parts of the distribution should be weighted against one another, I.Op. requires normative judgment also on what factors should be considered “circumstances”.
2. What is (in)equality of opportunity?

Illustrations

Source: Perry (2002): “Childhood Experience and Expression of Genetic Potential”, Brain and Mind
2. What is (in)equality of opportunity?

Illustrations

Figure 2: Opportunities are determined early.
Cognitive development for children ages three to five in Ecuador differs markedly across different family backgrounds.

Wealthiest and poorest quartiles

<table>
<thead>
<tr>
<th>Median score</th>
<th>Age in months</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>40</td>
</tr>
<tr>
<td>100</td>
<td>50</td>
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<tr>
<td>90</td>
<td>60</td>
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<tr>
<td>80</td>
<td>70</td>
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</tbody>
</table>

Wealthiest 25%

<table>
<thead>
<tr>
<th>Median score</th>
<th>Age in months</th>
</tr>
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<tbody>
<tr>
<td>100</td>
<td>40</td>
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<tr>
<td>90</td>
<td>50</td>
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<td>80</td>
<td>60</td>
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<tr>
<td>70</td>
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</table>

Poorest 25th%

<table>
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<th>Median score</th>
<th>Age in months</th>
</tr>
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<tbody>
<tr>
<td>80</td>
<td>40</td>
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<tr>
<td>70</td>
<td>50</td>
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<tr>
<td>60</td>
<td>60</td>
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<tr>
<td>50</td>
<td>70</td>
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</table>

Maternal education

<table>
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<th>Median score</th>
<th>Age in months</th>
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<td>110</td>
<td>40</td>
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<td>100</td>
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<td>90</td>
<td>60</td>
</tr>
<tr>
<td>80</td>
<td>70</td>
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</tbody>
</table>

12 or more years

<table>
<thead>
<tr>
<th>Median score</th>
<th>Age in months</th>
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</thead>
<tbody>
<tr>
<td>80</td>
<td>40</td>
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<tr>
<td>70</td>
<td>50</td>
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<tr>
<td>60</td>
<td>60</td>
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<tr>
<td>50</td>
<td>70</td>
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</tbody>
</table>

0–5 years

<table>
<thead>
<tr>
<th>Median score</th>
<th>Age in months</th>
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<tbody>
<tr>
<td>80</td>
<td>40</td>
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<td>70</td>
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<td>60</td>
<td>60</td>
</tr>
<tr>
<td>50</td>
<td>70</td>
</tr>
</tbody>
</table>

Source: Paxson and Schady (2005a).
Note: Median values of the test of vocabulary recognition (TVIP) score (a measure of vocabulary recognition in Spanish, standardized against an international norm) are plotted against the child’s age in months. The medians by exact month of age were smoothed by estimating fan regressions of the median score on age (in months), using a bandwidth of 3.

Source: Paxson and Schady, JHR, 2007
2. What is (in)equality of opportunity? 
Illustrations

Distribution of p.c.h. consumption conditional on mother’s education, in five LAC countries

Source: Ferreira and Gignoux, 2011
3. Can inequality of opportunity be measured?

A simple “canonical” model

- Let each and every individual be fully characterized by the triple \((x, C, e)\).
- Let all elements of the vector \(C\), as well as \(e\), be discrete.
- Let \(x_{ij} = g(C_i, e_j)\)
- Let a *type* consist of all individuals with identical circumstances
- Let a *tranch* consist of all individuals with identical effort levels
- Let there be \(n\) types and \(m\) tranches
- Then the population can be represented by the \(n \times m\) matrix \([X_{ij}]\) below.
- To \([X_{ij}]\), let there be associated another \(n \times m\) matrix \([P_{ij}]\), whose elements \(p_{ij}\) denote the proportion of the total population with circumstances \(C_i\) and effort level \(e_j\).

