A Risk Quantification Model for Public Debt Management

Antonio Velandia

The opinions expressed in this work are those of the author and should not be attributed to the World Bank, affiliated organizations, or members of its Board of Directors or the countries they represent. I am grateful to Fred Jensen for his guidance in developing the paper and for his ideas about the application of an ALM framework to sovereign debt management. Naturally, I am solely responsible of errors remaining.

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ABSTRACT

There is widespread agreement among leading debt management practitioners about the adequacy of debt servicing cash flow simulation technique to estimate the cost and risk of a sovereign liability portfolio. This practice gains a solid theoretical platform when re-cast into an asset liability management (ALM) framework. The issue then becomes how to incorporate the government's assets into cost and risk estimation. In a model developed in The World Bank, it is proposed that such a link be built by incorporating an analysis of the financial features of those assets, namely their sensitivity to currency and interest rate changes, as the metrics to measure the cost and risk of the liabilities. Within the ALM framework additional alternatives such as the joint modeling of assets and liabilities could be explored.

I. Introduction

On behalf of the World Bank I would like to extend our thanks to ECLAC for the invitation to this year's Regional Seminar on Fiscal Policy. Today I would like to discuss a few basic ideas on how to model risk for a sovereign liability portfolio. The basis for this presentation is a model developed in the World Bank for training of debt officers of client countries. It also incorporates potential avenues to further develop the methodology to measure risk in an Asset Liability Management (ALM) environment.

From the start it is important to emphasize that as the title of this first session suggests, governments should try their best to formulate a debt management policy that helps contain the bias to financial and economic instability typical of emerging market economies. In Southeast Asia, Eastern Europe and Latin America recent economic crises, the management of debt either was a direct contributor to the crisis, or at least seriously impaired the government's financial stability, severely curtailing the degrees of freedom needed to cope with the downturn of the economy.

Vulnerability, as we know, is rooted in weak institutional arrangements and a macroeconomic context heavily exposed to external shocks, and much has been said on its causes. The point here is that by properly analyzing the cost and risk of the sovereign debt portfolio –including contingent liabilities- governments can set up debt policy guidelines that can protect them better in times of crisis.

The basic technique for risk assessment can be learnt from the practice of countries that not so long ago carried a heavy and risky debt burden and have successfully tackled their debt problem[1]. Most of these countries approach the measurement of risk by simulating debt servicing cash flows and examining the potential impact of these payments on the government's budget. However, even though there is widespread recognition of the adequacy of the cash flow simulation technique, and significant

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[1] See Kari, Nars (1997). The book brings the examples of Belgium, Denmark, Finland, Ireland, Sweden, and New Zealand that borrowed heavily during the period 1989 to 1995.

discussion has been devoted to the notions of cost and risk, and to the different ways they both can be measured, the empirical modeling has lacked a theoretical framework that provides a firm underpinning to the measurement discussion.

It is on this ground that the work done at The World Bank may contribute by recasting these methodologies for risk analysis in a more formal ALM framework long in use by the banking industry. In line with the practice of leading Debt Management Offices, we envision risk as the potential destabilizing impact financial shocks to debt service may have on the budget. But we link this disturbance explicitly to the impact on taxes, or on government spending. Because government revenues or spending also may be correlated with the same financial variables that drive debt service, risk analysis requires a comparison of the impact of financial variables to debt service -the main government liability- as well as the impact on government net revenues –the main government asset: Hence, our approach is based on an explicit ALM framework[2].

The World Bank model, with which we illustrate this methodology is still work in progress. Different aspects could be improved and we would greatly benefit from your comments. This is by no means the only modeling approach for assessing risk of a sovereign liability portfolio in an ALM framework, are we are considering a number of ways to extend it. A few Debt Management Offices (DMO) also are exploring.

These notes are divided into 5 sections apart from this introduction. Section II focuses on the merits of the ALM framework. It highlights the complexities arising from dealing with non-financial assets such as the government revenues and expenditures with "blurred" financial features. Section III draws on the World Bank's model to illustrate the type of methodology used by a number of countries to model risk and evaluate strategies for managing risk. Although this type of modeling is consistent with an ALM framework, it doesn't model assets and liabilities simultaneously. Instead, it uses a separate analysis of the financial characteristics of the assets to construct the metrics that will be used to measure both the cost and risk of the liability portfolio: this link is

[2] See Jensen (2001).

explained in section IV. In section V a few alternative avenues to explicitly incorporate the government's assets in the model are explored. Finally, section VI concludes

II. Asset Liability Management Framework

The ALM application for a financial institution has been in use for a few decades and has subsequently been adopted by other private sector agents[3]. Its main proposal is that risk can be contained by matching the financial features (e.g. interest rate characteristics) of the assets and liabilities, as then one side of the balance sheet will be hedged –or immunized- by the other side.

A simplified view of the ALM application can be drawn from the risk management practice of a bank. As a financial intermediary, a bank takes deposits from the public and makes loans to individuals and businesses at interest rates that usually exceed the cost of raising such funds. Because of the particular needs of borrowers and depositors, deposit and loans may differ in maturity, duration and currency composition, leaving the bank exposed to changes in interest or exchange rates and also to the potential shortage of loanable funds. To oversee and manage the financial risks resulting from this intermediation, banks have implemented Asset Liability Committees. Such Committees periodically review the features of the main institution's assets and liabilities, analyze currency and interest rate mismatches, and on this basis, decide on possible adjustments to the balance sheet structure. In some cases, Committees opt to maintain the current market exposure or even increase it, pursuing the profit potential of a given mismatch if a forecasted move in the financial variables materializes. In others, Committees prefer to reduce the bank's market exposure because they feel uncomfortable with the risk/return profile of the position. Risk reduction can be achieved in a number of ways: reducing the outright exposure, hedging or diversifying.

