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The evaluation of the approaches for the linking of the regions at the aggregated levels

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The evaluation of the approaches for the linking of the regions at the aggregated levels

The ICP is carried out from the 1980 Round in accordance with the Regional principle. This is, first of all, due to the situation that some “ICP “Regions” are in the fact the international organizations (EU, OECD, CIS, etc.) which need the regional PPP results (usually without the impact of non-members) for the internal purposes and, secondly, due to the fact that the Regions or sub-Regions comprised usually more or less homogeneous sets of the countries. The last circumstances allow to produce shorter and more comparable item lists, in effect, more reliable regional results.

In accordance with the regional approach, the regional results are calculated firstly and, secondly, these regional results are linked in such way that the within-regional results are unchanged. There are several potential approaches for the linking of the regions. The ICP 2005 used for the 1st time the Ring comparison and the between-regional PPPs at the BH as well as aggregated levels (the between-regional PPPs were proposed by E.Diewert – see Diewert, 2004), to link the Regions in a Worldwide comparison and to keep simultaneously the within-regional results as unchanged.

The respective proposals / recommendations to the ICP Global Office on the linking in the ICP 2011 should be elaborated on the basis of the analysis of the experience in the ICP 2005, different approaches should be tested and, in effect, the linking of the ICP regional comparisons can be optimized. The linking at the BH level was analysed in a former notice – see Sergeev (2009b). The main aim of the present note is to evaluate the ICP 2005 approach used for the between-regional aggregation, to propose some modifications, to eliminate the drawbacks of the original approach, and to evaluate other possible aggregation methods.

I. EKS method for the calculation of between-Regional aggregated PPPs used in the ICP 2005 (Diewert 2 Approach) and proposed modifications

The between-regional PPPs for the aggregation were proposed by E.Diewert (Diewert, 2004 and 2008). This approach was described in the Ch.15 of the ICP Manual and was used in the official ICP 2005. The Diewert’s concept allows to keep the intra-regional PPPs as fixed figures and simultaneously to obtain the between-regional PPPs in a compact and elegant form.

The concept of the between-regional aggregated PPPs introduced by E.Diewert means that the regions but not the individual countries participate in the calculations (each region as a “super-country” = a combination of respective countries), i.e. input data of the countries should be aggregated within regions. It means the world comparison should be carried out principally in a symmetrical way "one Region - one set of input data" (a set of inter-regional BH-PPP and a

set of BH-weights for each Region). The main features of the Diewert's approach (as they were described in Diewert 2004 and 2008, and in the ICP manual) are the following:

- To use the between-regional BH-PPPs "Regional numeraire / ICP numeraire" from the Ring comparison
- To calculate the regional BH expenditure in Regional numeraires with the intra-regional BH-PPPs as

Sum(BH expenditure in national currency / BH-PPP "National currency/ Regional numeraire")

- To combine the between-regional BH-PPPs with the regional expenditure in Regional numeraires and to run the EKS aggregation procedure¹ for the Regions, to obtain between-regional aggregated PPPs.

These PPPs "Regional numeraire / ICP numeraire" are the conversion factors that permit the PPPs in each region to be denominated in a common ICP currency:

PPP "Country X / ICP numeraire" =

PPP "Country X/Regional numeraire" * PPP "Regional numeraire/ICP numeraire"

(from Regional comparison)

(from ICP comparison)

Fixity of respective intra-regional PPPs is done automatically by this procedure.

Generally the Diewert's regional approach seems to be straightforward. Nevertheless this original Diewert's approach used in the official ICP 2005 has two drawbacks:

1) First of all, original Diewert approach is not invariant relatively the choice of the Regional numeraires

2) Secondly, the countries within the Regions are treated in a non-symmetrical way in the Global comparison. This was not in the spirit of the EKS method officially adopted for the ICP 2005.

I.1 Problem with non-invariance

E.Diewert indicated (see Diewert, 2008, p.16): "... *P.Hill (2007e) shows that the overall procedure does not depend on the choice of numeraire countries, either within regions or between regions; i.e., the relative country parities will be the same no matter what the choices are for the numeraire countries.*"

However the Ch.15 of the ICP manual does not contain in reality the proof. There is only a general consideration on Page 3:

¹ The EKS method was used in the ICP 2005 but, the Diewert approach can be applied in each aggregation procedure (G-K, Ikle, etc.).

The results are also invariant to the choice of reference country and numeraire currency for each region. For example, consider the GDP PPP between two countries located in two different regions such as country t in region B and country s in region A . The GDP PPP for t on s , denoted by $GDPPPP^{s,t}$, is the following chain PPP:

$$(1) \text{ } GDPPPP^{s,t} = GDPPPP^{s,A1} \cdot GDPPPP^{A1,B1} \cdot GDPPPP^{B1,t}$$

□ where $GDPPPP^{s,A1}$ is the within-region GDP PPP for country s on the regional reference country $A1$ as calculated by region A .