Discussion draws on Ferreira and Peragine (2016)
3a. A ‘canonical model’

### Table 1

<table>
<thead>
<tr>
<th></th>
<th>$e_1$</th>
<th>$e_2$</th>
<th>$e_3$</th>
<th>…</th>
<th>$e_m$</th>
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</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>$x_{11}$</td>
<td>$x_{12}$</td>
<td>$x_{13}$</td>
<td>…</td>
<td>$x_{1m}$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$x_{21}$</td>
<td>$x_{22}$</td>
<td>$x_{23}$</td>
<td>…</td>
<td>$x_{2m}$</td>
</tr>
<tr>
<td>$C_3$</td>
<td>$x_{31}$</td>
<td>$x_{32}$</td>
<td>$x_{33}$</td>
<td>…</td>
<td>$x_{3m}$</td>
</tr>
<tr>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
<tr>
<td>$C_n$</td>
<td>$x_{n1}$</td>
<td>$x_{n2}$</td>
<td>$x_{n3}$</td>
<td>…</td>
<td>$x_{nm}$</td>
</tr>
</tbody>
</table>

Discussion draws on Ferreira and Peragine (2016)
3a. A ‘canonical model’

Discussion draws on Ferreira and Peragine (2016)

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<th>...</th>
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</tr>
</thead>
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<td>$x_{13}$</td>
<td>...</td>
<td>$x_{1m}$</td>
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<td>...</td>
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<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$C_n$</td>
<td>$x_{n1}$</td>
<td>$x_{n2}$</td>
<td>$x_{n3}$</td>
<td>...</td>
<td>$x_{nm}$</td>
</tr>
</tbody>
</table>
3b. Scalar measurement

In essence, the measurement of inequality of opportunity can be thought of as a two-step procedure:

• **first**, the actual distribution \([X_{ij}]\) is transformed into a counterfactual distribution \([\tilde{X}_{ij}]\) that reflects *only and fully* the unfair inequality in \([X_{ij}]\), while all the fair inequality is removed.

• In the **second** step, a measure of inequality – satisfying the usual axioms - is applied to \([\tilde{X}_{ij}]\).

• The first step can be taken in many different ways, depending on the specific form of the compensation (e.g. ex-ante vs. ex-post) and reward principles one chooses to adopt.
  
  – In what follows I focus on a single example, an *ex-ante approach known as between-types inequality.*

Discussion draws on Ferreira and Peragine (2016)
3b. Scalar measurement

The between-types approach

*Between types (\( \bar{x}_{BT} \)): For all \( j \in \{1, \ldots, m\} \) and for all \( i \in \{1, \ldots, n\} \), \( \bar{x}_{ij} = \mu_i \).*

**Table 2: Between-types inequality \((n=m=3)\)**

<table>
<thead>
<tr>
<th></th>
<th>e1</th>
<th>e2</th>
<th>e3</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>( \mu_1 )</td>
<td>( \mu_1 )</td>
<td>( \mu_1 )</td>
</tr>
<tr>
<td>C2</td>
<td>( \mu_2 )</td>
<td>( \mu_2 )</td>
<td>( \mu_2 )</td>
</tr>
<tr>
<td>C3</td>
<td>( \mu_3 )</td>
<td>( \mu_3 )</td>
<td>( \mu_3 )</td>
</tr>
</tbody>
</table>

Draws on the *min of means* approach.

Discussion draws on Ferreira and Peragine (2016)
3b. Scalar measurement
The between-types approach

- This approach - \( I(\tilde{x}_{BT}) \) - has been applied sufficiently widely so as to permit international comparisons.

- There are two versions of this index, both of which are subject to a downward bias due to the partial observability of circumstances. Using a slightly different notation:
  
  - IOL: \( \theta_a = I(\tilde{x}_{BT}) \)
  
  - IOR: \( \theta_r = \frac{I(\tilde{x}_{BT})}{I(x)} = \frac{I(\mu_i)}{I(x_{ij})} \)

- These indices can be computed non-parametrically (Checchi and Peragine, 2010), but precision weakens and biases arise when \( n \) is large relative to the sample size.

