# Fine Tuning Global Linking Procedures

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# Least Squares Fixity

The least-squares fixity (LSF) method can be used to aggregate at either above or below basic heading level.

Above basic heading level it is very similar to the CAR method.

Below basic heading level it is an alternative to the region-product-dummy (RPD) method.

For each basic heading, the method links regions as follows:

$$P_{Kj}^{Global} = P_{Kj}^{Region} \left[ \prod_{k=1}^{K} \left( \frac{P_{Kk}^{Core}}{P_{Kk}^{Region}} \right)^{1/N_K} \right]$$

$P_{Kj}^{Region}$ denotes a within-region CPD comparison.

$P_{Kj}^{Core}$ denotes a global CPD comparison over the core products.

# Spatial Chaining in International Comparisons of Prices and Real Incomes

Joint work with
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Chaining along a shortest path reduces the variance of a price index comparison when one of Diewert's weighted relative price dissmilarity (WRPD) metrics is used:

$$W1_{jk} = \sum_{n=1}^{N} \left\{ \left( \frac{s_{j,n} + s_{k,n}}{2} \right) \left[ \left( \frac{p_{k,n}}{P^F_{jk} \times p_{j,n}} - 1 \right)^2 + \left( \frac{P^F_{jk} \times p_{j,n}}{p_{k,n}} - 1 \right)^2 \right] \right\}$$

Two options for improving on GEKS are:

(i) Replace each Fisher bilateral with its chained shortest path equivalent prior to applying GEKS.

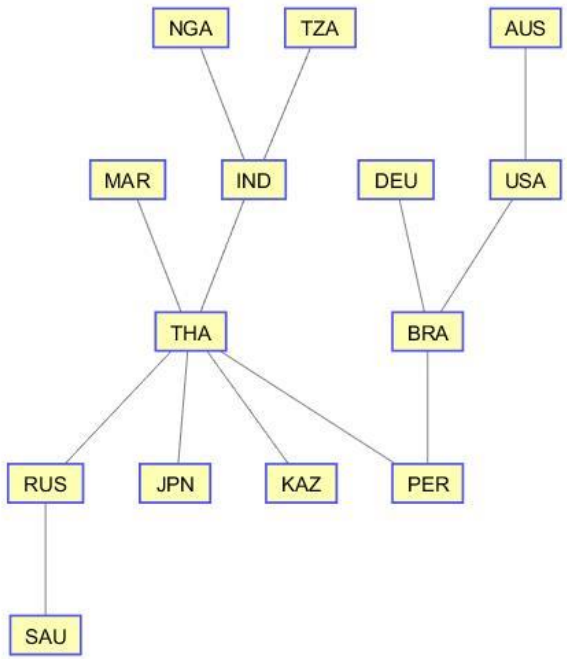
(ii) Take the union of the shortest path spanning trees as a weights matrix in the weighted-GEKS method.

Each country has its own shortest path spanning tree.

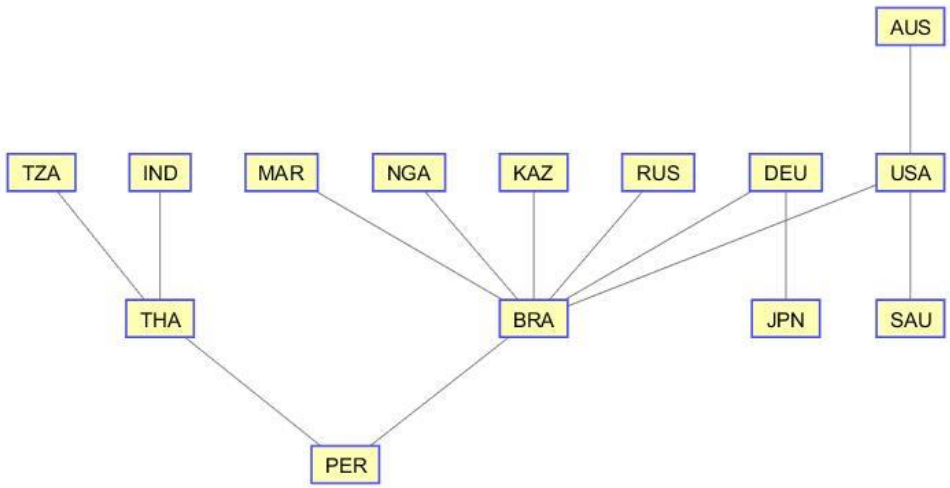
$$F = \begin{pmatrix} 1 & P^F_{1,2} & \cdots & P^F_{1,I} \\ P^F_{2,1} & 1 & \cdots & P^F_{2,I} \\ \vdots & \vdots & & \vdots \\ P^F_{I,1} & P^F_{I,2} & \cdots & 1 \end{pmatrix}$$

$$SP(F) = \begin{pmatrix} 1 & SP(P^F_{1,2}) & \cdots & SP(P^F_{1,I}) \\ SP(P^F_{2,1}) & 1 & \cdots & SP(P^F_{2,I}) \\ \vdots & \vdots & & \vdots \\ SP(P^F_{I,1}) & SP(P^F_{I,2}) & \cdots & 1 \end{pmatrix}$$

