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Approaches to Linking the Regions

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Table of Contents

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1. Introduction

In these notes, we will discuss various possible methods for linking the five regions in the ICP in such a way that the relative country volumes within each region are preserved.

Diewert (2004; 46-47) suggested a method for linking the regions in the 2005 ICP round that would preserve relative volumes for country aggregates within a region but at the same time would link the various regions so that volumes could be compared across all countries in the ICP comparison in a consistent manner. This is Diewert's Approach 2 and he made the following assertion about the method:

"The resulting set of country parities will respect the within region parities that have been determined by the regions. The overall procedure does not depend on the choice of numeraire countries, either within regions or between regions; i.e., the relative country parities will be the same no matter what the choices are for the numeraire countries." W.E. Diewert (2004; 47).

Sergey Sergeev (2009b) noted that Diewert's suggested method was not invariant to the choice of numeraire countries. Unfortunately, Diewert's exposition of his Approach 2 is lacking in precision and it is difficult to reconstruct exactly what he had in mind[1] and so we will attempt to lay out his suggested method in a more precise manner. In fact, there are two possible variants to his suggested method and we discuss both variants in section 2 below. The first variant of his method is invariant to the choice of the numeraire countries provided that the method of aggregation satisfies certain properties but the second variant is not independent of the choice of numeraire countries within the regions.. We will also discuss a third variant of Diewert's basic approach to linking the regions which was suggested by Sergeev (2009b) and this third approach is invariant to the choice of the numeraire countries and the numeraire region. However, all three variants are subject to substitution bias and so for ICP 2011, other methods for linking the regions should be considered.

In section 3, we look at another possible method for constructing interregional parities. Yuri Dikhanov (2007) suggested that we simply use GEKS[2] method for all countries in the comparison and this will generate shares of world output (or a subaggregate) for each country. Now simply add up the shares of world output of the countries in each region in order to obtain regional shares. These between region shares plus the within region shares

[1] His exposition has some typos and the verbal exposition does not always agree with the algebra in the equations.
[2] See Gini (1924) (1931), Eltetö and Köves (1964) and Szulc (1964).

(determined independently by the regions) will generate an overall set of shares that respect within region parities.

In section 4, we look at a variant of the method used in section 3: instead of using GEKS to generate individual country shares for each country in the comparison, we could use Robert Hill's spatial linking method[3] to form the shares of each country's output in the world aggregate. The basic idea behind this method is to link countries through a series of bilateral comparisons where the overall comparison rests on bilateral comparisons between countries which are most similar in their (relative) price structures.

In section 5, we compute the regional parities using the methods suggested in sections 2-4 for a small artificial numerical example.

In section 6, we present another numerical example based on a subset of the 2005 ICP data to again illustrate the fact that different methods can give quite different results.

Section 7 concludes.

An Appendix discusses some issues raised by Angus Deaton when countries are consuming quite different baskets.

2. Diewert's Suggested Method for Linking the Regions

We start out by defining what basic data are needed in order to link the regions in a fashion that will respect the parities and relative volumes that have been determined within each region. We assume that there are R regions in the comparison and region r has C(r) countries in it for r = 1,2,...,R. In ICP 2005, R was equal to 5 with varying numbers of countries in each region. We assume that there are N commodity groups and that each country in the comparison has collected expenditure data on these N commodity groups in their own currencies. In ICP 2005, N was equal to 155. Let E_{rcn} be the expenditure (in the currency of country c in region r) on commodity class n for the reference year for r = 1,...,R; c = 1,...,C(r); n = 1,...,N. We assume that these country expenditure data have been collected.

We assume that each region r has constructed a *Purchasing Power Parity* (PPP) for each commodity group n and each country c in the region, α_{rcn} say, for r = 1,...,R; c = 1,...,C(r) and n = 1,...,N. We assume that country c = 1 is the numeraire country in each region and so the PPP for this numeraire country is set equal to unity; i.e., we have:

$$(1)\ \alpha_{r1n} = 1\ ; \qquad r = 1,...,R;\ n = 1,...,N.$$

Thus for $c \neq 1$, α_{rcn} is the price in the currency of country c in region r of a bundle of commodity n which costs one currency unit in the currency of the numeraire country (country 1) for region r.

[3] See Robert Hill (1999a) (1999b) (2001) (2004). Fisher (1922; 271-274) is a precursor to the work of Hill.

Our next assumption is that the central office has constructed a set of *interregional linking PPPs*, β_{rn}, that link the PPPs of the numeraire country in each region with the numeraire country in the numeraire region, which we assume to be region 1. Thus we assume that

$$(2)\ \beta_{1n} = 1\ ; \qquad n = 1,...,N.$$

For $r \neq 1$, β_{rn} is the price in the currency of country 1 in region r of a bundle of commodity n which costs one currency unit in the currency of the numeraire country 1 in region 1, the numeraire region.

Now the two sets of PPPs can be multiplied together to form a consistent set of *world PPPs*, γ_{rcn}, defined as follows:

$$(3)\ \gamma_{rcn} \equiv \beta_{rn}\alpha_{rcn}\ ; \qquad r = 1,...,R;\ c = 1,...,C(r);\ n = 1,...,N.$$

Thus γ_{rcn} is the price in the currency of country c in region r of a bundle of commodity n which costs one currency unit in the currency of the numeraire country (country 1) in the numeraire region (region 1).

If we want to change the numeraire region, what happens to the world PPPs γ_{rcn} defined by (3)? Thus suppose we want region 2 to become the numeraire region in place of region 1. In this case, the original linking PPPs (the β_{rn}) should be replaced by the new linking PPPs defined as

$$(4)\ \beta_{rn}^{*} \equiv \beta_{rn}/\beta_{2n}\ ; \qquad r = 1,...,R;\ n = 1,...,N$$

and the original set of world PPPs, the γ_{rcn} defined by (3) should be replaced by the following new set of world PPPs:

$$(5)\ \gamma_{rcn}^{*} \equiv \beta_{rn}^{*}\alpha_{rcn} = \gamma_{rcn}/\beta_{2n}\ ; \qquad r = 1,...,R;\ c = 1,...,C(r);\ n = 1,...,N.$$

Thus the new set of world PPPs are equal to the old set of world PPPs except for a divisor that depends on the commodity n under consideration. Hence the net effect of switching the numeraire region is to leave the PPPs unchanged except that the new set of PPPs are measured in a *new system of units*; instead of measuring commodity units in terms of a dollar's worth of purchases of commodity n in the numeraire country of region 1, we now measure commodity units in terms of a peso's worth of purchases of commodity n in the numeraire country of region 2.