The lesson here is that the ALM approach helps firms analyze situations that can threaten their main objective of profit maximization, by processing the information

[3] See Fabozzi, F. (1990) and Bitner, J. (1992)

contained in the balance sheet, deriving from there the potential risks, quantifying them, suggesting ways to reduce risk and providing guidelines to manage residual risk.

Although applying the lesson to the risk analysis of a sovereign liability portfolio is not straightforward, the ALM principles remain relevant. Governments worry about their liabilities because of the financial and economic costs that result from an unexpected increase in debt servicing flows. Faced with such a shock, governments have two policy alternatives: for a given level of government spending, taxes can be raised, or for a given level of taxes government spending cuts can be implemented. If the unexpected shocks are so severe that the government is unable or unwilling to raise taxes or cut spending enough, there is a third alternative which is for the government to default on its obligations: the economic costs of a default are dire.

On the first alternative, the tax-smoothing literature proposes that taxes, other than lump sum, create deadweight welfare losses[4], and that volatility of tax rates increases these losses. In this context, it could also be argued that higher long-term growth is more likely to be achieved if tax variability is minimal, since it reduces uncertainty. Tax volatility creates inefficiencies as it distorts economic decision-making, complicates long-term investment decisions, depresses consumption and possibly channels excess savings into short-term financial instruments. The second alternative of reducing government spending may entail severely curtailing services and programs the government was expected to deliver involving high social and economic costs.

In general, volatility in tax rates or government programs creates risk for tax payers which they cannot hedge, and thereby leads to economic and social welfare losses.

Finally, if the volatility in debt service is so great the government is either unable or unwilling to raise taxes and/or cut spending enough to pay it, they must default, which

[4] See Barro, R (1979). The neutrality of taxes is one of the critical assumptions required for the Ricardian equivalence to hold. Beyond the Ricardian equivalence, the cost of public debt could have an impact on the possibilities of consumption of the economic agents and on the general welfare of society. This gives theoretical justification to the management of public debt along the lines of minimizing its cost for an acceptable level of risk as by the modern portfolio theory.

adds severe economic costs. These include output losses from economic recession and financial institutions and private sector bankruptcies, either directly because the entities hold government debt in their portfolios, or indirectly, as a consequence of the economic recession and higher costs of future borrowing both for the public and private sector as a result of the government's reputational loss[5].

In this context, the risk of the government's debt portfolio relates to the government being forced to recur to tax increases or spending cuts, or worst of all to default on its obligations when debt can no longer be serviced from the regular budget. An analysis of this risk requires a joint analysis of the impact of financial shocks on the government debt services together with the impact of these shocks on government revenues and other expenditures.

However, because of the nature of the government's assets, the application of the traditional ALM framework to the analysis of risk of sovereign debt is more complex. Unlike the financial assets of banks, this less tangible fiscal asset lacks explicit financial features that are the basis for risk identification and quantification in the ALM framework described above. The stream of government revenues and expenditures are stochastic in nature and highly responsive to macroeconomic policies which makes it difficult to discover their response to changes in inflation, interest or exchange rates[6]. For instance, depending on whether inflation is caused by a demand or a supply shock, the net fiscal position could improve or worsen. Short term interest rates could be positively correlated with government revenues alongside the economic cycle, but may have lagging effects bearing a negative correlation[7]. A currency devaluation may increase government revenues if a real depreciation boosts economic activity, but may trigger a contraction as a result of inflation or a severe curtailment on imports.

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[5] See Alesina, A (1992).

[6] Finding the true financial features of the government assets is a major prerequisite for the application of The World Bank model as explained in section III. The inherent difficulties in discovering such financial features is further discussed in section IV, whilst section V presents a few ideas to deal with this problem.

[7] This correlation also change depending on whether the shock being considered is demand or supply driven.

In the next section, I use the model built by the World Bank to illustrate the type of methodology used by a number of countries to model risk for a sovereign liability portfolio and to evaluate strategies for managing risk.

III. Methodology for Quantification of Risk in a Sovereign Liability Portfolio

The model developed by The World Bank draws on the experience of a handful of industrialized countries who had a heavy debt burden in the 80's and 90's and substantially revamped their management of debt in order to control the risk debt service could cause on the budget and on the macro adjustment program.

In essence, these countries approached risk of the liability portfolio by simulating future debt service cash flows. The simulation provides in the first place an expected path for future debt service, which is associated with the notion of cost, (or expected cost). It also identifies the potential deviation of debt servicing flows from that expected path due to shocks in interest rates, exchange rates or shortage of loanable funds in the domestic or the international markets, providing a measure of risk, (i.e. the potential increase in debt servicing costs). Different countries use somewhat different measures for cost and risk, involving the different time horizons, whether or not to include present values, how the risk scenarios were generated, etc., but the cash-flow projection technique was the core of many of them (see diagram 1).

Whatever the measures chosen, the simulation technique is used to measure the cost and risk of a given debt strategy. When this application is extended to all relevant debt strategies, and the cost and risk measures are expressed as single values (i.e. as present values, or, some aggregate measure of the cash flows), every debt strategy can be located as a point in the cost-risk surface. This allows us to compare different strategies, measure the cost-risk trade-off, and in the end, select a debt strategy that better reflects the government's objectives and its tolerance to risk (see diagram 2).