- Common parametric approximation:
  
  \[ y = C\psi + \varepsilon \quad \tilde{\mu}_i = C_i\hat{\psi} \quad IO_p = \frac{I(\tilde{\mu}_i)}{I(y)} \]
### 3c. An application to Latin America

#### TABLE 1

**Household Survey Names, Dates, and Sample Sizes**

<table>
<thead>
<tr>
<th></th>
<th>Brazil</th>
<th>Colombia</th>
<th>Ecuador</th>
<th>Guatemala</th>
<th>Panama</th>
<th>Peru</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Survey</strong></td>
<td>PNAD</td>
<td>ECV</td>
<td>ECV</td>
<td>ENCOVI</td>
<td>ENV</td>
<td>ENAHO</td>
</tr>
<tr>
<td><strong>Sample of 30 to 49 year-olds</strong></td>
<td>85,692</td>
<td>22,517</td>
<td>12,650</td>
<td>6,956</td>
<td>6,339</td>
<td>17,030</td>
</tr>
<tr>
<td><strong>Sample of heads and spouses, aged 30 to 49 years</strong></td>
<td>73,847</td>
<td>18,069</td>
<td>10,719</td>
<td>6,067</td>
<td>5,105</td>
<td>13,947</td>
</tr>
<tr>
<td><strong>Of those, observations with income/consumption and circumstances (share of original sample)</strong></td>
<td>70,521</td>
<td>17,979</td>
<td>10,719</td>
<td>5,988</td>
<td>4,556</td>
<td>13,621</td>
</tr>
</tbody>
</table>

Source: Ferreira and Gignoux, 2011
3c. An application to Latin America

<table>
<thead>
<tr>
<th>Ethnicity</th>
<th>Brazil</th>
<th>Colombia</th>
<th>Ecuador</th>
<th>Guatemala</th>
<th>Panama</th>
<th>Peru</th>
</tr>
</thead>
<tbody>
<tr>
<td>Category 1</td>
<td>Self-reported white ethnicity</td>
<td>Other</td>
<td>Self-reported ethnicity: white, mixed blood (&quot;mestizo&quot;) or other</td>
<td>European maternal language</td>
<td>Other</td>
<td>European maternal language</td>
</tr>
<tr>
<td>Category 2</td>
<td>Self-reported black (&quot;negro&quot;) and mixed blood (&quot;pardo&quot;) ethnicity</td>
<td>Self-reported minority ethnicity: &quot;indígena, gitan, archipiélago o palenquero&quot;</td>
<td>Self-reported ethnicity: indigenous, black (&quot;negro&quot; or &quot;mulato&quot;)</td>
<td>Indigenous maternal language</td>
<td>Speaks indigenous language</td>
<td>Indigenous maternal language</td>
</tr>
</tbody>
</table>

| Father’s occupation           |                                     |                     |                                 |                                      |                                 |                                    |
| Category 1                    | Agricultural worker                 | Missing             | Agricultural worker or domestic worker | Agricultural worker                  | Agricultural worker              | Missing                            |
| Category 2                    | Other                               |                      | Other                           |                                      | Other                           |                                    |

| Mother’s and father’s education|                                     |                     |                                 |                                      |                                 |                                    |
| Category 1                    | None or unknown                     | None or unknown     | None or unknown                 | None or unknown                      | None or unknown                 |                                    |
| Category 2                    | Completed grade 1 to 4              | Primary incomplete  | Primary complete or more        | Primary incomplete                   | Primary complete or more        |                                    |
| Category 3                    | Completed grade 5 or more           | Primary complete or more | Secondary or more                |                                      | Secondary or more               |                                    |

| Birth region                  |                                     |                     |                                 |                                      |                                 |                                    |
| Category 1                    | Sao Paulo and Federal district      | Departments at the periphery | Sierra and Amazonia provinces | Guatemala City, Northeast departments and El Petén | Cities and intermediate urban centers | Inland non-southern departments |
| Category 2                    | South East, Center-West, and South  | Central departments(a) | Costa and Insular provinces    | North and Northwest departments      | Other urban centers             | Southern and other coastal departments |
| Category 3                    | North-East, North or missing        | Bogota, San Andres, and Providencia islands and foreign country | Pichincha province (with Quito) and Azuay province | Southeast, Southwest, and Center departments | Rural areas                     | Arequipa, Callao, and Lima |

Note: Central departments are Boyaca, Caklas, Caqueta, Cundinamarca, Huila, Meta, Norte de Santander, Quindio, Risarakda, Santander, Tolima, and Valle del Cauca.