Now suppose that we want to change the numeraire country in a region. If the change of numeraire country is outside region 1, (say in region 2 where we switch from country 1 to country 2 as the numeraire country), then it can be seen that nothing will happen to the world PPPs γ_{rcn}: the old $\gamma_{rcn} = \beta_{rn}\alpha_{rcn}$ will remain the same for all regions r except for r = 2 and for the second region, α_{2cn} will be replaced by $\alpha_{2cn}/\alpha_{22n}$ for n = 1,...,N and c =

1,...,C(2) and β_{2n} will be replaced by $\beta_{2n}\alpha_{22n}$ for n = 1,...,N. Thus the new γ_{rcn}^{*} will still equal the old γ_{rcn} for all r including r = 2.

If the change of numeraire country is within region 1 so that the new numeraire country in region 1 is country 2 in place of country 1, then the within region parities for region 1 become $\alpha_{1cn}/\alpha_{12n}$ for n = 1,...,N and the new set of interregional linking PPPs become β_{2n}/α_{12n} for n = 1,...,N. Thus in this case, the new set of world PPPs becomes:

$$(6)\ \gamma_{rcn}^{*} = \gamma_{rcn}/\alpha_{12n}\ ; \qquad r = 1,\ldots,R;\ c = 1,\ldots,C(r);\ n = 1,\ldots,N.$$

Thus in this case of a change of the numeraire country within the numeraire region, the new set of world PPPs are equal to the old set of world PPPs except for a divisor that depends on the commodity n under consideration. Hence as was the case with a change in the numeraire region, the net effect of switching the numeraire country within the numeraire region is to leave the PPPs unchanged except that the new set of PPPs are measured in a *new system of units*.

Recall that we have assumed a knowledge of the expenditures (in the local currencies) of country c in region r by commodity class n, E_{rcn}, for r = 1,...,R; c = 1,...,C(r); n = 1,...,N. We use this expenditure information along with the consistent set of world PPPs, the γ_{rcn} defined by (3) above, in order to define consistent (across countries) *imputed quantities or volumes*, Q_{rcn}, for each commodity n and each country c in each region r as follows:

$$(7)\ Q_{rcn} \equiv E_{rcn}/\gamma_{rcn}\ ; \qquad r = 1,\ldots,R;\ c = 1,\ldots,C(r);\ n = 1,\ldots,N.$$

These imputed quantities are measured in units which are comparable across countries and regions. Thus these quantities can be added across countries within a region and the resulting regional totals, Q_{rn}, are also comparable across regions; i.e., define *regional total volumes or quantities by commodity class* as follows:

$$(8)\ Q_{rn} \equiv \textstyle\sum_{c=1}^{C(r)} Q_{rcn}\ ; \qquad r = 1,\ldots,R;\ n = 1,\ldots,N.$$

Now form *regional quantity or volume vectors* from the Q_{rn}:

$$(9)\ Q^{r} \equiv [Q_{r1},\ldots,Q_{rN}]\ ; \qquad r = 1,\ldots,R.$$

Note that if we change the numeraire region or the numeraire countries within a region, then provided that we change the world PPPs in a consistent manner, the regional quantity vectors will be identical to the initial regional quantity vectors defined by (7)-(9) except that the units of measurement for these vectors may be have been changed by the change in numeraires; i.e., a change in the numeraire region or numeraire country within a region will lead to identical regional quantity vectors Q^{r} or to a new set of Q^{r*} which are equal to $\hat{\delta}\,Q^{r}$ where $\hat{\delta}$ is a diagonal matrix where the diagonal elements reflect changes in the units of measurement of the N commodity groups.

We now need regional price vectors P^r to match up with the above regional quantity vectors Q^r defined by (9). There are at least three possible strategies at this point and we will discuss each in turn.

Option 1: The Conversion to Regional Currency Unit Values Option:

Our strategy using this option will be to convert country expenditures in each commodity classification category into common regional expenditures (in a numeraire country's currency) using market exchange rates for the reference year and then deflate these regional commodity expenditures by the corresponding regional quantities defined by (8) in order to form regional unit value prices, which will be used as the regional prices. This is the same type of strategy that is used by national income accountants in forming annual price and quantity vectors from subannual information.

Suppose the *reference year exchange rate* for country c in region r is ε_{rc} for r = 1,...,R and c = 1,...,C(r). We assume that country 1 in each region is the numeraire region as usual so that

$$(10)\ \varepsilon_{r1} = 1\ ; \qquad r = 1,\ldots,R.$$

Thus ε_{rc} for $c \neq 1$ tells us how many units of the numeraire (country 1) currency in region r is equal to one unit of the currency unit of country c in region r. These market exchange rates can be used to convert country expenditures (in the country's currency) on commodity class n within region r into region r numeraire currency units, $V_{rcn} \equiv \varepsilon_{rc}E_{rcn}$, and then these *country expenditures on commodity class n in a common regional currency* can be summed to regional totals V_{rn} defined as follows:

$$(11)\ V_{rn} \equiv \textstyle\sum_{c=1}^{C(r)} \varepsilon_{rc}E_{rcn}\ ; \qquad r = 1,\ldots,R\ ;\ n = 1,\ldots,N.$$

Now *regional unit value prices* P_{rn} can be formed by taking the regional values defined by (11) and dividing them by the corresponding regional quantity totals Q_{rn} defined by (8):

$$(12)\ P_{rn} \equiv V_{rn}/Q_{rn}\ ; \qquad r = 1,\ldots,R\ ;\ n = 1,\ldots,N.$$

Finally, form *regional unit value price vectors* from the components, P_{rn}:

$$(13)\ P^r \equiv [P_{r1},\ldots,P_{rN}]\ ; \qquad r = 1,\ldots,R.$$

We now consider what happens to the regional total expenditures on commodity class n, V_{rn}, if we change a numeraire country within a region or change the numeraire region. If we change the numeraire region, nothing happens to regional expenditures defined by (11) since we do not need to relate the regional exchange rates across regions. However, if we change the numeraire country within a region r, it can be seen that that all of the regional values for this region will change by a scalar factor; i.e., if in region r, we change the numeraire country from country 1 to country 2, then the new market exchange rates will be $\varepsilon_{rc}/\varepsilon_{r2}$ for c = 1,...,C(r) and hence the new regional totals for region r will be

$$(14)\ V_{rn}^{*} \equiv \sum_{c=1}^{C(r)} [\varepsilon_{rc}/\varepsilon_{r2}]E_{rcn} = V_{rn}/\varepsilon_{r2}\ ; \qquad n = 1,....,N.$$

Thus the new regional expenditure totals on the N commodity groups in region r, V_{rn}^{*}, will be equal to a scalar multiple $(1/\varepsilon_{r2})$ times the old regional expenditure totals in region r, V_{rn}.