Diagram 1 - Modeling Cost and Risk

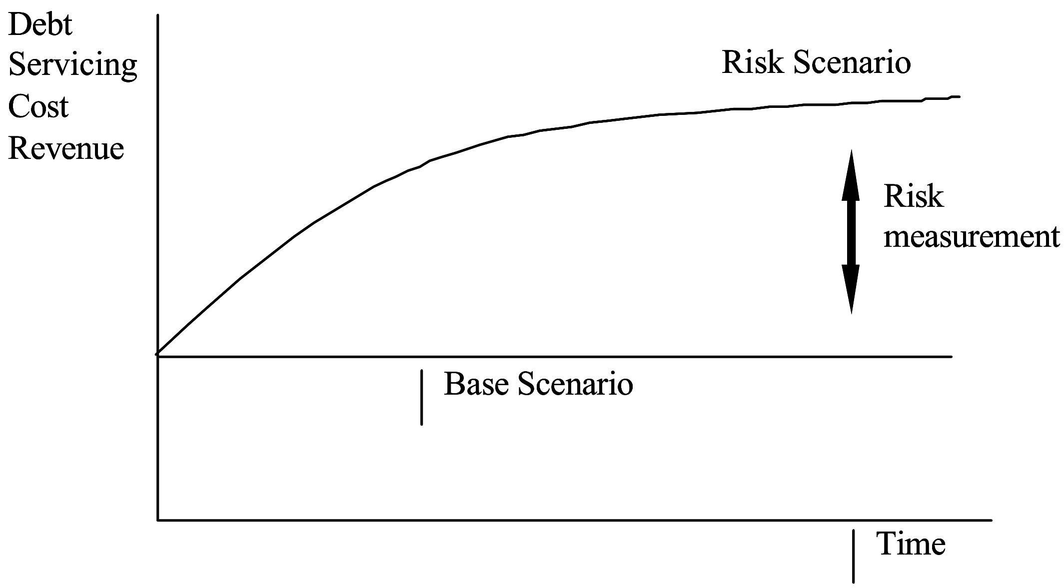
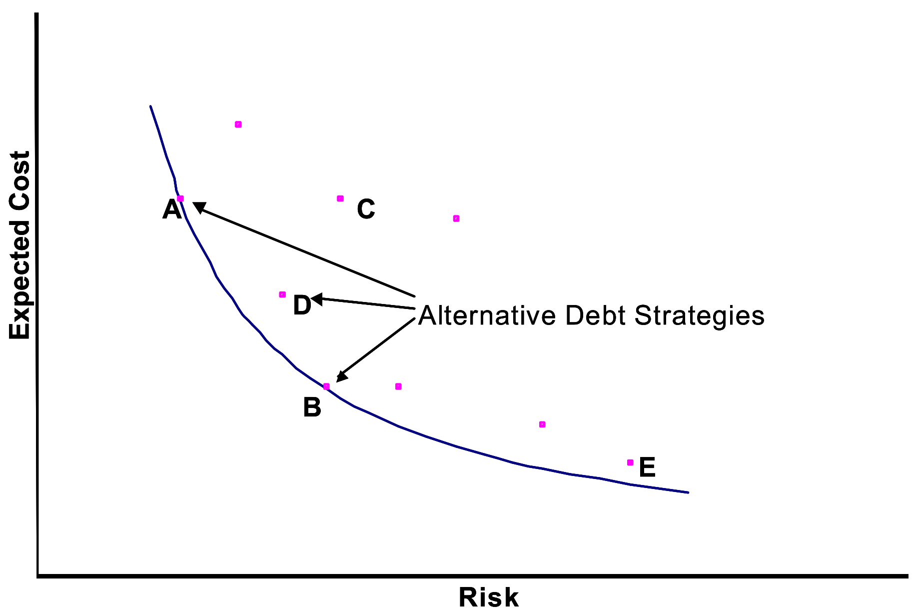


Diagram 2 - Efficient Frontier



The World Bank model shares these basic principles with leading practitioners in the debt management field. However, what is particular to this model are the conditions required for the simulation of the debt servicing flows which we discuss next, and the explicit link with the government assets that is considered in section IV.

In the World Bank model, debt servicing flows are projected based on the principal repayment profile, interest type of loans outstanding, and on assumptions regarding new borrowing requirements and future interest and exchange rates. For existing debt, the liability portfolio composition is known, so only the projected financial variables need to be provided[8]. For rolling over existing debt as well as for new debt, a debt strategy has to be assumed (i.e. the interest rate and currency composition, maturity profile, etc.) in order to compute the debt servicing flows of the new loans.

Since the new borrowing requirements are exogenous to the model, future debt servicing flows depend entirely on the projections of interest and exchange rates (or such variables as inflation if the debt is inflation indexed). In the World Bank model, these projections could be either deterministic or stochastic. In the former, cost is determined by a baseline scenario designed to be market-neutral, whereas risk is given by debt servicing costs under various scenarios of future interest and exchange rate shocks to the baseline scenario. In the case of stochastic simulation, a module uses Monte-Carlo simulation to generate a large number of possible interest and exchange rate paths, each of which generates a particular projection of future debt servicing flows[9]. Cost is then given by the mean of all possible scenarios and risk is measured based on the dispersion of debt servicing paths around the mean (see diagram 3). The mean can also be restricted to ensure the expected costs are market-neutral.

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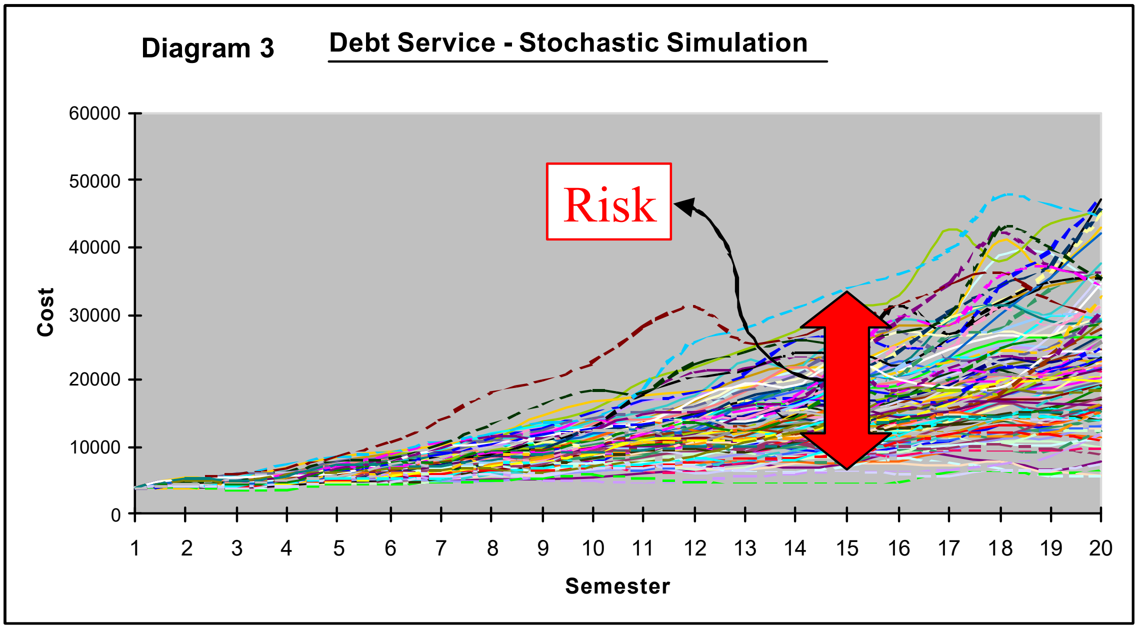
[8] If the country has access to derivatives, it would be possible to change the composition of existing debt. Accordingly, a debt strategy needs to be assumed for the entire liability portfolio.