Source: Ferreira and Gignoux, 2011
3c. An application to Latin America

### TABLE 6

**Scalar Indices of Inequality of Opportunity**

<table>
<thead>
<tr>
<th></th>
<th>Brazil</th>
<th>Colombia</th>
<th>Ecuador</th>
<th>Guatemala</th>
<th>Panama</th>
<th>Peru</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Household income (per capita)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total inequality ($E_0$)</td>
<td>0.692</td>
<td>0.572</td>
<td>0.580</td>
<td>0.593</td>
<td>0.630</td>
<td>0.557</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.033)</td>
<td>(0.028)</td>
<td>(0.036)</td>
<td>(0.029)</td>
<td>(0.022)</td>
</tr>
<tr>
<td>Non-parametric estimates</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IOL</td>
<td>0.227</td>
<td>0.144</td>
<td>0.164</td>
<td>0.213</td>
<td>0.213</td>
<td>0.163</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.023)</td>
<td>(0.022)</td>
<td>(0.031)</td>
<td>(0.024)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>IOR</td>
<td>0.329</td>
<td>0.252</td>
<td>0.283</td>
<td>0.359</td>
<td>0.338</td>
<td>0.293</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.026)</td>
<td>(0.023)</td>
<td>(0.030)</td>
<td>(0.026)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>Parametric estimates</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IOL</td>
<td>0.223</td>
<td>0.133</td>
<td>0.150</td>
<td>0.199</td>
<td>0.190</td>
<td>0.156</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.019)</td>
<td>(0.020)</td>
<td>(0.028)</td>
<td>(0.023)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>IOR</td>
<td>0.322</td>
<td>0.232</td>
<td>0.259</td>
<td>0.335</td>
<td>0.301</td>
<td>0.279</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.023)</td>
<td>(0.023)</td>
<td>(0.030)</td>
<td>(0.028)</td>
<td>(0.018)</td>
</tr>
<tr>
<td><strong>Panel B: Household consumption expenditures (per capita)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total inequality ($E_0$)</td>
<td>0.462</td>
<td>0.359</td>
<td>0.415</td>
<td>0.381</td>
<td>0.351</td>
<td></td>
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<tr>
<td></td>
<td>(0.018)</td>
<td>(0.015)</td>
<td>(0.025)</td>
<td>(0.018)</td>
<td>(0.013)</td>
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<td>IOL</td>
<td>0.123</td>
<td>0.124</td>
<td>0.221</td>
<td>0.156</td>
<td>0.123</td>
<td></td>
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<tr>
<td></td>
<td>(0.015)</td>
<td>(0.013)</td>
<td>(0.024)</td>
<td>(0.016)</td>
<td>(0.010)</td>
<td></td>
</tr>
<tr>
<td>IOR</td>
<td>0.265</td>
<td>0.346</td>
<td>0.532</td>
<td>0.409</td>
<td>0.351</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
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<td>(0.023)</td>
<td>(0.025)</td>
<td>(0.018)</td>
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<tr>
<td>Parametric estimates</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IOL</td>
<td>0.114</td>
<td>0.117</td>
<td>0.213</td>
<td>0.144</td>
<td>0.119</td>
<td></td>
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<tr>
<td></td>
<td>(0.014)</td>
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<td>0.247</td>
<td>0.326</td>
<td>0.514</td>
<td>0.377</td>
<td>0.339</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.022)</td>
<td>(0.022)</td>
<td>(0.026)</td>
<td>(0.017)</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** Sample: household heads and spouses, aged 30–49, with positive income and information on a set of circumstances; bootstrap standard errors (taking into account stratification and clustering) in parentheses; father’s occupation missing for Colombia and Peru.
3c. An application to Latin America

1. In Latin America, inequality of economic opportunity:
   • ranges from 23% to 35% for income per capita.
   • ranges from 24% to 50% for consumption per capita.