The above is a careful exposition of the regional unit value method for forming regional price and quantity vectors that could be used to link the regions by using a multilateral index number method with the above regional price and quantity vectors as the input vectors to the method.

We now consider linking the R regions using a *multilateral index number method*. Diewert (1999) defined such methods in terms of share functions for the R regions; i.e., he looked at a system of *shares of world output*, $s_1(P^1,....,P^R;Q^1,...,Q^R)$,..., $s_R(P^1,....,P^R;Q^1,...,Q^R)$, where $s_r(P^1,....,P^R;Q^1,...,Q^R)$ is region r's share of world output and considered the axiomatic properties of such multilateral systems. The final question we want to address in this note is: what properties do we require on the multilateral method in order to ensure that the regional shares are independent of the choice of the numeraire region and the numeraire countries within the regions?

From the above discussion, it is clear that the multilateral method must satisfy the following two properties:

Property 1: Invariance to changes in the units of measurement.

Thus let $\delta_n > 0$ for n = 1,...,N and define $\hat{\delta}$ as the N by N matrix with the elements δ_n running down the main diagonal. Then this property requires that the multilateral share system satisfy the following equations:

$$(15)\ s_r(\hat{\delta}P^1,....,\hat{\delta}P^R;\hat{\delta}^{-1}Q^1,...,\hat{\delta}^{-1}Q^R) = s_r(P^1,....,P^R;Q^1,...,Q^R)\ ; \qquad r = 1,...,R.$$

Property 2: Homogeneity of degree zero in the regional price vectors.

Let $\lambda_1 > 0$, ..., $\lambda_R > 0$. Then this property requires that the multilateral share system satisfy the following equations:

$$(16)\ s_r(\lambda_1P^1,....,\lambda_RP^R;Q^1,...,Q^R) = s_r(P^1,....,P^R;Q^1,...,Q^R)\ ; \qquad r = 1,...,R.$$

This homogeneity property means that it is relative regional prices that affect the interregional volume comparisons and not the absolute level of regional prices.

It can be seen that the above two properties are also sufficient to imply that a multilateral method using the unit value regional price and quantity vectors defined above will generate regional shares that are independent of the choice of the numeraire region and the numeraire countries within the regions.

A drawback of this method for linking the regions is that it brings market exchange rates into the picture. Since market exchange rates are often far removed from their corresponding PPPs, it would be good to avoid their use in constructing the PPPs! The following two methods make use of the regional quantity vectors Q^r defined by (8) but the corresponding regional price vectors P^r do not make use of exchange rates.

Option 2: The Use of Regional Numeraires as Regional Price Weights

It is not necessary to bring in market exchange rates to convert regional expenditures into a common currency. Rather than doing this, we can simply use the PPPs for each numeraire country 1 in each region r, relative to the numeraire country in the numeraire region, the β_{rn} described above, as the price for commodity n in region r; i.e., define the regional price for commodity n in region r as follows:

$$(17)\ P_{rn} \equiv \beta_{rn}\ ; \qquad r = 1,\ldots,R\ ;\ n = 1,\ldots,N.$$

Then use (13) to form the *regional price vectors* P^r in the usual way. The regional quantity vectors, Q^r, are defined as before by (7)-(9) and these equations do not involve exchange rates. Finally, use these regional price and quantity vectors, P^r and Q^r, in order to calculate the share functions for the R regions using a favoured multilateral method, $s_1(P^1,\ldots,P^R;Q^1,\ldots,Q^R),\ldots, s_R(P^1,\ldots,P^R;Q^1,\ldots,Q^R)$, where $s_r(P^1,\ldots,P^R;Q^1,\ldots,Q^R)$ is region r's share of world output.

Unfortunately, as Sergeev (2009b) pointed out, this method is not invariant to the choice of the numeraire countries within the regions. Thus this method should not be used in ICP 2011.

Option 3: The Use of the Geometric Average of the Regional Numeraires as Regional Price Weights

Sergeev (2009b) suggested a way to avoid the lack of numeraire invariance in Option 2: within each region, take the *geometric mean of the country parities* over all countries in the region. Thus equations (17) are replaced by the following equations:[4]

$$(18)\ P_{rn} \equiv \Pi_{c=1}^{C(r)}\, \gamma_{rcn}^{1/C(r)}\ ; \qquad r = 1,\ldots,R\ ;\ n = 1,\ldots,N.$$

[4] This method of aggregation within a region is related to the geometric average price multilateral method originally suggested by Walsh (1901; 381 and 398) which was noted by Gini (1924; 106) and implemented by Gerardi (1982; 387). These authors used reference world prices that were the geometric mean over all countries in the world that were applied to all countries so that the resulting volume estimates are additive over all countries and all regions. In section 6 when we implement the Sergeev method using a subset of the ICP 2005 data, the regional parities price parities P_{rn} are equal to a regional constant times the parities defined by (18). This will not affect the final regional parities.

Then use (13) to form the *regional price vectors* P^r in the usual way. Finally, use these regional price and quantity vectors defined by (9), P^r and Q^r, in order to calculate the share functions for the R regions using a favoured multilateral method, $s_1(P^1,....,P^R;Q^1,...,Q^R)$,..., $s_R(P^1,....,P^R;Q^1,...,Q^R)$, where $s_r(P^1,....,P^R;Q^1,...,Q^R)$ is region r's share of world output. If the multilateral method satisfies Property 1 above (Invariance to changes in the units of measurement), then the resulting regional shares will be invariant to the choices of both numeraire countries and the numeraire region.

As mentioned above, we can rule out Option 2 as a method for linking the regions for ICP 2011 due to its lack of country numeraire invariance within the regions. However, there is a good case for ruling out Options 1 and 3 as well. This is due to the fact that all of the methods in this section *impose a set of common prices to add up the quantities within a region*; i.e., the methods impose a form of *additivity*. Additive multilateral methods are subject to *substitution bias* if there are three or more countries in the comparison.[5]

Thus in the following two sections, we turn to *nonadditive methods* for linking the regions which will avoid the substitution bias that is inherent in the methods discussed in the present section.