□ $GDPPPP^{A1,B1}$ is the GDP PPP for region B on region A expressed in terms of the currencies of the two regional reference countries $A1$ and $B1$ as calculated at the global level.

□ $GDPPPP^{B1,t}$ is the within-region PPP for country t on the reference country $B1$ as calculated by region B .

At the 1st look, no doubts about the invariance of the approach should occur. However it seems that the E.Diewert and the ICP Manual considered the invariance relatively regional numeraires in the Ring comparison only. However the problem is not the choice of a numeraire in the Ring (the invariance is really guaranteed here). The actual problem is the choice of Regional numeraires for the calculations of the Regional Totals. **When one calculates the Regional GDP Total in a regional numeraire (a reference country) then the choice of the regional numeraires changes the structure of Regional Total GDP.** Speaking roughly, the use of a country as numeraire means that the regional GDP evaluated by price structure of this country.² So, **in reality, the original method proposed by Diewert is not invariant relatively the regional numeraires.**

The author of this notice demonstrated the invariance of the original Diewert approach on the Dikhanov's example which was presented during the ICP meeting of the Regional coordinators (Feb'07). Dikhanov's example contained 3 Regions (OECD with US\$ as numeraire, Asia with HKG\$ as numeraire, Africa with Botswana as numeraire). The choice of other regional numeraires (LUX - for OECD, BGD - for Asia, SLE - for Africa) led to the changes in the inter-regional aggregated results, i.e. they were not invariant.

Similar simulations were made with the example from the 1st version of respective Chapter of ICP Manual concerning the World aggregation (Chapter 13) - see Tables 1 and 2 below.

² In his recent paper (Jan 2010) E.Diewert proposed to use market exchange rates instead of intra-regional BH-PPPs to calculate regional expenditure. This proposal brings invariance because structures of nominal regional expenditure do not depend on any regional numeraire (the same XRs are used for all BHs) but simultaneously brings the XRs in the play which should not have any impact on the PPP results in accordance with the main ICP aim.

Table 1

ICP Handbook, former Chapter 13 (numeraire countries = A1, B1, C1)												
Expenditures in Regional numeraires												
BH	Region A					Region B				Region C		
	Countries				Total A	Countries			Total B	Countries		Total C
	1	2	3	4		1	2	3		1	2	
1	2000.0	100.0	200.0	80.0	2380.0	380.0	500.0	7500.0	8380.0	4500.0	450.0	4950.0
2	500.0	20.0	40.0	20.0	580.0	70.0	103.6	1440.0	1613.6	1200.0	95.7	1295.7
3	800.0	70.0	70.0	40.0	980.0	180.0	225.0	2100.0	2505.0	1500.0	300.0	1800.0
4	300.0	60.0	35.0	30.0	425.0	70.0	94.4	714.3	878.7	800.0	120.0	920.0
1-4 = C	3600.0	250.0	345.0	170.0	4365.0	700.0	923.0	11754.3	13377.3	8000.0	965.7	8965.7
5	700.0	31.0	50.0	35.0	816.0	180.0	179.2	3600.0	3959.2	1500.0	150.0	1650.0
6	700.0	40.0	53.8	40.0	833.8	130.0	126.7	2560.0	2816.7	2400.0	111.4	2511.4
5+6 = I	1400.0	71.0	103.8	75.0	1649.8	310.0	305.8	6160.0	6775.8	3900.0	261.4	4161.4
Total = GDP	5000.0	321.0	448.8	245.0	6014.8	1010.0	1228.8	17914.3	20153.1	11900.0	1227.1	13127.1
		EKS Inter_Regional PPP			1				2.62296			11.12929
		Regional Real Value			6014.8				7889.3			1179.5
		Volume index (A=1)			1				1.277			0.196
Shares of expenditures in regional numeraires (%)												
BH	Region A					Region B				Region C		
	Countries				Total A	Countries			Total B	Countries		Total C
	1	2	3	4		1	2	3		1	2	
1	40.0	31.2	44.6	32.7	39.6	37.6	40.7	41.9	41.6	37.8	36.7	37.7
2	10.0	6.2	8.9	8.2	9.6	6.9	8.4	8.0	8.0	10.1	7.8	9.9
3	16.0	21.8	15.6	16.3	16.3	17.8	18.3	11.7	12.4	12.6	24.4	13.7
4	6.0	18.7	7.8	12.2	7.1	6.9	7.7	4.0	4.4	6.7	9.8	7.0
1-4 = C	72.0	77.9	76.9	69.4	72.6	69.3	75.1	65.6	66.4	67.2	78.7	68.3
5	14.0	9.7	11.1	14.3	13.6	17.8	14.6	20.1	19.6	12.6	12.2	12.6
6	14.0	12.5	12.0	16.3	13.9	12.9	10.3	14.3	14.0	20.2	9.1	19.1
5+6 = I	28.0	22.1	23.1	30.6	27.4	30.7	24.9	34.4	33.6	32.8	21.3	31.7
Total = GDP	100	100	100	100	100	100	100	100	100	100	100	100
Regional expenditure shares and inter-regional BH-PPPs												
BH	Region A		Region B		Region C							
	PPP	Sh.-Tot	PPP	Sh.-Tot	PPP	Sh.-Tot						
1	1	39.6	2.5	41.6	9.0	37.7						
2	1	9.6	1.6	8.0	12.0	9.9						
3	1	16.3	3.0	12.4	15.0	13.7						
4	1	7.1	2.4	4.4	8.0	7.0						
5	1	13.6	3.0	19.6	15.0	12.6						
6	1	13.9	3.2	14.0	12.0	19.1						
Total-GDP	xxx	100	xxx	100	xxx	100						