[9] The financial variable x follows a random path determined as: $x_t = x_{t-1} \cdot e^{\mu \Delta t + \sigma \sqrt{\Delta t \cdot z}}$

where $E(x_t / x_{t-1} - 1) = \mu \cdot \Delta t$ and $VAR(x_t / x_{t-1} - 1) = \sigma^2 \cdot \Delta t$



Clearly, the measures of cost and risk depend critically on the way future interest and exchange rates are projected, and the output of the model is as good as the assumptions used for projecting these key financial variables. Let me turn now to a few basic considerations incorporated into the model for the projection of the financial variables.

In determining the cost, either through stochastic or deterministic simulation, it is essential that financial variables projections be in line with market prices. Indeed, where liquid securities markets permit, interest and exchange rate projections should be taken from the forward markets. Choosing values other than the forwards is tantamount to betting against the market and introduces a bias on the cost estimate[10].

This is why the projection of exchange rates in the baseline case for deterministic modeling, and the mean of the exchange rate distribution in the stochastic modeling, are

[10] Explicit risk premia can be estimated to allow for a possible sloped yield curve and/or risk premia built into the domestic rates of certain currencies.

compliant with the interest rate parity condition[11]. This condition implies that expected exchange rates reflect the interest rate differential of the currencies involved. Although the parity condition can be applied to any pair of currencies, a risk premium can also be added to account for situations in which parity does not hold[12].

The same principle of arbitrage pricing can be applied to obtain market-neutral projections of interest rates at different maturities. Where bond markets over the entire range of maturities are liquid, implied forward rates can be derived from the spot yield curve providing unbiased estimators of the relevant future interest rates. In a similar way as with the exchange rates, this parity condition can be broken if the market is segmented with investors demanding a liquidity premium to purchase securities in the back end of the curve. The model allows one to incorporate such a premium.

When moving to the analysis of risk by assessing the potential impact of financial shocks on debt servicing flows, market neutrality and parity have to be abandoned. The input for these "risk scenarios" is provided on the basis of historical information.

For the deterministic simulation, past shocks - in the country, or in the region- can be used directly as estimates of future interest and exchange rates in worst case scenarios. In the stochastic simulation, historical volatilities can be applied to the baseline case projections, and used as parameters of an interest or exchange rate generating process following a specified statistical probability distribution. The specification of a statistical distribution (usually assumed to be log-normal) allows the dispersion about the mean to be associated with a probability level. This allows the specification of a confidence interval for the potential increase in debt servicing costs to be used as a meaningful

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11 The interest rate parity condition can be expressed as:
$\left(E\left(\Delta x_t / x_{t-1}\right)\right) = \left(1 + i_{t-1}{}^{d}\right)/\left(1 + i_{t-1}{}^{f}\right) - 1$, where the left hand side show the expected devaluation of the local currency x at time t as a function of the domestic i^d and foreign i^f interest rates

12 This may be the case of the local currency in emerging market economies when investors demand a premium to compensate for the risk that the government inflate its debt away by excessive printing the local currency. If interest rate parity holds for all foreign currencies, but the local currency has a built-in premium, all external borrowing will bear the same expected cost no matter the currency chosen, and this expected cost will be lower than the expected cost of borrowing in the domestic currency by the amount of the risk premium.

measure of risk. When more than one variable is simulated, projections need to be consistent either with historical correlations, both between exchange and interest rates or among interest rates of different maturities. The model uses the Cholesky decomposition technique to simulate random paths of exchange and interest rates maintaining such correlations[13].

To conclude this discussion let me summarize the methodology to measure cost and risk for a sovereign liability portfolio:

Step 1: Debt service cost flows are projected forward for a specific time horizon under base case assumptions of the funding strategy and future market rates. The base case assumptions of future market rates should be market-neutral. The stream of debt servicing flows gives the expected cost of the debt strategy which may be expressed directly as a cash flow or converted to present value;

Step 2: New projections are made under alternative market rate assumptions. These alternative cases can be generated using statistical techniques, historical analysis, worst case scenarios, etc. These new projections deviate from the most likely path computed in step 1. Again, the cost of these risk scenarios can be expressed as cash flow or discounted to present value;

Step 3: Risk could be measured as the difference between the cash flow or present value of the base case in step 1, and the range of cash flows or present values of alternative scenarios in step 2. In stochastic simulation a measure of risk is obtained by relating a confidence level to the potential increase in debt service. Measures of risk using confidence intervals are commonly known as VaR, or CaR. Whereas VaR has to