3. The Global Comparison GEKS Method

Recall definitions (7) which defined the quantity of commodity n, Q_{rcn}, that was finally demanded by purchasers in country c in region r. Define the country c in region r quantity or volume vector, Q^{rc}, in the usual way, using definitions (7) to define the components, Q_{rcn}:

$$(19)\ Q^{rc} \equiv [Q_{rc1},...,Q_{rcN}]\ ; \qquad r = 1,...,R;\ c = 1,...,C(r).$$

The country price vectors P^{rc} that correspond to the country quantity vectors Q^{rc} defined by (19) are defined using the global set of PPPs, γ_{rcn}, defined by equations (3) above. Recall that γ_{rcn} is the price in the currency of country c in region r of a bundle of commodity n which costs one currency unit in the currency of the numeraire country (country 1) in the numeraire region (region 1). The basic heading price vector for country c in region r, P^{rc}, is defined as follows:

$$(20)\ P^{rc} \equiv [\gamma_{rc1},...,\gamma_{rcN}]\ ; \qquad r = 1,...,R;\ c = 1,...,C(r).$$

Thus we have country price and quantity vectors (P^{rc},Q^{rc}) for all C(1) + C(2) + ... + C(R) countries in the ICP.

At this stage, any multilateral method could be used in order to form price comparisons between each individual country participating in the International Comparisons Project.

[5] For an explanation of the problem, see Marris (1984; 52) and Diewert (1999; 48-50).

One such multilateral method which has good axiomatic and economic properties is the GEKS method.[6] The algebra for this method works as follows. First define the *Fisher* (1922) *quantity index* for country c in region r relative to country d in region s as follows:[7]

$$(21)\ Q_F(rc/sd) \equiv [P^{rc}\cdot Q^{rc}\ P^{sd}\cdot Q^{rc}/P^{rc}\cdot Q^{sd}\ P^{sd}\cdot Q^{sd}]^{1/2}\ ;$$
$$r = 1,\ldots,R;\ c = 1,\ldots,C(r)\ ;\ s = 1,\ldots,R\ ;\ d = 1,\ldots,C(s)$$

where $P^{rc}\cdot Q^{rc} \equiv \sum_{n=1}^{N} P_{rcn}Q_{rcn}$ denotes the inner product between the vectors P^{rc} and Q^{rc}. If we fix the base country (i.e., fix the region s and the country d in region s) and let r = 1,...,R and c = 1,...,C(r), then the Fisher indexes defined by (21) will give the volume of each country rc in the comparison relative to the base country sd and then these relative volumes can be normalized into a set of shares of world product using country sd as the base country; i.e., we obtain a set of country "star" shares for each rc with country sd as the "star" country.[8] The GEKS method then takes the geometric mean of all of these country parities over all possible "star" bases. Thus define these geometric mean relative parities as follows:

$$(22)\ Q(rc) \equiv [\prod_{s=1}^{R}\prod_{d=1}^{C(s)} Q_F(rc/sd)]^{1/[C(1)+\ldots+C(R)]}\ ; \qquad r = 1,\ldots,R;\ c = 1,\ldots,C(r).$$

Now normalize the relative parities given by (22) into country shares of world product. Thus define the sum of the parities defined by (22) as σ:

$$(23)\ \sigma \equiv \sum_{r=1}^{R} \sum_{c=1}^{C(r)} Q(rc).$$

The *GEKS share of world product for country c in region r* can now be defined as Q(rc) divided by σ:

$$(24)\ s_{rc} \equiv Q(rc)/\sigma\ ; \qquad r = 1,\ldots,R;\ c = 1,\ldots,C(r).$$

Following Dikhanov (2007), we can aggregate over the individual country shares of world product defined by (24) within each region in order to obtain the following *GEKS regional shares of world output*:

$$(25)\ S_r \equiv \sum_{c=1}^{C(r)} s_{rc}\ ; \qquad r = 1,\ldots,R.$$

The regional shares $S_1, S_2,\ldots,S_R$ defined by (25) can be used in conjunction with the regional shares in each region in order to obtain a system of world product shares for

[6] See Gini (1924) (1931), Eltetö and Köves (1964) and Szulc (1964) and Diewert (1999) for the properties of this method.
[7] Instead of using the Fisher ideal quantity index as the basic building block for this method, any other superlative quantity index could be used in this multilateral method. However, we prefer to use the Fisher index due to its strong axiomatic properties; see Diewert (1992). See Diewert (1976) for the definition of a superlative (bilateral) index number formula.
[8] Kravis (1984; 10) introduced this "star" terminology.

each country that respect the regional parities that are independently determined by the regions.

Why are the GEKS regional shares defined by (25) to be preferred over the regional shares defined in the previous section? The reason for this preference is that the GEKS shares are consistent with broader patterns of substitutability between commodities; i.e., if the preferences of each country can be represented by certain homothetic preferences (that can approximate arbitrary homothetic preferences to the second order), then the GEKS country shares will give exactly the "right" relative volumes across countries.[9]

In the following section, we turn to a variant of the method used in this section.

4. Spatial Comparisons Based on Similar Price Structures

The GEKS multilateral method treats each country "star" parity as being equally valid and hence an averaging of the parities is appropriate under this hypothesis. However, is it really the case that all bilateral comparisons of volume between two countries are equally accurate? One could argue that the answer to this question is that the answer is no: if the relative prices in countries A and B are very similar, then the Laspeyres and Paasche quantity indexes will be very close to each other and hence it is likely that the "true" volume comparison between these two countries (using the economic approach to index number theory) will be very close to the Fisher volume comparison. On the other hand, if the structure of relative prices in the two countries is very different, then it is likely that the structure of relative quantities in the two countries will also be different and hence the Laspeyres and Paasche quantity indexes will likely differ considerably and we can no longer be certain that the Fisher quantity index will be close to the "true" volume comparison. The above considerations suggest that a more accurate set of world product shares could be constructed if we started out making a bilateral comparison between the two countries which had the most similar relative price structures. We could then look for a third country which had the most similar relative price structure to the first two countries and link in this third country to the comparisons of volume between the first two countries and so on. At the end of this procedure, we would have a *minimum spanning tree*: a path between all countries that minimized the sum of the relative price similarity measures. This linking methodology has been developed by Robert Hill (1999a) (1999b) (2001) (2004) (2009).