Table 2

ICP Handbook, former Chapter 13 (numeraire countries = A2, B2, C2)												
Expenditures in Regional numeraires												
BH	Region A					Region B				Region C		
	Countries				Total A	Countries			Total B	Countries		Total C
	1	2	3	4		1	2	3		1	2	
1	1200.0	60.0	120.0	48.0	1428.0	760.0	1000.0	15000.0	16760.0	2250.0	225.0	2475.0
2	450.0	18.0	36.0	18.0	522.0	196.0	290.0	4032.0	4518.0	840.0	67.0	907.0
3	400.0	35.0	35.0	20.0	490.0	252.0	315.0	2940.0	3507.0	600.0	120.0	720.0
4	120.0	24.0	14.0	12.0	170.0	126.0	170.0	1285.7	1581.7	240.0	36.0	276.0
1-4 = C	2170.0	137.0	205.0	98.0	2610.0	1334.0	1775.0	23257.7	26366.7	3930.0	448.0	4378.0
5	700.0	31.0	50.0	35.0	816.0	432.0	430.0	8640.0	9502.0	750.0	75.0	825.0
6	560.0	32.0	43.1	32.0	667.1	195.0	190.0	3840.0	4225.0	1680.0	78.0	1758.0
5+6 = I	1260.0	63.0	93.1	67.0	1483.1	627.0	620.0	12480.0	13727.0	2430.0	153.0	2583.0
Total = GDP	3430.0	200.0	298.1	165.0	4093.1	1961.0	2395.0	35737.7	40093.7	6360.0	601.0	6961.0
		EKS Inter_Regional PPP			1				7.38033			8.60170
		Regional Real Value			4093.1				5432.5			809.3
		Volume index (A=1)			1				1.327			0.198
Shares of expenditures in regional numeraires (%)												
BH	Region A					Region B				Region C		
	Countries				Total A	Countries			Total B	Countries		Total C
	1	2	3	4		1	2	3		1	2	
1	35.0	30.0	40.3	29.1	34.9	38.8	41.8	42.0	41.8	35.4	37.4	35.6
2	13.1	9.0	12.1	10.9	12.8	10.0	12.1	11.3	11.3	13.2	11.1	13.0
3	11.7	17.5	11.7	12.1	12.0	12.9	13.2	8.2	8.7	9.4	20.0	10.3
4	3.5	12.0	4.7	7.3	4.2	6.4	7.1	3.6	3.9	3.8	6.0	4.0
1-4 = C	63.3	68.5	68.8	59.4	63.8	68.0	74.1	65.1	65.8	61.8	74.5	62.9
5	20.4	15.5	16.8	21.2	19.9	22.0	18.0	24.2	23.7	11.8	12.5	11.9
6	16.3	16.0	14.5	19.4	16.3	9.9	7.9	10.7	10.5	26.4	13.0	25.3
5+6 = I	36.7	31.5	31.2	40.6	36.2	32.0	25.9	34.9	34.2	38.2	25.5	37.1
Total = GDP	100	100	100	100	100	100	100	100	100	100	100	100
Regional expenditure shares and inter-regional BH-PPPs												
BH	Region A		Region B		Region C							
	PPP	Sh.-Tot	PPP	Sh.-Tot	PPP	Sh.-Tot						
1	1	34.9	8.3	41.8	7.5	35.6						
2	1	12.8	5.0	11.3	9.3	13.0						
3	1	12.0	8.4	8.7	12.0	10.3						
4	1	4.2	10.8	3.9	6.0	4.0						
5	1	19.9	7.2	23.7	7.5	11.9						
6	1	16.3	6.0	10.5	10.5	25.3						
Total-GDP	xxx	100	xxx	100	xxx	100						

The between-regional Volume indices with different regional numeraires (A1, B1, C1 in the original version and A2, B2, C2 - in experimental version) are different:

Reg. A = 1; Reg. B = 1.277; Reg. C = 0.196 (numeraires: A1, B1, C1)

Reg. A =1; Reg. B = 1.327; Reg. C = 0.198 (numeraires: A2, B2, C2)

To obtain the clear invariance in Diewert's approach, the author of this paper proposed (TAG meeting, Sept'07) **to operate with the average Regional numeraires** (like PPS in Eurostat comparison) instead of the regional country's numeraires (like HKD or UK Pound). This is the standardized presentation of PPPs with the base "Region = 1".³

The prices of Ring countries should be recalculated in the average Regional numeraires. The regional expenditure are calculated also in the average regional currencies instead of the use of base regional currencies for the calculation of between-regional BH-PPPs. This approach does not change the BH results but this is important for the further aggregation above the BH level and the problem with the invariance of the aggregated results will be avoided.