Source: Ferreira and Gignoux, 2011
3c. A broader international comparison
Drawing on 8 “first-generation” studies covering 51 countries; x = income

Total inequality GE(0) and IOp (absolute)

ordered according to IOL

IOR: 3% Norway - 40% Malawi

Source: Brunori, Ferreira, Peragine (2013) – with updates
3c. Recent evolution of I.Op. in the US
An example from the Equalchances database
3d. “Second generation” studies

1. ‘Second-generation’ between-types approach: looking for upper-bound estimates (Niehues and Peichl, SCW 2014)

• Two-stage estimator using panel data:
  
i. Estimate \( \ln w_{it} = \beta E_{it} + c_i + u_t + \varepsilon_{it} \)

  ii. Back in cross-section, estimate \( \ln w_{is} = \phi \hat{c}_i + v_{it} \)

  Construct \( \tilde{\mu}^{UB} = \exp(\phi \hat{c}_i + \sigma^2 / 2) \)

  – Application to Germany (SOEP) and the US (PSID), for both current and permanent incomes
3d. “Second generation” studies

1. ‘Second-generation’ between-types approach: looking for upper-bound estimates (Niehues and Peichl, SCW 2014)

Source: Own calculations based on SOEP and PSID. The two graphs on the top illustrate IOp shares in annual incomes (2009 for Germany, 2007 for the US); the graph at the bottom IOp shares in permanent incomes.
3d. “Second generation” studies

2. ‘Second-generation’ between-types approach: **enlarging the circumstance set through admitting an “age of consent”** (Hufe, Peichl, Roemer and Ungerer; SCW 2017)

- Use National Longitudinal Survey of Youth (NLSY -79) for the US and British Cohort Study (BCS – 70) for the UK

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Circumstance Set</th>
<th>Circumstance Var.</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>Base</td>
<td>Sex, Country of Birth, Ethnic Affiliation, Cohort, Age, Academic Achievement Mother, Occupation Code Mother, Rural/Urban, Height (16), Family Income</td>
</tr>
<tr>
<td></td>
<td>Ability</td>
<td>PIAT Math, PIAT Reading</td>
</tr>
<tr>
<td></td>
<td>Behavioral Problems</td>
<td>Behavioral Problems Index (BPI)</td>
</tr>
<tr>
<td></td>
<td>Child-Parent Relationship</td>
<td>Play/Schoolwork w/ Parents, Perceived Quantity of Time w/ Parents, Parents Split, Parental Income</td>
</tr>
<tr>
<td></td>
<td>Health-Related Behavior</td>
<td>Smoking Habits Mother, Drinking Habits Mother, Health Restrictions Child</td>
</tr>
<tr>
<td></td>
<td>Survey Specifics</td>
<td>Specific to NLSY79 and BCS70. See text for more information.</td>
</tr>
</tbody>
</table>
3d. “Second generation” studies

2. Hufe, Peichl, Roemer and Ungerer (2017) find that the lower-bound IOR can be as high as 45% in the US and 31% in the UK when using this extended circumstance set.

Figure 2: IOp with varying circumstance sets (NLSY79), comparable sample, average income
3d. “Second generation” studies

3. An alternative approach to “let the data choose the model specification” is proposed by Brunori, Hufe and Mahler (2018), using conditional inference trees and forests.

– A conditional inference tree consists of a set of terminal nodes (leaves) obtained by recursive binary splitting, as follows.