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[13] The method consists in finding a triangular matrix A with the following properties: $A \cdot A^T = R$, such that R is the correlation matrix of the financial variables X. In the case of 2 variables we have: $R = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$ and $A = \begin{pmatrix} 1 & 0 \\ \rho & \sqrt{(1-\rho^2)} \end{pmatrix}$. If Y is a vector of independent variables with the desired statistical properties, a random variable path can be generated for Y following the process described in footnote 5. This allows to generate a random variable path for the vector X since $X = A \cdot Y$. In the 2 by 2 matrix the transformation gives us $x_1 = y_1$, and

$$x_2 = \rho \cdot y_1 + \sqrt{(1-\rho^2)} \cdot y_2$$

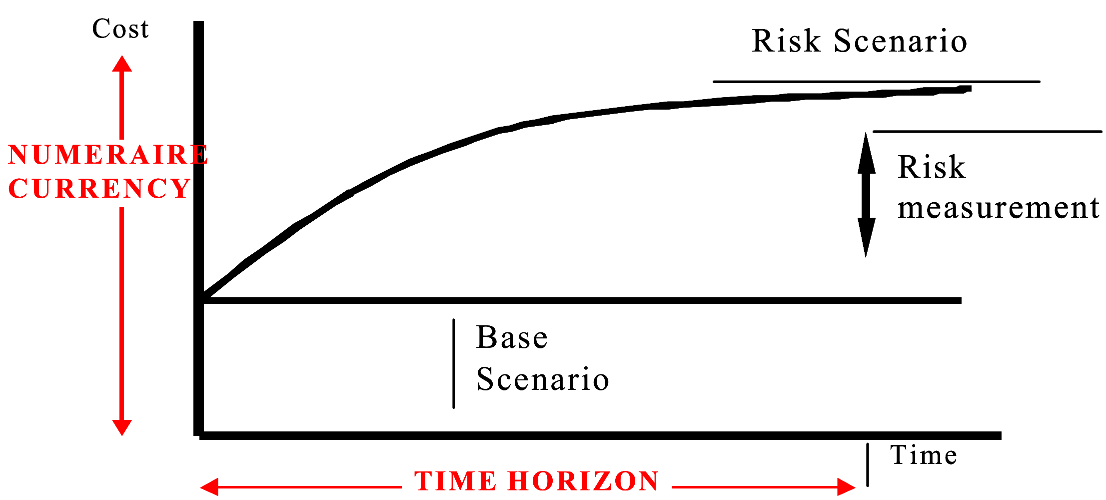
do with the volatility of the present value of the debt, CaR deals with the volatility of future debt servicing flows, and express the maximum expected increase in annual costs relative to the mean with a given probability on a given period[14];

Step 4: Steps 1-3 are repeated for alternative funding strategies, and cost-risk trade-offs are assessed.

IV. World Bank Model Links To The Assets

So far The World Bank's debt model has been used to illustrate a general methodology to estimate cost and risk of a sovereign liability portfolio without reference to the government's assets. In this section, we show how to use the model as an ALM application. The simplest way to link the modeling of the liabilities with the government's assets is by using the financial characteristics of the assets as the metrics to measure cost and risk for the liabilities (see diagram 4). This metrics consists of: (i) the currency numeraire, that is the basis to measure currency risk, and (ii) the time horizon to carry out the simulation which is the basis to measure interest rate risk.

Diagram 4 - Cost and Risk Metrics



[14] The following example helps illustrate the concept . Let us assume a $100 loan, an expected interest rate of 5% normally distributed with a standard deviation of 1%. One can expect with 95% probability the rate will be is smaller than 6.65% therefore 95% of the time the service won't surpass 6.65, this is the absolute CaR. Since budgeted debt service was 5, relative CaR is 1.65.

By currency numeraire, we mean the one which is most closely correlated with government revenues and expenditures. If a government issues most of its liabilities in that particular currency its budget would be less affected by changes in exchange rates and this would minimize currency risk. The fact that many industrialized countries have pursued the reduction of foreign currency debt as a main component of their debt management strategy can be explained on the grounds that this minimizes risk as their revenues are generated in their own currency. However, choosing a numeraire for risk measurement is not necessarily this straightforward. Even if government revenues and expenditures are all denominated in the local currency, they may be responsive to the changes in exchange rates. Consider for instance the case of emerging market economies, less diversified than their industrialized partners, with heavy dependence of exports and GDP upon a few commodities. When prices of these commodities are linked to a particular currency (such as oil), one would expect some correlation between the exchange rate of that currency and the domestic currency value of government revenues and expenditures. Assuming the degree of responsiveness of government revenues and expenditures to the relevant exchange rate can be identified, the model may use as a numeraire a basket composed of local and foreign currencies that reflects such sensitivity.

Another approach would be to measure cost as the ratio of debt servicing flows to government revenues, both expressed in local currency. The projected revenues then can be based on an equation which includes the elasticity of government revenues to the exchange rate, and a different projection of revenues would be produced for each exchange rate scenario on simulation.

The currency numeraire could also be real or nominal depending on the link between government fiscal surpluses and inflation. Although government revenues are primarily in nominal terms, they do not always move in synchrony with inflation, that is, inflation shocks impact real government revenues. In the long-term, for instance, there is a well known negative correlation between inflation and real growth. In the short term, inflationary shocks may be associated both with higher or lower government revenues. Inflation caused by demand shocks tend to increase real government revenues, so that inflation indexed debt acts as a hedge. Supply shocks, on the contrary, reduce the supply of goods and services and tend to shrink real government revenues. Nominal debt, in this case isn't as harmful as inflation indexed debt. Clearly, one needs to know the source of inflation before reaching a definite conclusion on the most convenient type of debt. In terms of the model numeraire, a pragmatic solution again is to measure cost as the ratio of debt servicing flows to government revenues, both expressed in nominal terms, and to include in the model a separate equation linking government revenues to inflation[15].

Under any of the approaches, a thorough analysis of the sensitivity of government revenues and expenditures to a foreign currency -or basket of currencies- is essential in order to determine the numeraire with which the model will measure the cost of the sovereign liability portfolio.