Y.Dikhanov proposed recently two other procedures to avoid the non-invariance of the original Diewert's approach:

- *To use an additive index [e.g. GK] to aggregate countries of a region into a super-country entity"*

It difficult to find any advantage of this approach relatively the use of Regional Average Numeraires. The use of Regional Average Numeraires is a kind of the Gerardi method, i.e. this also additive aggregation method allowed to aggregate countries of a region into super-country entity. The use of the G-K for the regional aggregation is very ambiguous. If we eliminate the G-K method from the ICP aggregation because of the Gerschenkron effect (poorer countries receive higher Volumes) then it is unclear – Why we accept this method for the regional aggregation? It is possible to use other methods of the G-K type (free of 2Geschenkron effect") like the Ikle but do we need really this additional aggregation procedure of the G-K type? The use of GM Regional numeriares keeps the Erwin's original efficient idea and "repairs" the non-invariance in a simple and straightforward way. What is the necessity to do the process more complicated without a clear advantage?

- *To use varying base countries and taking a GM of the results"*

³ See for more detail, S. Sergeev (2001c) „Presentation of results of international comparisons (Some considerations about the scaling procedures)“; Consultation on the European Comparison Programme (Geneva, 12-14 November) http://siteresources.worldbank.org/ICPINT/Resources/Selection_of_numeraire.doc

Firstly, this approach is very cumbersome. For example, 146 countries from 5 Regions participated in the ICP 2005:

ICP 2005	
Regions	No. of countries
Eurostat-	
OECD-CIS	55
Africa	48
ADB	23
LA	10
WA	10
Total no. of variants	6 072 000

$$(6\ 072\ 000 = 55 \cdot 48 \cdot 23 \cdot 10 \cdot 10)$$

It means that more than 6 mio. versions with different base-countries in the Regions had to be calculated. Of course, it is not problematic technically for modern computers. However, there is more serious problem - Would be all of these 6 mio. versions meaningful? If the GM of the results of all versions is taken at the end, then - Why not to use the Regional Average GM Numeraires from the beginning?

So, the methods proposed by Y.Dikhanov are workable. However, as it was attempted to demonstrate above, the use of Regional Average Numeraires (GM) is the simplest and, probably, the most straightforward way to keep the main Diewert's idea on the between-regional PPPs and to guarantee the regional-base invariance.

I.2 What regional expenditure weights should be used in the Diewert approach

The next problematic point of the Diewert's aggregation method is the kind of regional expenditure data (weights). The situation is ambiguous - the kind of BH-weights can depend partly on the method selected for the aggregation in the worldwide comparison:

a) If the **G-K** method is selected then the original Diewert's proposal – to use Regional expenditure in an (invariant) numeraire is unambiguously straightforward and is fully in spirit of the G-K method: aggregated regional expenditure data of in regional numeraire should be used as "super country" data of Regions.

b) If the **EKS** method is selected then there is a dilemma:

- to use **plutocratic weights** (as proposed initially by Diewert)

or

- to use **democratic weights** (regional average shares calculated from country's shares)?

The use of plutocratic weights (aggregated regional expenditure as a "super country") is not in spirit of the EKS method because the Regional expenditure would be dominated by large countries and small countries (as Luxembourg, Malta, Honkong, Singapore, etc.) would have no impact on the Regional data. The large countries will have higher impact on regional weights and this is not in accordance with the EKS principles. Additionally, it is unclear – why the countries are treated symmetrically in the Regional comparisons and non-symmetrically in the World comparison. Therefore a modification for the calculations of the "Between-regional PPPs using the EKS method", where the regions and the countries within regions are treated symmetrically is desirable.