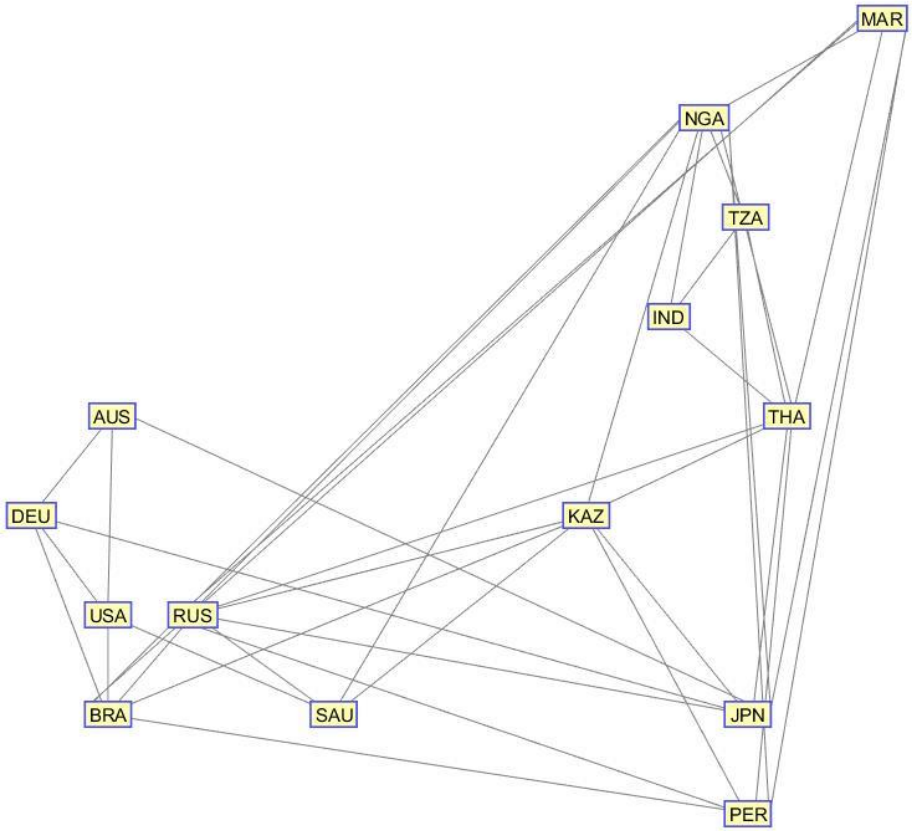
Figure 1 : Shortest Path Spanning Trees – 14 Country Example (ICP 2011)




(a) Shortest Paths for India – W1 Metric (3 of 13 direct)




(b) Shortest Paths for Brazil – W1 Metric (7 of 13 direct)




Union of Shortest Paths – W1 Metric

$$W_{W1}^{SP} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \end{array} \begin{array}{c} \begin{array}{cccccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \end{array} \\ \left( \begin{array}{cccccccccccccc} 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{array} \right) \end{array}.$$

# Performance Criteria

| | L-P Bounds | GL-GP Bounds |
|---|---|---|
| MST(LPS) | 9 296 | 4 460 |
| SP-GEKS(LPS) | 10 506 | 5 584 |
| SP-Union(LPS) | 11 292 | 6 624 |
| SP-Sum(LPS) | 10 792 | 5 885 |
| MST(W1) | 10 816 | 6 129 |
| SP-GEKS(W1) | → **11 316** | 7 212 |
| SP-Union(W1) | → **11 357** | → **7 553** |
| SP-Sum(W1) | → **11 350** | → **7 309** |
| MST(W2) | → **11 352** | → **7 497** |
| SP-GEKS(W2) | → **11 342** | → **7 360** |
| SP-Union(W2) | 10 648 | 5 816 |
| SP-Sum(W2) | → **11 387** | → **7 426** |
| MST(W3) | → **11 360** | → **7 439** |
| SP-GEKS(W3) | → **11 359** | → **7 327** |
| SP-Union(W3) | 10 634 | 5 781 |
| SP-Sum(W3) | →**11 381** | → **7 381** |
| GEKS | 11 305 | 7 269 |
| Maximum possible | 11 994 | 12 090 |