– Given a set of circumstance variables and categories, the algorithm splits the sample in all possible partitions [C], and computes the p-value for the null hypothesis that the statistic of interest (e.g. the mean) in the two sub-samples is identical.

– [C]* is chosen as \([C]^* = \{[C] : \text{argmin} \, p_{adj}^{[C]}\}\), where the adjustment is a Bonferroni correction (for multiple hypothesis testing).

– A critical significance level \(\alpha\) can be chosen so that if \(p_{adj}^{[C]^*} > \alpha\), one exit the algorithms, and otherwise [C]* is chosen as splitting variable.

– Repeat the algorithm for each node (sub-sample), until one has exited everywhere.

– A conditional inference forest is basically a set of trees estimated on random subsamples of the original data, in each case using a different subset of circumstance variables. The size of the subsets of circumstances is chosen by minimizing the “out-of-the-bag” MSE.
3d. “Second generation” studies

3. Although forests outperform trees in terms of out-of-sample prediction, trees can be visually informative of the ‘structure’ of inequality of opportunity in different countries.

Figure 3: Opportunity Tree: Sweden

Note: Opportunity tree for Sweden. White rectangular boxes indicate terminal nodes. The first number inside the rectangular boxes indicates the share of the population belonging to this group, while the second number indicates the predicted income.
3d. “Second generation” studies

3. Although forests outperform trees in terms of out-of-sample prediction, trees can be visually informative of the ‘structure’ of inequality of opportunity in different countries.

- IGM:
  \[ y_c = \beta y_p + \varepsilon \]
  \[ R^2 \]

- IOp:
  \[ y = C \psi + \varepsilon \]
  \[ IOR = \frac{I(\tilde{\mu}_i)}{I(y)} \]

- Inequality of opportunity (at least in the ex-ante approach) is very close to origin-independent measures of IGM.
  - Recent literature on the US (building on Chetty, Hendren, Kline and Saez, QJE 2014) almost seems to equate the two concepts.
  - Difference: more circumstances
    - In “steady state” \( Var y_c = Var y_p \), Chetty et al.’s IGE of 0.45 implies \( R^2 = 0.2 \)
  - Omitted variables: I.Op. is explicitly not a causal estimate for any individual circumstance.

The Great Gatsby Curve

Source: Corak (2012)

Inequalities of outcome and opportunity: strong correlation

- Beyond measurement: once quantified, the variable can be used in causal analysis.

- A possible answer to the lengthy and inconclusive literature on inequality and growth.
  - Marrero and Rodriguez (2013) for the US.
  - Ferreira, Lakner, Lugo, Özler (2018) – cross country

- Key challenge: comparable data on advantages and circumstances

- A growing number of studies evaluates policies and programs in terms of their impacts not only on means, or on the outcome distribution, but also on the distribution of opportunities (e.g. on $\tilde{X}_{ij}$).

  i. Impact of Progresa on family types defined by indigenous status and parental education (van de Gaer et al., *WBER* 2014)

  ii. Impact of Progresa on ECD indicators (e.g. PPVT, child behavior checklist, etc.) for children from different family types (Figueroa, *Soc. Sci. & Med.* 2014)

  iii. Impact of a child care reform in Norway, using gap curves and QTEs. (Andreoli et al., *REStat*, 2018)

Algorithmic fairness, discrimination, and equality of opportunity

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Source: Michele Loi’s lecture at ISWT14, Canazei, January 8, 2019

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The equality of opportunity idea.

1. Moritz Hardt


Equal odds for binary classifiers:

\[
\Pr\left(\hat{Y} = 1 \mid A = 0, Y = y\right) = \Pr\left(\hat{Y} = 1 \mid A = 1, Y = y\right), \quad y \in \{0, 1\}
\]

Prediction and protected variable (A) are independent conditional on Y (actual label)
5. Conclusions

• There is reason to think of IOp as the “active ingredient of inequality”, both intrinsically and instrumentally.
  – Arguably it focuses on the inequalities that matter most.