The second dimension to look at is the time horizon relevant to measure cost and risk, which is essential for estimating the liability portfolio interest rate risk. In general, this time horizon is dictated by the sensitivity of the government's revenues to interest rates, that is, the duration of government's assets. If the government's revenues were highly responsive to short-term interest rates, or in other words, the assets have a short duration, a short time horizon would be appropriate, and long-term fixed rate instruments would appear risky. If, on the other hand, revenues were rather insensitive to short-term interest rates -the assets have a long duration-, a long time horizon would seem more adequate and the main interest rate risk would come from the refinancing of short term instruments or from the resetting of short term interest rates.

[15] When the relationship between government revenues and inflation is not stable, historical information can be used to narrow down the number of relationships or equations that are worth analyzing. The debt model can then be run for different paths of government revenues and borrowing requirements, each one corresponding to one particular relationship between government revenues and inflation. Each debt strategy will have as many estimates for cost and risk as the number of equations being tried. However by focusing on the ranking of different debt strategies a sense of robustness can be found.

When the sensitivity of the stream of net revenues to interest rates is known, this implies that the interest rate risk of the government debt portfolio can be measured as the volatility of debt service over a time horizon which corresponds to the duration of the government's assets. This interest rate risk will be minimized if the liability portfolio is structured to have the same sensitivity, or, duration as the assets.

In practice, government's assets correlation to interest rates could be positive or negative depending on whether interest rate shocks are demand or supply driven. Since such shocks tend to be random over the long time horizon relevant to the government's objectives, that average is for revenues to have a low or zero correlation to interest rates. This is equivalent of an asset with long duration and implies a long time horizon for analyzing the risk of the debt portfolio.

The time horizon establishes the valuation rules applicable to debt servicing cash flows[16]. Once these rules are established, debt servicing costs can be expressed either in terms of cash flows or discounted to present value; the key is to use the same time horizon whichever method is selected.. Although both measures are equivalent, DMOs often find the cash flow expression more natural and meaningful given their concern on the potential impact debt service may have on the government budget which is also a cash flow. In addition, the cash flow expression of debt servicing costs serves better to capture liquidity and refinancing risk whose importance has been highlighted in the most recent debt crisis.

But no matter whether cost is expressed in cash flow or present value terms, the fundamental parameter determining risk is the time horizon. Since for most countries the

[16] Short term loans, and long term ones indexed to short term interest rates, are refinanced or reset during the projection period, and their debt service is thereby subject to the volatility of interest rates. If there are loans whose original maturity exceed the time horizon, the cash flows falling beyond are discounted to the end of the time horizon. In this case, such loans debt service is also subject to the volatility of the discount rates applicable at the end of the time horizon.

duration of the government's assets tend to be long, the time horizon should also be long[17].

In a way, the metrics for the currency and the time horizon provides the equivalent of a "notional benchmark" for the liabilities. To illustrate this point let us suppose that the stream of future fiscal surpluses the government will use to service the debt is denominated in local currency and has a long duration. Ideally, liabilities should bear those two main characteristics. If they do, the volatility of projected debt service measured in local currency over the time horizon will be low. If this ideal portfolio is not feasible, -e.g. because of the lack of a developed domestic market- projected debt service will be more volatile and the debtor has to bear with currency and interest rate risks. But the closer the portfolio comes to the desired metrics the lower the projected volatility and the lower the risk. Hence, by using the "notional benchmark" implied in the metrics, the simulation model allows the debtor to quantify such risks and ultimately to look for an adequate strategy to manage them.

The former discussion puts forth the importance of discovering the financial features of the government's assets. But no matter how careful this analysis, it is possible that in the end some ambiguity about those true financial features remains. Indeed, not only are the financial features of the government revenues stochastic, but the events driving these financial features are themselves stochastic and difficult to model. That is the case of supply or demand driven shocks on the financial variables, terms of trade shocks and policy response to both type of shocks. So, even if it is possible to estimate on average what those sensitivities are over a long time horizon, there might be periods in which those sensitivities significantly depart from their long term averages in a way that government's financial position could be at risk.

[17] For DMO's who actively trade their portfolio, in addition of using a long-term horizon for risk measurement and for the liability benchmark, the actual portfolio should be marked to market in order to measure the performance and trading risk incurred by the debt manager. However, such trading activity is inconsistent with the objectives of most DMOs and with most governments tolerance to risk.

To deal with the degree of uncertainty on the metrics for cost and risk measurement, a possible solution consists in running the World Bank simulation model using a few different metrics selected around identified values. With this technique there will not be a single debt strategy that performs consistently better under all metrics. Rather, strategies with a portfolio of liabilities diversified across the selected time horizons or alternative currency compositions should perform reasonably well, giving a sense of robustness to the exercise. Nonetheless, dealing with assets whose financial features are stochastic may be a difficult challenge for the type of ALM models that rely on the metrics for building the links between assets and liabilities. A better approach in this case is the simultaneous modeling of assets and liabilities that will be described in the next section.

A final comment related to the uniqueness of this type of risk analysis and its implications for cost and risk measures is worth making. Given the particular conditions of different country economies, one would expect that every government has distinctive sensitivities of its fiscal surpluses to the financial variables, or, in other words, its own metrics for the cost and risk estimates[18].

V. Alternatives Ways To Incorporate the Government's Assets In An ALM Approach

So far, the methodology for risk analysis has focused on the liability side, treating the government's assets as exogenous to the model. As explained in the previous section, treating them as exogeneous poses a number of problems starting by the inherent difficulty in discovering the true financial features of those assets. Furthermore, not only are assets as affected by financial shocks as liabilities are, but in the case of the assets, their sensitivity to the change of interest or exchange rates varies depending on the nature

[18] For instance, cost could be nominal when measured as debt servicing flows, or, real when scaled by government revenues or GDP. The first one suits a country where government revenues are not very sensitive to inflation. Cost can also be measured in terms of cash flows over a particular time span that may differ from one country to another. It may partially involve marking the portfolio to market, or ignore it completely. As long as risk is estimated as a cost increase under different scenarios for the financial variables, each measure of cost implies a corresponding one for risk thus varying from one country to another.

of the shock. In this context, assuming the financial features for the assets and using them as the metrics for the liabilities is just a rough approximation. Three different ways to better incorporate government's assets in the analysis are considered in this section.