The use of democratic weights allows to calculate the "between-regional aggregated PPPs" using the EKS method where the regions and the countries within regions are treated symmetrically. The "Between-regional aggregated EKS-PPPs" are calculated on the basis of matrix of between-regional aggregated Fishers PPPs using appropriate aggregated regional input data. To calculate the between-regional binary Fishers PPPs, between-regional BH-PPPs and the regional BH expenditure (weights) are needed. The between-regional BH-PPPs are obtained within the Ring comparison. The situation with the regional BH expenditure is less straightforward - use simply the sum of values for the countries (even in a common regional currency) would be not in accordance with the EKS principles. However it is easy to solve this problem taking into account that the Laspeyres and Paasche PPPs do not need absolute expenditure but the weights (shares) can be used instead. It is sufficient to calculate the average regional BH-shares as simple arithmetic mean from national shares (percentages) of the countries belonging to this Region (all countries - large and small - will contribute equally in this case). If region A has C(A) countries then average shares of the Region A are calculated as arithmetic mean from the values⁴:

$$w_{Aj} = \frac{e_{Aj}}{\sum_{i=1}^n e_{Ai}} \quad j=1, 2, \dots, C(A)$$

and if Region B has C(B) countries then average shares of the Region B are calculated as arithmetic mean from the values

$$w_{Bk} = \frac{e_{Bk}}{\sum_{i=1}^n e_{Bi}} \quad k=1, 2, \dots, C(B)$$

The EKS procedure is be applied further to the set of between-regional Fishers PPPs (where each regional country is included in an equal way) to obtain the between-regional EKS-PPPs. So, all Regions and the countries within the Regions will be treated fully symmetrically by this approach in a natural transparent way in accordance with the underlying EKS principles⁵ and

⁴ If a region has only two countries then average regional shares will be the same as in Tornqvist method, if more than two countries – then the same as by the Walsh method.

⁵ Y.Dikhanov indicated that if there are "bad countries" then there are problems with "plutocratic" as well as with "democratic" weightings: if "bad countries" are large countries then "plutocratic" weighting is worse, if small countries then – "democratic" weighting is worse. This is true but this is rather a general problem for all approaches – how to treat the countries with lower quality of input data. For example, the Ikle method uses the country's

democratic weights do not impose a set of common prices⁶ to add up the quantities within a region (no bias).

An illustration of the proposed method is given below (Table 3) on the basis of the example from an initial version of Chapter 13 of the ICP manual.

expenditure shares as the weights. Therefore the use of data of “bad countries” like SRI or TAJ in the Ikle aggregation will have the same higher negative effect as by “democratic” weighting.

⁶ If we use democratic regional weights then Regional (GM) average numeraires seem to be not necessary - between-regional BH-PPPs are transitive and democratic regional weights do not depend on regional BH-PPPs.

Table 3

Calculation of between-Regional PPPs by the EKS method

(input data are from the example done in former Chapter 13)

Expenditures in national currencies

BH	Region A				Region B			Region C	
	Countries				Countries			Countries	
	1	2	3	4	1	2	3	1	2
1	2000	60	1200	9600	380	1000	3000	4500	225
2	500	18	400	2000	70	290	720	1200	67
3	800	35	700	4000	180	315	1260	1500	120
4	300	24	245	3300	70	170	500	800	36
1-4 = C	3600	137	2545	18900	700	1775	5480	8000	448
5	700	31	600	2800	180	430	1080	1500	75
6	700	32	700	2800	130	190	1280	2400	78
5+6 = I	1400	63	1300	5600	310	620	2360	3900	153
Total = GDP	5000	200	3845	24500	1010	2395	7840	11900	601

National shares of expenditures (%)

BH	Region A				Region B			Region C	
	Countries				Countries			Countries	
	1	2	3	4	1	2	3	1	2
1	40.0	30.0	31.2	39.2	37.6	41.8	38.3	37.8	37.4
2	10.0	9.0	10.4	8.2	6.9	12.1	9.2	10.1	11.1
3	16.0	17.5	18.2	16.3	17.8	13.2	16.1	12.6	20.0
4	6.0	12.0	6.4	13.5	6.9	7.1	6.4	6.7	6.0
1-4 = C	72.0	68.5	66.2	77.1	69.3	74.1	69.9	67.2	74.5
5	14.0	15.5	15.6	11.4	17.8	18.0	13.8	12.6	12.5
6	14.0	16.0	18.2	11.4	12.9	7.9	16.3	20.2	13.0
5+6 = I	28.0	31.5	33.8	22.9	30.7	25.9	30.1	32.8	25.5
Total = GDP	100								

Regional expenditure shares & between-regional BH-PPPs						
BH	Region A		Region B		Region C	
	PPP	Av.Sh.	PPP	Av.Sh.	PPP	Av.Sh.
1	1	35.1	2.5	39.2	9	37.6
2	1	9.4	1.6	9.4	12	10.6
3	1	17.0	3.0	15.7	15	16.3
4	1	9.5	2.4	6.8	8	6.4
5	1	14.1	3.0	16.5	15	12.5
6	1	14.9	3.2	12.4	12	16.6
Total-GDP	xxx	100	xxx	100	xxx	100

The RESULTS by EKS-method (with weights in %) for GDP

MATRIX of BINARY LASPEYRES's PPPs

(... currency units of row-country per 1 currency unit of column-country)