• Inequality of opportunity can be measured rigorously, but:
  – Not precisely: lower and upper bounds, because of partial observability of circumstances.
  – The broader information requirements place considerable demands on data (particularly on circumstance variables)
  – Contingency on normative judgments (Atkinson, 1970) is greatly amplified: Not only can different indices be used to measure inequality on $X_{ij}$, but the matrix itself can be constructed in different ways.

• Current empirical estimates range from 3% of total inequality to lower bounds as high as 51% (in Guatemala), and upper bounds above 60% (in the US and Germany!)

• Increasingly used in empirical analysis, and apparently also in the design of machine learning tools used both by governments and companies
5. Conclusions

A number of challenges remain...

1. Conceptually:
   - Clashes between different versions of the compensation and reward principles (largely absent from our discussion today)
   - The materialist causal thesis and the “incompatibilist” view: “There is no such thing as free will; hence all inequality is I.Op.”
   - Requires a discussion of what society chooses to classify as circumstances, particularly as data on (epi)genetics become more widely available.
     - Ways forward: ‘Bergen experiments’? Rawlsian ‘Reflective equilibrium’?

2. Empirically, because of data limitations:
   - Partial observability of circumstances (downward bias)
   - Sample size limits and sampling variation (upward bias)

But progress is being made on many fronts!
Thank you!
Appendix: A ‘canonical model’

When effort is continuous, n=3
Appendix: Allocation rules

van de Gaer’s “min of means”

Roemer’s “mean of mins”

Conditional equality

Egalitarian equivalence
5. **Extensions: Development objectives**

- What is the policy objective for opportunity egalitarians?

\[
\max_{\phi \in \Phi} \min_i \int_t^\infty e^{\delta(t-s)} \mu_{i,s} \, ds
\]

\[
s.t. \ x_{ij,s} \geq z_s, \forall i, j, s
\]

- The choice of policies from a feasible set so as to maximize the future stream of ‘advantage’ for the most disadvantaged type, subject to a no-deprivation constraint and to a policy acceptability constraint.

Source: Bourguignon, Ferreira and Walton, JEI 2007.
5. **Extensions: Development objectives**

- ‘Deconstructing’ the equitable development policy problem:

\[
\max_{\phi \in \Phi} \min_i \int_t^\infty e^{\delta(t-s)} \mu_{i,s} \, ds
\]

subject to

\[
x_{ij,s} \geq z_s, \quad \forall i, j, s
\]

“Rawlsian” criterion. All weight on the least advantaged.

“Growth matters”

Permissible Policy Set: Technical feasibility and social acceptability

Poverty eradication as a ‘constraint’.

Source: Bourguignon, Ferreira and Walton, JEI 2007.
3d. “Second generation” studies

Brunori, Peragine and Serlenga (2018) note that sampling variation can produce an upward bias in IOp estimates when cell partition is too fine for a given sample size (or regression model is overfitted).

They propose choosing the specification $\hat{g}(C)$ that minimizes the mean squared error of out-of-sample predictions:

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{g}(C_i))^2$$

Which is decomposable as follows:

$$E \left( y_0 - \hat{g}(C_0) \right)^2 = Var(\hat{g}(C_0)) + \left[ Bias (\hat{g}(C_0)) \right]^2 + Var(u)$$

- Captures the upward bias from sampling variation
- Captures the downward bias from misspecification
3d. “Second generation” studies

3. The procedure uses $k$-fold cross-validation. The average MSE for the $k$ test samples is computed for each model specification, and the specification with the lowest MSE is chosen.

![Graph showing IOp in 31 European countries under different model specifications. The Figure shows each country’s IOp measure obtained with the three alternative methods: (i) the linear, most parsimonious case (linear), (ii) the fully interacted model (full); (iii) the best model selected (best). Countries are ordered according to the IOp level based on the best model specification with 95% confidence intervals. Table 2 in the Appendix contains IOp estimates and relative bootstrapped standard errors based on 500 replications for the three alternative model specifications. Source: EU-SILC, 2011.](image_url)