A first alternative involves dividing the government's assets into its constituent parts, or sub-portfolios, and exploring the financial features of these sub-portfolios separately and looking for natural hedges. Because finding the sensitivity to interest and exchange rates is easier for some assets and liabilities than for others, it makes sense to analyze them separately instead of aggregating the corresponding cash flows. Those asset classes with clear financial characteristics then can be hedged by configuring a matching amount of the government's debt to have the same financial characteristics. The costs and risks of any remaining liabilities then can be modeled as described above. This sub-portfolio approach reduces the problem of the indeterminacy of the assets financial features and hence reduces the margin of error of the risk estimate.

Among the main government assets are the foreign reserves, on-lending to the rest of the public sector, and in some countries, the government's stake in companies devoted to the production of commodities that constitute a main source for foreign exchange for the country as a whole e.g. oil in Venezuela, copper in Zambia, etc.

To illustrate the sub-portfolio approach let us assume that: (i) the only liability the government has is its external debt -contingent liabilities can be neglected[19]-, (ii) foreign reserves and the stake in the oil state company represent all foreign currency assets, with identifiable sensitivity to interest rates and a known currency composition, and (iii) other assets are net revenues whose financial characteristics cannot be precisely determined. By following the sub-portfolio approach, a portion of the government foreign debt, bearing the same financial features found in the foreign assets, could be matched to those

[19] Contingent liabilities can seldom be neglected particularly after their role in recent economic crises like Asia in 1998 and more recently in Argentina. Implicit or explicit commitments of the central government to SOE, sub-nationals or other agents in the economy, often times pose a serious currency risk as a substantial part of these agents liabilities is denominated in foreign currencies whereas their assets are largely denominated in local currency. Also, market risks may cause severe liquidity problems when governments lack budgetary procedures that account for the expected cost and reserves against unexpected costs, as there are no assets readily available to match those liabilities.

assets. The remainder of the foreign liability portfolio is matched with other assets, possibly domestic, but whose financial features cannot be precisely determined. This "residual" sub-portfolio represents the government's net exposure and the debt manager can now concentrate on this portion to identify and quantify the government's risk[20] (see diagram 5).

Diagram 5 - Government's Balance Sheet

Assets	**Liabilities**
PV of future net revenues	Debt
Foreign reserves	
Stake in the oil company	

A second avenue to improve the risk analysis in a ALM context consists in allowing a more explicit interaction of the simulation of the debt servicing flows with the macro projections, in particular, with net fiscal revenues and new borrowing requirements. This can be done by building an interface between the World Bank debt model interface with a macro-programming one.

In a typical macro-programming exercise, the set of macro variables is projected forward ensuring both arithmetic consistency and economic plausibility. Take for

[20] When Contingent Liabilities (CL) are brought into the picture, aside from determining the CL's elasticity to interest and exchange rates, the value of the CL itself has to be found. CL behave like options that are exercised if a certain event occurs, for instance, the government may have to bail out a state owned enterprise (SOE) if the latter go belly up. One approach to value these options is to simulate the distribution of the underlying variable that triggers the government's intervention, in our case, the SOE performance, which then allows to simulate the corresponding government's payout. Often, this simulation is a function of the same economic and financial variables that generate risk for direct debt, so the technique for valuing CL is similar to the one described in section II and used to quantify the risk of a sovereign liability portfolio. Once the value of the CL and its elasticity to changes in interest and exchange rates are known, it is feasible to incorporate them in the sub-portfolio approach described before.

instance RMSM-X, the model used by the Bank and the IMF[21]. It uses four sectors: public, private, monetary and foreign. Based on the fundamental account identity of standard national income accounts, together with behavioral equations and a set of assumptions, the model builds the flow of funds between sectors: consumption and investment of public and private sectors, the current and capital flows in the balance of payments, and credit to the public and private sector in the monetary sector. The model may be closed by selecting the government spending and borrowing, or if the governments accounts are specified, the closure is given by the selecting the level of private spending and investment. Alternatively, if borrowing restrictions are imposed, the model closes selecting the growth that is consistent with such financing restrictions.

Clearly, for every macro scenario projected there is a corresponding projection of interest and exchange rates that are consistent. By construction, the baseline case in the deterministic simulation (or, the mean of debt servicing cost in the stochastic one) should correspond to the macro context that is most likely to occur.

Once the projections of interest and exchange rates depart from the baseline case various sets of macro variables and debt service paths can be consistent with the new levels of the market variables. This is because there is more than one way for the economy to adjust to unexpected shocks on interest or exchange rates. Accordingly, by choosing a particular "response" to the shock, a new "macro environment" can be found. For instance, in the public closure of RMSM-X, a real depreciation of the exchange rate increases the foreign sector contribution to GDP at the expense of government and private sector consumption, reducing government borrowing requirements. Foreign borrowing is then adjusted to offset the marginal surplus generated in the current account of the balance of payments.

The input of the shocks on the financial variables on the macro programming model allows us to estimate new government borrowing requirements which, in effect, become endogenous to the joint projection of the two models. The new borrowing projections

[21] RMSM-X (1997) Reference Guide.

are the fed into the debt model for a new simulation of cost and risk which then are fed back into the macro model, etc. The iterations shall continue until all conditions of arithmetic and economic consistency are satisfied.

Clearly, this type of iterative analysis could become quite tedious so the number of risk scenarios would have to be limited. And the choice of policy responses for closure is somewhat arbitrary. But it does allow for a direct analysis of the sustainability of government debt under the risk scenarios –e.g. certain policy choices might no be feasible and this can be represented as limits on maximum taxes or minimum government spending requirements. If shocks cause these limits to be exceeded the debt is not sustainable. And even if the debt is sustainable, this type of analysis also gives a qualitative insight to the economic costs of a risky debt portfolio because of the adjustments to taxes or government spending and the impact on economic growth.