A key aspect of this methodology is the choice of the measure of similarity (or dissimilarity) of the relative price structures of two countries. Various measures of the similarity or dissimilarity of relative price structures have been proposed by Aten and Heston (2009), Diewert (2009), Hill (2009) and Sergeev (2001) (2009a). We will focus

[9] On the other hand, the additive methods discussed in the previous section are consistent with homothetic preferences that can provide only a first order approximation to arbitrary homothetic preferences. For further explanation of this point, see Diewert (1999; 31) who introduced the concept of a superlative multilateral system. The GEKS system is a superlative method, whereas the additive methods are not. Balk (2009;82) provided a recent overview of various multilateral methods and endorsed the GEKS-Fisher method as a centre stage method, particularly from the economic approach to international comparisons.

here on one of Diewert's (2009; 207) measures of relative price similarity, the *weighted log quadratic measure of relative price dissimilarity*, $\Delta_{PLQ}(p^1,p^2,q^1,q^2)$, (the smaller the measure, the more similar is the structure of relative prices between the two countries):

$$(26)\ \Delta_{PLQ}(p^1,p^2,q^1,q^2) \equiv \sum_{n=1}^{N} (1/2)(s_n^{\,1} + s_n^{\,2})[\ln(p_n^{\,2}/p_n^{\,1}P_F(p^1,p^2,q^1,q^2))]^2$$

where $P_F(p^1,p^2,q^1,q^2) \equiv [p^2\cdot q^1\ p^2\cdot q^2/p^1\cdot q^1\ p^1\cdot q^2]^{1/2}$ is the Fisher ideal price index between countries 2 and 1 and $s_n^{\,c} \equiv p_n^{\,c}q_n^{\,c}/p^c\cdot q^c$ is the country c expenditure share on commodity n for c = 1,2 and n = 1,...,N.

It can be seen that if prices are proportional for the two countries so that $p^2 = \lambda p^1$ for some positive scalar λ, then $P_F(p^1,p^2,q^1,q^2) = \lambda$ and the measure of relative price dissimilarity $\Delta_{PLQ}(p^1,p^2,q^1,q^2)$ defined by (26) will equal its minimum of 0. Thus the smaller is $\Delta_{PLQ}(p^1,p^2,q^1,q^2)$, the more similar is the structure of relative prices in the two countries.

We will illustrate Hill's method of spatial linking using the relative price dissimilarity measure defined by (26) in the following two sections using small numerical examples. Basically, instead of using the GEKS country shares s_{rc} defined by (24) in the previous section, we use the shares generated by the minimum spanning tree to link all of the countries in the International Comparison Project. Once these country shares s_{rc} have been defined, we again use equations (25) in order to form the regional shares S_r.

The narrowing of Paasche and Laspeyres spreads by the use of a spatial linking method is not the only advantage of this method of linking countries. There are advantages at *lower levels of aggregation* in that if we compare similar in structure countries, we will find that product overlaps are maximized:

> "Many differences in quality and proportion of high tech items discussed above are likely to be more pronounced between countries with very different economic structures. If criteria can be developed to identify countries with similar economic structure and they are compared only with each other, then it may overcome many of the issues of quality and lowest common denominator item comparisons. Economically similar countries are likely to have outlet types in similar proportions carrying the same types of goods and services. So direct comparisons between such countries will do a better job of holding constant the quality of the items than comparisons across more diverse countries." Bettina Aten and Alan Heston (2009; 251).

The above quotation suggests that perhaps the similarity criterion should not be based only on the similarity of the structure of relative prices across the two countries being compared. In addition, we should look at the degree of similarity in the structure of absolute per capita quantity vectors and take a sum of the two measures of similarity as our overall measure of similarity in structure.

There are some disadvantages to the spatial linking method. The two most important disadvantages are:

- The path of bilateral links between countries generated by the method tends to be unstable; i.e., the most similar tree linking the countries tends to change when we move from one cross sectional comparison between countries to another cross sectional comparison.

- Some countries in the comparison will inevitably have lower quality data than other countries and if these poorer data quality countries end up having many bilateral links with many countries in the minimum spanning tree, then the quality of the entire comparison may be low.

Hill (2009) discusses both of these problems and offers “reasonable” solutions to these difficulties. The first difficulty is not really a difficulty if the overall volume comparisons remain more or less the same even if the particular bilateral links change. In particular, it may be the case that countries break up into two or more relatively homogeneous groups. Within each group, the bilateral dissimilarity measures are all low so even if the links within each group change, the relative volume indexes within each group remain roughly unchanged. The key problem then boils down to the bilateral links between the various groupings. In order to get more stability between these groupings, it may be advisable to have more than one link between the groupings and this constraint can readily be imposed. The second difficulty can be dealt with by specifying that countries with lower quality data are not allowed to have more than one link in the overall tree of comparisons.

Of course, a problem with the above “solutions” to the problems associated with spatial linking is that the solutions appear to have an ad hoc character and this may lead to charges by outside observers that the ICP is being manipulated. This potential problem could be mitigated by experimentation with the ICP 2005 data set so a firm a priori strategy could be put in place before the results of ICP 2011 were calculated.

In order to get an impression as to how the different methods suggested in this section and the previous 2 sections perform in practice, we will consider two small numerical examples in the following two sections. The first example uses an artificial data set and the second example uses a small subset of the ICP 2005 data.

5. An Artificial Data Set Numerical Example

We construct an example where there are only 4 countries and only 2 commodities. There are two regions where region 1 consists of the first two countries and region 2 consists of the second two countries. The basic data for the countries is the within region r Basic Heading PPP for commodity n for country c in region r, α_{rcn}, the expenditure on commodity class n for country c in region r in domestic currency, E_{rcn}, and the market exchange rate for country c in region r, ε_{rc} (which does not depend on the commodity n), for r = 1,2, c = 1,2 and n = 1,2. We also need the Basic Heading interregional PPPs for the numeraire countries in each region, the β_{rn}, which is the parity for commodity n for region r relative to region 1. See Table 1 below for a listing of the data.

