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CAUSAL INFERENCE

Technical Track Session I

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These slides were developed by Christel Vermeersch and modified by Phillippe Leite for the purpose of this workshop

Policy questions are causal in nature

Cause-effect relationships are a part of what policy makers do:

- Does school decentralization **improve** school quality?
- Does one more year of edu
cause higher income?
- Does conditional cash trans
better health outcomes in c
- How do we **improve** studer

But the statistics you learnt in school/university **do not address this...**

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Standard Statistical Analysis

Tools

Likelihood and other estimation techniques.

Aim

To infer parameters of a distribution from samples drawn of that distribution.

Uses

With the help of such parameters, one can:

- Infer association among variables,
- Estimate the likelihood of past and future events,
- Update the likelihood of events in light of new evidence or new measurement.

Standard Statistical Analysis

Condition

For this to work well, experimental conditions **must** remain the same.

But our policy questions were:

- If I decentralize schools, will quality improve?
- If I find a way to make a child go to school longer, will she earn more money?
- If I start giving cash to families, w
  become healthier?
- If I train teachers, will students lea

The conditions **change!!**

Causal Analysis

- For causal questions, we need to infer aspects of the data generation process.

- We need to be able to deduce:
  1. the likelihood of events under *static conditions*, (as in Standard Statistical Analysis) and also
  2. the dynamics of events under *changing conditions.*

Causal Analysis

- *"dynamics of events under changing conditions"* includes:
  1. Predicting the effects of interventions.
  2. Predicting the effects of spontaneous changes.
  3. Identifying causes of reported events.

Causation vs. Correlation

- Standard statistical analysis/probability theory:
  - The word "*cause*" is not in its vocabulary.
  - Allows us to say is that two events are mutually *correlated*, or dependent.

- This is not enough for policy makers
  - They look at rationales for policy decisions: *if we do XXX, then will we get YYY?*
  - We need a vocabulary for *causality*.

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THE RUBIN CAUSAL MODEL

Vocabulary for Causality

Population & Outcome Variable

- Define the population by *U*.
  Each unit in *U* is denoted by *u*.
- The outcome of interest is *Y*.
  Also called the response variable.
- For each $u \in U$, there is an associated value *Y(u)*.

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Causes/Treatment

“

Causes are those things that could be treatments in hypothetical experiments.

- Rubin

For simplicity, we assume that there are just two possible states:

- Unit *u* is exposed to treatment and
- Unit *u* is exposed to comparison.

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The Treatment Variable

- Let D be a variable that indicates the state to which each unit in U is exposed.

$$D = \begin{cases} 1 & \text{If unit } u \text{ is exposed to treatment} \\ 0 & \text{If unit } u \text{ is exposed to comparison} \end{cases}$$

- Where does D come from?
  - In a controlled study: constructed by the experimenter.
  - In an uncontrolled study: determined by factors beyond the experimenter's control.

Linking Y and D

- Y=response variable
- D= treatment variable
- The response Y is potentially affected by whether u receives treatment or not.
- Thus, we need two response variables:
  - $Y_1(u)$ is the outcome if unit u is exposed to treatment.
  - $Y_0(u)$ is the outcome if unit u is exposed to comparison.

The effect of treatment on outcome

- Treatment variable *D*

$$D = \begin{cases} 1 & \text{If unit } u \text{ is exposed to treatment} \\ 0 & \text{If unit } u \text{ is exposed to comparison} \end{cases}$$

- Response variable *Y*

  $Y_1(u)$ is the outcome if unit *u* is exposed to treatment
  $Y_0(u)$ is the outcome if unit *u* is exposed to comparison

- For any unit *u*, treatment causes the effect

  $\delta_u = Y_1(u) - Y_0(u)$

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But there is a problem:

- For any unit *u*, treatment causes the effect

  $\delta_u = Y_1(u) - Y_0(u)$

- Fundamental problem of causal inference
  - For a given unit *u*, we observe either $Y_1(u)$ or $Y_0(u)$,
  - it is impossible to observe the effect of treatment on *u* by itself!

- We do not observe the **counterfactual**

  If we give *u* treatment, then we cannot observe what would have happened to *u* in the absence of treatment.

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So what do we do?