	Reg.A	Reg.B	Reg.C
Reg.A	1.00000	.39001	.09163
Reg.B	2.66608	1.00000	.23959
Reg.C	11.50288	4.41811	1.00000

NUMBER of LASPEYRES-PPPs = 0 in the matrix = 0

INITIAL MATRIX of DIRECT BINARY FISHER's PPPs

(1st line: PPP = ... currency units of row-country per 1 currency unit of column-country,
2nd line: L/P ratio)

	Reg.A	Reg.B	Reg.C
Reg.A	1.00000	.38247	.08925
	1.00	1.04	1.05
Reg.B	2.61458	1.00000	.23287
	1.04	1.00	1.06
Reg.C	11.20436	4.29419	1.00000
	1.05	1.06	1.00

NUMBER of the MISSING VALUES in the Fisher's-matrix = 0

Matrix of PPPs by EKS-method

(... currency units of row-country per 1 currency unit of column-country)

	Reg.A	Reg.B	Reg.C
Reg.A	1.00000	.38273	.08919
Reg.B	2.61278	1.00000	.23303
Reg.C	11.21207	4.29124	1.00000

Between-regional PPPs for GDP

	R. Hill version	S. Sergeev version
A	1.000	1.000
B on A	2.624	2.613

The Chapter 15 of the ICP Manual describes one other possibility based on binary between-regional Fishers PPPs which are obtained with the intermediate use of within-regional EKS-PPP as it was proposed by R.Hill⁷. A set of transitive aggregate PPPs by this approach is derived from a set of the binary PPPs which are calculated as GM of PPPs between all possible pairs of countries belonging to different regions (this is like country's prices are recalculated to regional numeraires by within-regional EKS-PPP). However the use of data of individual countries by the R. Hill's proposal does not bring any advantage (neither conceptual nor technical) for the calculation of the between-regional PPPs. Conceptually the R.Hill method is not transparent and not straightforward – if country's input data are used then the standard country approach (so called, universal approach - all countries together) is much simple and understandable (in principle, between-regional PPPs are not necessary by this approach). From the technical side, if the calculation will be expanded on the whole set of the countries then this is a cumbersome procedure because this does not avoid the calculations of numerous bilateral between-country PPPs⁸. Obviously modern computers do these calculations very quickly but it is unclear - Why one needs the between-regional aggregated PPPs in the case when the between-country PPPs have been calculated? The EKS-PPP can be calculated by an universal approach (all countries from all Regions - country approach) and a simple procedure can be used after this for the fixity of the between-country PPPs obtained within the regional comparisons. It is visible from Table 3 that numerical differences between Hill's and Sergeev's results for the artificial example below are rather small. Probably this will occur also in the practical calculations. However the Sergeev's approach as a modification of the Diewert original approach seems to have methodological and technical advantages.

⁷ See R.J.Hill, T.P.Hill (2007).

⁸ One of additional disputable points is the following – Does play some role the differences in the no. of the countries from different regions in the Ring comparison? This aspect can be considered from different points of view. R.Hill & P.Hill (see paper, 2008) believe that this factor does not play any role. They analyse an example for two regions A and B with 4 countries (M=4) and 2 countries (N=2). They wrote: „Suppose now that the base country in region A is country a1, while the base country in region B is b1. The eight components of the untransitized price index between regions A and B can now be simplified to the following:

$$\begin{aligned}
 & \mathbf{1} \times \text{PFa1,b1} \times \mathbf{1} \\
 & \text{PEKSa1,a2} \times \text{PFa2,b1} \times \mathbf{1} \\
 & \text{PEKSa1,a3} \times \text{PFa3,b1} \times \mathbf{1} \\
 & \text{PEKSa1,a4} \times \text{PFa4,b1} \times \mathbf{1} \qquad (4) \\
 & \mathbf{1} \times \text{PFa1,b2} \times \text{PEKSb2,b1} \\
 & \text{PEKSa1,a2} \times \text{PFa2,b2} \times \text{PEKSb2,b1} \\
 & \text{PEKSa1,a3} \times \text{PFa3,b2} \times \text{PEKSb2,b1} \\
 & \text{PEKSa1,a4} \times \text{PFa4,b2} \times \text{PEKSb2,b1}
 \end{aligned}$$

Sergeev's concern arises from the fact that six within-region-A EKS indexes will appear in (1) as compared with only four within-region-B EKS indexes. Does this imply that region A exerts greater influence than region B on the resulting between-region price index? The answer is no. Region A seems to exert more influence in (4) only because more of the region B EKS indexes in (3) equal one and hence drop out of (4).“

A possible contra-argument is that the indices which are automatically equal 1 do not bring any information in the comparison. No. of such indices by M=4 and N=2 is (M+N)=6. If one wants to minimize the share of them in the total no. of bilateral regional indices (=2*M*N) and, in effect, to maximize the use of input data, then this is obtained if M=N. It means that it is desirable that the regions A and B have equal no. of Ring countries.