Table 1: Data for Four Countries in Two Regions

Region 1		Region 2		Regional
Country 1	Country 2	Country 1	Country 2	BH Parities

n	α_{11n}	E_{11n}	ε_{11}	α_{12n}	E_{12n}	ε_{12}	α_{21n}	E_{21n}	ε_{21}	α_{22n}	E_{22n}	ε_{22}	β_{1n}	β_{2n}
1	1	10	1	20	2	5	1	20	3	10	20	18	1	2
2	1	10	1	4	8	5	1	20	3	4	160	18	1	4

There are 8 world Basic Heading PPPs that are obtained using equations (3), $\gamma_{rcn} \equiv \beta_{rn}\alpha_{rcn}$. Thus we obtain for commodity class 1 the following world BH PPPs: $\gamma_{111} = 1$; $\gamma_{121} = 20$; $\gamma_{211} = 2$; $\gamma_{221} = 20$. For commodity class 2, we obtain the following world BH PPPs: $\gamma_{112} = 1$; $\gamma_{122} = 4$; $\gamma_{212} = 4$; $\gamma_{222} = 16$.

Working our way through the algebra in section 2, we find that the Option 1 (regional currency unit value method) shares of world output for Regions 1 and 2 turns out to equal 0.45134 for Region 1 and 0.54866 for Region 2. As expected, we get the same regional shares no matter which region is chosen as the numeraire region and no matter which country is chosen as the numeraire country within a region.

The Option 2 regional shares turn out to depend on the choice of the regional numeraire countries as expected. Letting Country 1 in each region be the numeraire country, we obtain 0.45676 and 0.54324 as the two regional shares of world product. Letting Country 2 in Region 1 be the numeraire country and maintaining Country 1 in Region 2 as the numeraire country leads to 0.46287 and 0.53713 as the new regional shares of world product. Letting Country 1 in Region 1 be the numeraire country and letting Country 2 in Region 2 be the numeraire country leads to 0.46041 and 0.53959 as the new regional shares of world product. The bottom line is that the Option 2 regional shares are not invariant to the choice of the numeraire countries in the regions.

The Option 3 regional shares[10] (Sergeev Option) turn out to be S_1 equal to 0.46186 and S_2 equal to 0.53814. Recall that the Option 1 regional shares were 0.45134 for Region 1 and 0.54866 for Region 2. Thus the Option 1 and 3 regional shares differ by about 2.3%.

We turn our attention to the methods explained in section 3 above. The individual country shares of world output using each of the four countries as the “star” in bilateral Fisher index number comparisons are listed in the first four rows of Table 2. It can be seen that there are some relatively large differences between these world shares, particularly for the small country, Country 2 in Region 1. The GEKS country shares of world product are listed in the last line of Table 2.

Table 2: Star and GEKS Country Shares of World Product

	Region 1		Region 2	
	Country 1	Country 2	Country 1	Country 2
Region 1, Country 1 Star	0.43523	0.02879	0.30775	0.22823
Region 1, Country 2 Star	0.47078	0.03114	0.32258	0.17550

[10] The Sergeev regional price and quantity vectors P^r and Q^r for r = 1,2 turn out to be P^1 = [4.472, 2.000], P^2 = [6.325, 8.000], Q^1 = [10.1, 12.0] and Q^2 = [11.0, 15.0].

Region 2, Country 1 Star	0.42439	0.02897	0.30009	0.24655
Region 2, Country 2 Star	0.44331	0.04125	0.28296	0.23248
GEKS	0.44436	0.03226	0.30387	0.21951

The GEKS volume shares in the last row of the above table are defined by equations (24); i.e., the entries for this last row are the GEKS shares s_{11}, s_{12}, s_{21} and s_{22} respectively. Using these country shares, the *regional GEKS shares* S_r are defined by (25) so that using the entries in the last line of Table 2, we have:

(27) $S_1 = s_{11} + s_{12} = 0.47662$; $S_2 = s_{21} + s_{22} = 0.52338$.

Thus the GEKS share of world output for Region 1 is 0.47662 which is higher than the corresponding Region 1 shares for the Option 1 method (0.45134) and for the Option 3 method (0.46479).

Finally, we turn to the spatial linking method. Recall that the weighted log quadratic measure of relative price dissimilarity between countries 1 and 2, $\Delta_{PLQ}(p^1,p^2,q^1,q^2)$, was defined by (26) above. The 4 by 4 matrix of relative price dissimilarity measures is listed in Table 3 below. For convenience in labeling the countries, we have set country 1 equal to Country 1 in Region 1, country 2 to Country 2 in region 1, country 3 to Country 1 in Region 2 and country 4 to Country 2 in Region 2.

Table 4: Weighted Log Quadratic Relative Price Dissimilarities between Countries i and j

	Country 1	Country 2	Country 3	Country 4
Country 1	0.00000	0.59465	0.12011	0.01057
Country 2	0.59465	0.00000	1.22741	0.25253
Country 3	0.12011	1.22741	0.00000	0.17933
Country 4	0.01057	0.25253	0.17933	0.00000

From the above Table, it can be seen that countries 1 and 4 have the most similar structures of relative prices with a dissimilarity measure equal to 0.01057. The next pair of countries that have most similar structure of relative price is countries 1 and 3 with a dissimilarity measure equal to 0.12011. Thus countries 3 and 4 can be linked to country 1 using the bilateral Fisher quantity index between 4 and 1 and between 3 and 1. The next most similar in structure pair of countries is 3 and 4 with a dissimilarity measure equal to 0.17933. But these two countries are already linked so we move on to the next lowest measure of relative price dissimilarity. The next most similar pair of countries is 2 and 4 with a dissimilarity measure equal to 0.25253. We use the bilateral Fisher quantity index to link country 2 to country 4 and now we have linked all of the countries using bilateral links. The volumes of all 4 countries relative to country 1 turn out to be 1.00000, 0.09305, 0.70711 and 0.52440. This leads to the following vector of country shares of world output: 0.43019, 0.04003, 0.30419, 0.22559. Adding up the shares of the countries in each region leads to the following *spatially linked regional shares of world output*:

(28) $S_1 = s_{11} + s_{12} = 0.47022$; $S_2 = s_{21} + s_{22} = 0.52978$.

Thus the spatial linking method leads to 0.47022 as the estimate of Region 1's share of world output compared to the GEKS estimate of 0.47662.

A summary of the Region 1 share of world output using the various methods can be found in Table 5 below.