Instead of measuring the treatment effect on unit *u*, we identify the *average treatment effect* for the population *U* (or for sub-populations)

$$\delta_u = Y_1(u) - Y_0(u)$$

$$\Downarrow$$

$$\begin{aligned} ATE_U &= E_U\ [Y_1(u) - Y_0(u)] \\ &= E_U[Y_1(u)] - E_U[Y_0(u)] \\ &= \overline{Y}_1 - \overline{Y}_0 \\ &= \overline{\delta} \qquad (1) \end{aligned}$$

Estimating the ATE

- So, (1) Replace the impossible-to-observe treatment effect of *D* on a specific unit *u*,
  (2) with the possible-to-estimate *average* treatment effect of *D* over a population *U* of such units.
- Although $E_U(Y_1)$ and $E_U(Y_0)$ cannot both be calculated, they can be estimated.
- Most econometrics methods attempt to construct from observational data consistent estimators of:

  $E_U(Y_1) = \bar{Y}_1$ and $E_U(Y_0) = \bar{Y}_0$

A simple estimator of ATE_U

- So we are trying to estimate:

$$ATE_U = E_U(Y_1) - E_U(Y_0) = \bar{Y}_1 - \bar{Y}_0 \qquad (1)$$

- Consider the following simple estimator:

$$\hat{\delta} = [\hat{\bar{Y}}_1 \mid D = 1] - [\hat{\bar{Y}}_0 \mid D = 0] \qquad (2)$$

Note

- Equation (1) is defined for the whole population.
- Equation (2) is an estimator to be computed on a sample drawn from that population.

An important lemma

Lemma: If we assume that

$$[\bar{Y}_1 \mid D = 1] = [\bar{Y}_1 \mid D = 0]$$

$$and \ [\bar{Y}_0 \mid D = 1] = [\bar{Y}_0 \mid D = 0]$$

then

$$\hat{\bar{\delta}} = [\hat{\bar{Y}}_1 \mid D = 1] - [\hat{\bar{Y}}_0 \mid D = 0]$$

is a consistent estimator of

$$\bar{\delta} = \bar{Y}_1 - \bar{Y}_0$$

Fundamental Conditions

Thus, a sufficient condition for the simple estimator to consistently estimate the true *ATE* is that:

> $[\bar{Y}_1 | D=1] = [\bar{Y}_1 | D=0]$
>
> The average outcome under treatment $\bar{Y}_1$ is the same for the treatment (*D=1*) and the comparison (*D=0*) groups

And

> $[\bar{Y}_0 | D=1] = [\bar{Y}_0 | D=0]$
>
> The average outcome under comparison $\bar{Y}_0$ is the same for the treatment (*D=1*) and the comparison (*D=0*) groups

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When will those conditions be satisfied?

- It is sufficient that treatment assignment *D* be uncorrelated with the potential outcome distributions of Y_0 and Y_1
- Intuitively, there can be no correlation between (1) Whether someone gets the treatment and (2) How much that person potentially benefits from the treatment.
- The easiest way to achieve this uncorrelatedness is through random assignment of treatment.

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Another way of looking at it

With some algebra, it can be shown that:

$$\underset{\text{simple estimator}}{\hat{\bar{\delta}}} = \underset{\text{true impact}}{\bar{\delta}} + \underbrace{\left([\bar{Y}_0 \mid D=1] - [\bar{Y}_0 \mid D=0]\right)}_{\text{Baseline Difference}}$$

$$+ (1-\pi) \underbrace{\left(\bar{\delta}_{\{D=1\}} - \bar{\delta}_{\{D=0\}}\right)}_{\text{Heterogeneous Response to Treatment}}$$

Another way of looking at it (in words)

- There are two sources of bias that need to be eliminated from estimates of causal effects :
  - Baseline difference/Selection bias
  - Heterogeneous response to the treatment
- Most of the methods available only deal with selection bias.

Treatment on the Treated

Average Treatment Effect is not always the parameter of interest…

often, it is the average treatment effect ***for the treated*** that is of substantive interest:

$$
\begin{aligned}
TOT &= E\,[Y_1(u) - Y_0(u) \mid D = 1] \\
&= E\,[Y_1(u) \mid D = 1] - E\,[Y_0(u) \mid D = 1]
\end{aligned}
$$

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Treatment on the Treated

If we need to estimate Treatment on the Treated:

$$TOT = E\,[Y_1(u) \mid D = 1] - E\,[Y_0(u) \mid D = 1]$$

Then the simple estimator (2)

$$\hat{\hat{\delta}} = [\hat{\bar{Y}}_1 \mid D=1] - [\hat{\bar{Y}}_0 \mid D=0]$$

consistently estimates Treatment on the Treated if:

$$[\bar{Y}_0 \mid D=1] = [\bar{Y}_0 \mid D=0]$$

"No baseline difference between the treatment and comparison groups."

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References

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Thank You

Q & A