II. Analysis of other possible approaches for the linking

There are several other possible approaches for the between-regional aggregation.

E.Diewert gives the preferences to the Spatial Linking (like MST approach), some other expert – to the unrestricted EKS method (all countries together) with the further distribution of the Regional volumes in accordance with the shares obtained in the regional comparisons. Some considerations on these approaches.

II.1 Spatial Linking - MST approach

E.Diewert believes that the use of the Minimal Spanning Tree (MST) approach is a perspective way for the ICP. Maybe, this is too optimistic opinion. The use of bilateral linking approach like MST is interesting approach as a „scientific game" but, it seems, this is not very promising for the practical application in the ICP. The idea of chain linking in international comparisons is not a pioneering idea. This idea as idea (without technical algorithms) can be found already in the books by I.Fisher (1922), C.Clark (1940) and some Hungarian researchers (1950th – 1960th). P.Köves presented in his book "Index theory and economic reality"(Academia Kiado, Budapest. 1983) the MST for ICP 1970 for 10 countries (see §8.3.3). The chain linking methods were discussed actively during the preparation of the ICP 1980 (1st ICP exercise on the regional basis) but they were rejected in favor of multilateral comparisons. The drawbacks of the chain linking methods in the international comparisons are obvious:

- First of all, organizational difficulties: it is much easier to organize a multilateral comparison between N countries than to organize (N-1) best bilateral comparisons especially belonging to different Regions;

- The MST is oriented on the obtaining of a global minimum for the whole set of countries but the links for some individual countries can be not very reliable. Such examples can be found in each of the MST exercises. Let us give some examples from the article by R.Hill and M.Timmer "Standard Errors as Weights in Multilateral Price Indices", indicated by the author of the paper in question. This article uses FAO data for Agricultural sector. The results for Cameroon are immediately in the eyes: the ratio between Max-PPP (USA=1) and Min-PPP (USA=1) obtained by the different methods is extreme approx. 9 times! (from 488.5 by both MSTs till 4446.5 by the EKS). Generally such colossal differences occur due to very unusual price ratios⁹ or simply some rough mistakes in input data (like comma mistakes or incorrect measuring units) but one should believe that FAO input data were examined carefully. All comparisons for Cameroon are done only via Canada and only on the basis of 6 products all EKS methods had to use more prices for Cameroon than the MST methods. It seems that the EKS results for Cameroon are more reliable than the MST results. It would be much easier to organize the regional multilateral comparisons and to link them than to organize numerous bilateral comparisons for whole set of the countries (102 binary comparisons in Hill / Timmer example) - many of them are enough exotic. For example, who should organize and coordinate such very specific (especially in Agricultural Sector) comparisons as Norway - Niger, Norway - Indonesia, Norway - Albania, Norway - Peru, Norway - Nepal as it was "proposed" by the MST(1/Var)?

[Some of these problems are related to the selected measures of similarity but this is a separate topic for the discussions].

- It is not always possible to use in the practice the best bilateral links due to political reasons. The non-desirable bilateral links can be excluded *ex ante* by the MST algorithm but, in effect, the actual MST will be not the best but only the 2nd best choice.

- Very small (even accidental) changes in the measure of between-country's price / quantity similarities can lead to very significant changes in the MST. Additionally, as Y.Dikhanov indicated recently (email of 10.02.10): "*Another thought I am having relates to a different sort of stability: index numbers vary in terms of their sensitivity to disturbances in data [stochastic and systematic]. Not only in price data but also in expenditure data. And it would not be an overstatement to claim that standard errors in BH PPPs could exceed 50% in some cases, and on the expenditure side, a distortion of 10 times at the BH level is not unheard of. In view of such uncertainties, not only the indexes themselves will be affected, as they react differently to such distortions, but any similarity index would be affected as well [if one uses spanning trees]*". Therefore it is non-desirable to base the whole comparison for a country X on only one (partly accidental) bilateral link.

-The straightforward MST should be based on the bilateral "best" item lists¹⁰. It was agreed that the ICP 2011 will be linked on the basis of the Global Core list. It would be strange if all countries collect prices for this common list and only a small part of these data would be used in accordance with the MST approach.

So, it is unlikely (at least, for the author of this notice) that the Spatial Linking approach in the form of the Minimum Spanning Tree (MST) can be used in the ICP practice due to numerous organisational, methodological and technical problems. It seems that A.Heston and B.Aten made the same conclusions (see A.Heston, B.Aten – 2001 and 2009). The multilateral methods are more practicable for the ICP. Additionally, multilateral methods like the EKS also allow eliminating "weak" links: "weak" direct bilateral indices (e.g. with high L/P ratios or L/ P less than 1) can be replaced on the "good" indirect indices. This approach (so called "selective EKS procedure") was used, for example, in the official overall ECP 1993. The different weighting on the basis of the similarity indicators (weighted EKS) are also possible.¹¹

II. 2 Unrestricted EKS method

The use of an unrestricted (i.e. without fixity¹²) index such as EKS on all countries first, and then distributing the regional totals according to regional fixity was agreed initially by the TAG for the ICP 2005¹³.