Although the model cannot quantify the economic and social losses derived from sacrificing growth, the risk analysis provides insightful information regarding the burden debt presents for the macro program and, in the extreme in which where debt cannot be serviced by adjusting the government revenues or expenditures, it provides a warning about a potential government insolvency.

Although the suggested interaction between debt and macro-programming models allows a better incorporation of the government assets in the risk analysis, it also forces a specific type of correlation between financial and economic variables that is given by the "response" chosen to the shock, or in other words, by the selected closure of the macro model. In the actual event, different policy makers might choose different responses, the specified correlations will be wrong and so will the measures of cost and risk. In this context it might be better to directly model such "reaction function" using historical data.

A third approach , then, is to jointly model the government's assets and liabilities. These are uncharted waters for DMOs and only the Swedish National Debt Office has published its first attempt to such type of ALM modeling (the French Treasury is said to be experimenting with a joint modeling approach as well)[22]. The idea here is to explicitly model the links between the financial variables affecting future debt service and the variables affecting government revenues.

One way is by constructing an econometric model with behavioral equations that specify the effect of macro variables on the financial variables. For instance, specifying domestic short-term interest rates as a function of inflation and real growth in a way that higher growth brings about higher interest rates. This approach has the advantage of solving the interest rates for a given value of the macro variables guaranteeing consistency between the two.

The disadvantage however, as discussed before, is that behavioral equations impose "reaction functions" and correlations between financial and economic variables that are not always correct. Depending on whether the economy is in a boom or in a slump, or depending on the nature of a particular shock, those correlations may change. In addition, there is no general equilibrium theory of how real economy and financial variables are jointly determined, so models of this type necessarily contain some ad-hoc elements.

Instead of forcing the correlations through behavioral equations in an econometric model, another possibility is to derive those correlations from actual data, estimating a variance-covariance matrix containing all relevant variables. In this case, the problem lies in the large number of correlation coefficients that needs to be estimated –exchange rates with different currencies, interest rates with different maturities for different currencies, and a set of macro variables for the local economy and for the relevant foreign economies- since the data requirements increase exponentially with the number of unknowns for the estimates to be statistically reliable. In addition, a joint probability distribution is needed for that variance-covariance matrix in order to provide confidence intervals that can be used to derive meaningful risk measures.

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[22] See Bergstrom, P (2000)

A possible approach to reduce the number of coefficients to be estimated is to impose certain parameters. For instance, a factor model in which a smaller number of variables is used to drive the others. We are unaware of any debt office which has used this type of statistical approach.

VI. Conclusion

There is widespread consensus that poor debt management can have a destabilizing impact on the government budget through the unexpected increase in debt servicing costs. The risk modeling technique used by leading DMOs are based on implicit simplifying assumptions that allows them to model the sovereign liability portfolio in a tractable manner. In effect, government revenues are considered exogenous which permits a focus on the projection of the financial variables, limiting the complexity of the exercise to the simulation of the interaction of future interest and exchange rates and to the simulation of the interest rate term structure. Within this approach, a few scenarios for future government revenues, designed independently from the simulation of the liabilities, are included to check the robustness of the different debt strategies[23].

In attempting to build a more explicit link with the government's assets, the model developed by the World Bank moves one step further. The model uses an analysis of the financial characteristics of the government's revenues as an explicit metric to measure cost and risk of the liabilities. Keeping the simplifying advantage of modeling the government's liabilities only, the model relies on a careful analysis of the government's net revenues to quantify the interest and exchange rate risks of the debt portfolio. However, discovering the financial features of the government's revenues may prove a major challenge. Not only can assets have an implicit financial structure in the sense that they may be correlated with financial shocks, and therefore can be modeled in a manner similar to the financial liabilities, but these correlations may change depending on the source of the shock.

[23] See Matos, P (2001)

The alternative to improve the treatment of the assets is to introduce the idea of jointly modeling assets and liabilities. A first approximation consists in dividing the government's assets into its constituent parts, or sub-portfolios. Because finding the sensitivity to interest and exchange rates is easier for some assets and liabilities than for others, this sub-portfolio approach reduces the problem of the indeterminacy of the assets' financial features and hence reduces the margin of error of the risk estimate.

A second alternative is to build an interface between the World Bank debt model and a macro-programming one. The input of the shocks on the financial variables on the macro programming model allows us to estimate new government borrowing requirements which, in effect, become endogenous to the joint projection of the two models. The new borrowing projections are then fed into the debt model for a new simulation of cost and risk which then are fed back into the macro model, etc. The iterations are continued until all conditions of arithmetic and economic consistency are satisfied. Although the idea is for the interface between the two models to help discover the true correlations between the financial and economic variables, this approach has the disadvantage that the specification of the macro-programming model imposes particular policy responses to the financial shocks and therefore imposes the correlations that should result from the analysis.

A more comprehensive solution would indeed be the joint simulation of future debt servicing flows and future government budget accounts. The few DMOs that are experimenting in these uncharted waters have struggled with the specification of the model, since there is no general equilibrium theory of how the real economy and financial variables are jointly determined. Ad-hoc specifications of econometric models impose correlations between financial and economic variables that are not always correct. Statistical models that attempt to capture those correlations using only historical data through variance-covariance matrices, run into large data requirements that, if unfulfilled, turn the correlation coefficient estimates unreliable. A possible approach to use this statistical approach is to reduce the number of parameters to be estimated so as to recover degrees of freedom and bring the variance-covariance matrix to a manageable size. This can be done by estimating some parameters using an econometric model, or statistical Arima model, or finally using a factor model in which a smaller number of variables is used to drive the others. We are unaware of any debt office which has used this type of statistical approach.

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