Table 5: The Share of World Output for Region 1 using Various Methods

	Region 1 Share of World Output
Option 1 (Regional Unit Values Method)	0.45134
Option 2: Base Countries: 1 in Region 1; 1 in Region 2	0.45676
Option 2: Base Countries: 2 in Region 1; 1 in Region 2	0.46287
Option 2: Base Countries: 1 in Region 1; 2 in Region 2	0.46041
Option 3: Geometric Mean Average Prices in each Region	0.46186
GEKS	0.47662
Spatial Linking	0.47022

My own ranking of the methods would be Spatial Linking in first place, followed by GEKS, Option 3 and Option 1. The Option 2 methods are not suitable for ICP 2011 since they are not invariant to the choice of numeraire countries in the regions.

The differences between the various methods are fairly substantial: a 4.2 % difference in the share of Region 1 for the highest share (Spatial Linking) versus the lowest share (Regional Unit Values Method). In the following section, we consider another numerical example based on actual ICP 2005 data that lead to even bigger numerical differences between the various methods.

6. A Numerical Example Based on ICP 2005 Data

Yuri Dikhanov provided the author with some highly aggregated data (across Basic Heading groups) from ICP 2005 on 5 consumption components for 8 countries.
The 8 countries are:

- 1 = Hong Kong;
- 2 = Bangladesh;
- 3 = India;
- 4 = Indonesia;
- 5 = Brazil;
- 6 = Japan;
- 7 = Canada and
- 8 = US.

The 5 commodity groups are:

- 1 = durables;
- 2 = food, alcohol and tobacco;
- 3 = other nondurables excluding food, alcohol, tobacco and energy;
- 4 = energy and
- 5 = services.

The expenditure data (converted to US dollars) and the quantity data for the 8 countries are listed in the following Tables 6 and 7.

Table 6: Expenditures in US Dollars for Eight Countries and Five Consumption Categories

	Country 1	Country 2	Country 3	Country 4	Country 5	Country 6	Country 7	Country 8
1	14320	1963	23207	8234	52722	307547	94121	967374
2	10562	24835	176782	83882	105527	448995	82056	778665
3	14951	5100	60748	15158	60798	272875	69461	992761
4	2619	3094	42126	17573	39933	125835	43342	524288
5	62124	11627	166826	61248	273669	1736977	379629	5559458

Table 7: Quantities (Volumes) in Comparable Units for Eight Countries and Five Consumption Categories

	Country 1	Country 2	Country 3	Country 4	Country 5	Country 6	Country 7	Country 8
1	15523	2312	30189	9781	46146	280001	81021	967374
2	9164	47509	356756	138273	163868	251846	63689	778665
3	317564	10588	180964	29879	65274	200614	58261	992761
4	1095	3033	38377	22084	23963	59439	35714	524288
5	81148	47611	786182	223588	541236	1695136	417210	5559458

If we divide the entries in Table 6 (expenditures converted to US dollars at market exchange rates) by the entries in Table 7 (quantities in comparable units), we obtain the exchange rate converted (into US dollars) prices for each commodity class for each country. These prices are listed in Table 8 below.

Table 8: Prices of Consumption Components in US Dollars for Eight Countries and Five Consumption Categories

	Country 1	Country 2	Country 3	Country 4	Country 5	Country 6	Country 7	Country 8
1	0.92250	0.84905	0.76872	0.84184	1.14250	1.09838	1.16169	1.0
2	1.15255	0.52274	0.49553	0.60664	0.64398	1.78282	1.28839	1.0
3	0.85123	0.48168	0.33569	0.50731	0.93143	1.36020	1.19224	1.0
4	2.39178	1.02011	1.09769	0.79573	1.66644	2.11704	1.21359	1.0
5	0.76556	0.24421	0.21220	0.27393	0.50564	1.02468	0.90992	1.0

Thus the US price level for each commodity group is set equal to 1 and the other prices are the average domestic prices for the commodity group converted into US dollars at the

2005 market exchange rates. Note that for durables, Bangladesh has the lowest price level at 0.84 and Canada has the highest level at 1.16; for food, Bangladesh has the lowest prices at 0.52 while Japan has the highest at 1.78; for other nondurables, India has the lowest price level at 0.33 while Japan has the highest level at 1.36; for energy, Indonesia has the lowest price level at 0.79 while Japan has the highest at 2.12 and for services, India has the lowest price level at 0.21 while Japan has the highest level at 1.02. Thus the amount of price level variation across countries ranges from 38% for durables to 500% for services.

We first use the above data to compute relative consumption volumes for the 8 countries using various multilateral methods. Instead of normalizing the relative volumes into shares of "world" consumption, we will express the consumption of each country relative to the consumption of the US.

The GEKS method for comparing relative volumes was explained in section 3[11] and the Hill spatial linking method was explained in section 4 above. We also computed relative consumption volumes for the 8 countries using two other (additive) methods: the Geary-Khamis (GK) method[12] and the Iklé Dikhanov Balk (IDB) method.[13] The country consumption volumes relative to the US for the four multilateral methods are listed in Table 9 below.

Table 9: Country Consumption Volumes Relative to the US Using Four Multilateral Methods

Method	**HK**	**BGD**	**INDIA**	**INDO**	**BRA**	**JPN**	**CAN**	**US**
GEKS	0.01315	0.01332	0.15317	0.04966	0.09128	0.26556	0.07357	1.0
Spatial Linking	0.01349	0.01310	0.14720	0.04779	0.09214	0.27596	0.07429	1.0
GK	0.01386	0.01357	0.16258	0.05057	0.09613	0.27814	0.07431	1.0
IDB	0.01346	0.01392	0.16187	0.05143	0.09441	0.27076	0.07417	1.0

It can be seen that the relative consumption volumes generated by the two superlative methods (the GEKS and Spatial Linking Methods) are fairly close to each other and the relative consumption volumes generated by the two additive methods (GK and IDB) are also fairly close to each other but the additive methods tend to overstate the consumption levels of the poorer countries (Bangladesh, India, Indonesia and Brazil) relative to the US.[14]

[11] The Fisher star parities for the 7 countries relative to the US had the following relative volume ranges: Hong Kong: 0.012570 to 0.013550; Bangladesh: 0.012770 to 0.014370; India: 0.14351 to 0.16439; Indonesia: 0.046600 to 0.053220; Brazil: 0.089840 to 0.095040; Japan: 0.24629 to 0.27724 and Canada: 0.071290 to 0.074640.

[12] The GK method was suggested by Geary (1958) and Khamis (1972) provided an existence proof.