¹⁰ The GEKS method should be also based on the best bilateral PPPs (Fisher-PPP, etc.). To allow the production of the best bilateral PPPs from a multilateral list Eurostat introduced the asterisks. In effect, prices for products which are non-representative in both countries are not included in the calculation of bilateral PPPs.

¹¹ The ICP 2011 should be based on the Global Core List and therefore each country is involved in the between-regional comparisons. However, input data of other countries from different Regions can be different quality. In this aspect the use of weights or the selection of "good linking" countries by some criteria can be useful. The computation of weights or the rules/ criteria for the selection of "good linking" countries should be clearly formulated and each country should have a chance to be a "good linking" country still criteria do not demonstrate the opposite view.

¹² The regional fixity can have in some cases remarkable impact of the results but, generally, this factor does not play extremely important role: usually several percent points (the revision of NA data brings sometimes much more

This approach (with the G-K method) was used for the 1st time in the ICP 1980. The use of whole set of the countries in the aggregation (universal calculation) with further distribution of regional Volumes in accordance with the Regional shares (two-stage approach) was very logical for the G-K method. It is possible to use the 1980 approach (universal aggregation for all countries with further distribution of Regional Volumes) in the ICP 2011 where the EKS method should be used. However the advantages the ICP 1980 are not so clear if the EKS method should be used for the aggregation.

First of all, the EKS calculations for more than 150 (each country with each country) would be cumbersome and not very transparent. Many bilateral indices (for the countries with different price and quantity structures) would be not very realistic and useful. For example, it is very doubtful that the comparison “Brazil/LA – Moldova/CIS” is meaningful and will bring useful information. It is not accident that experts mentioned weighting or elimination if there are egregious bilateral indices. Both versions (weighting and elimination of non-reliable bilateral indices) are possible but these procedures need (non-arbitrary) weighting systems and criteria for the elimination (like LPS). It is very likely that it will be not easy to agree on the following points:

- On which indicator should be based weighting? (coefficient of similarity?)
- Which bounds should be used for LPS if this approach is accepted? Should be unique bounds for LPS for all pairs (in this case, some countries like Tajikistan can have no links at all) or these should be specific for different sets of countries?

All decisions here will be inevitable (at least, partly) arbitrary and disputable.

Secondly, the distribution of Regional Volumes in accordance with the intra-regional shares is straightforward for the two-stage G-K method. The situation for the two-stage EKS is not so clear. The distribution of Regional Volumes (fixity of Regional Volume shares) as well as the distribution of (product of) Regional PPPs (fixity of Regional PPPs) are possible. The between-regional results will be (slightly) different depending on the choice of the indicator for fixity (Volumes shares of PPPs). The results inside the regions are the same in both versions but the results between the countries from different regions are different - see one notice on the ICP WB web-site

http://siteresources.worldbank.org/ICPINT/Resources/FIXITY_PPP_vs_ShVol_SS53.DOC

So, it will be necessary to justify the choice of the distribution of volumes for the ICP 2011. This is not so obvious. For example, the present Eurostat-OECD practice is based on the use of fixity for PPPs in the aggregation.

significant changes – see, for example, the results for TUR and MEX in the ICP 2005 and the recent results of MEX and TUR for 2005 in the recent OECD publications.

¹³ However E.Diewert proposed later the between-regional PPPs and this proposal was adopted as a principal development for the BHs-PPP as well as for the aggregated PPPs

Conclusions

1)

The Diewert's concept allows to keep the intra-regional PPPs as fixed figures and simultaneously to obtain the between-regional PPPs in a straightforward, compact and elegant form.

However the original Diewert's approach at the aggregated levels used in the official ICP 2005 has two drawbacks:

- original Diewert's approach is not invariant relatively the choice of the Regional numeraires
- the countries within the Regions are treated in a non-symmetrical way by the obtaining of regional weights. This is not in the spirit of the EKS method.

Two modifications were proposed by the author of this notice for the "repairs":

- to use average regional numeraires instead country's regional numeraires in input data (this guarantee the invariance)
- to use of average democratic regional expenditure shares instead plutocratic regional expenditure weights (this guarantee the symmetrical treatment of all countries within the Regions in the spirit of the GEKS approach).

These proposals keep the main features of the original Diewert approach and allow to eliminate the drawbacks in a simple form.

2)

The use of the Spatial linking like the MST method seems to be inefficient in the ICP practice due to numerous organisational, methodological and technical problems

3)

The use of the unrestricted EKS method with further distribution of regional Volumes in accordance with the Regional shares is possible. However it seems that this would be a methodological step backwards relatively the use of the between-regional PPPs.

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