[13] Iklé (1972; 202-204) suggested the method in a very indirect way, Dikhanov (1994) (1997) suggested a much clearer system of equations which define the method and Balk (1996; 207-208) provided the first existence proof

[14] The GK volume relative to the GEKS volume for India was 6.1% higher and the IDB volume relative to the GEKS volume for India was 5.7% higher.

It is of some interest to determine which countries have the most similar structure of relative prices (and of course, this information is required in order to implement the Hill type spatial linking method). Recall that the weighted log quadratic measure of relative price dissimilarity between countries 1 and 2, $\Delta_{PLQ}(p^1,p^2,q^1,q^2)$, was defined by (26) above. The 8 by 8 matrix of these relative price dissimilarity measures is listed in Table 10 below.

Table 10: Weighted Log Quadratic Relative Price Dissimilarities between Eight Countries

	HK	**BGD**	**IND**	**INDO**	**BRA**	**JPN**	**CAN**	**US**
HK	0.00000	0.10056	0.11017	0.09067	0.07011	0.01381	0.03660	0.06143
BGD	0.10056	0.00000	0.01188	0.01165	0.05632	0.10506	0.13237	0.23223
IND	0.11017	0.01188	0.00000	0.03133	0.08980	0.13429	0.18955	0.29841
INDO	0.09067	0.01165	0.03133	0.00000	0.07084	0.07726	0.09610	0.19600
BRA	0.07011	0.05632	0.08980	0.07084	0.00000	0.09146	0.08770	0.14328
JPN	0.01381	0.10506	0.13429	0.07726	0.09146	0.00000	0.01904	0.05322
CAN	0.03660	0.13237	0.18955	0.09610	0.08770	0.01904	0.00000	0.02020
US	0.06143	0.23223	0.29841	0.19600	0.14328	0.05322	0.02020	0.00000

Looking at the above Table, it can be seen that the 8 countries group themselves into two groups that have similar price structures: the rich countries HK, JPN, CAN and US form one group and the poorer countries BGD, INDIA, INDO and BRA form the other group. The linking between the two groups took place via Hong Kong and Brazil.[15] The details of the spatial linking process are as follows. The rich countries are 1,6,7 and 8. Country 7 (CAN) is linked to 8 (US) (the dissimilarity measure Δ equals 0.0202) and 7 (CAN) is linked to 6 (JPN) as well (Δ = 0.019). Then country 6 (JPN) is linked to 1 (HK) (Δ = 0.0138) and this completes the linking of the rich countries. Country 2 acts as a star country for the poorer countries: 2 (BGD) is linked to 4 (INDO) (Δ = 0.0116); 2 (BGD) is linked to 3 (INDIA) (Δ = 0.0118) and 2 (BGD) is linked to 5 (BRAZIL) (Δ = 0.056). Finally, the two groups of countries are linked via countries 1 (HK) and 5 (BRAZIL) (Δ = 0.070).

We will now consider the problems associated with forming regional shares of "world" consumption where the "world" is simply the 8 countries listed above. Suppose that the first four countries form an "Asian" region, Region 1 and the remaining four countries form a G20 region, Region 2. Obviously, the Dikhanov method for forming regional shares can be applied to the data listed in Table 9 above. The consumption volumes (relative to the US) listed there for the GEKS, Spatial Linking, GK and IDB methods can be converted into shares of "world" consumption and then the first four country shares can be summed to form the Region 1 shares, S1 and S2. The resulting Region 1 shares for the four methods are listed below in Table 14.

[15] Note that another possible bilateral link between the two regions would be via Indonesia and Japan which have a dissimilarity measure equal to 0.07726 which is a bit higher than the Hong Kong and Brazil dissimilarity measure which was equal to 0.07011.

We also need to calculate the Region 1 shares of "world" consumption for the Option 1 (Diewert) and Option 3 (Sergeev) methods. Equations (7) and (8) are first used to calculate the country and regional quantity vectors where the country expenditures E_{rcn} are listed in Table 6 above and the country Basic Heading PPPs (which can be set equal to the γ_{rcn}) are listed in Table 8 above. The resulting matrix of regional consumption quantities, Q_{rn}, are listed in Table 11 below.

Table 11: Regional Consumption Quantities or Volumes Q_{rn} by Commodity

Commodity	**1**	**2**	**3**	**4**	**5**
Region 1	57805	551702	238995	64589	1138529
Region 2	1374542	1258068.	1316910	643404	8213040

The regional unit value commodity prices n for each region r, P_{rn}, defined by (12) are listed in Table 12 below.

Table 12: Regional Unit Value Consumption Prices P_{rn} by Commodity

Commodity	**1**	**2**	**3**	**4**	**5**
Region 1	0.82560	0.53663	0.40150	1.01274	0.26510
Region 2	1.03435	1.12493	1.05998	1.13987	0.96794

As could be expected, the Region 2 unit value commodity prices are all relatively close to the US prices (which are all equal to unity) since 3 out of 4 of the Region 2 countries are "rich" and hence have price structures similar to the US structure. The Region 1 unit value prices are all lower than the corresponding Region 2 prices and for Commodity 5, Services, the Region 1 unit value price is considerably lower. Now the Fisher quantity index for Region 2 relative to Region 1 can be calculated using the regional price and quantity data listed in Tables 12 and 11 and the resulting index is equal to 6.26177. Thus the relative regional consumption volumes are 1 and 6.17739 and when we convert these volumes into shares, we find that the Region 1 share of world consumption is 0.13771 and the Region 2 share is 0.86229.

We now turn our attention to the Option 3 additive method which was suggested by Sergeev (2009). In order to implement this method, we need to compute the geometric mean of the regional Basic Heading prices that are listed in Table 8; recall equations (18) above. These regional geometric mean prices P_{rn}^* are listed in Table 13 below.

Table 13: Regional Geometric Mean Prices P_{rn}^* by Commodity

Commodity	**1**	**2**	**3**	**4**	**5**
Region 1	0.84377	0.65236	0.51405	1.20824	0.32287
Region 2	1.09882	1.10282	1.10861	1.43846	0.82863

The pattern of regional commodity prices is fairly similar in Tables 12 and 13. To complete the analysis for this case, we calculate the Fisher quantity index for Region 2 relative to Region 1 using the regional price and quantity data listed in Tables 13 and 11 and the resulting index is equal to 6.17739, which is very similar to the corresponding Option 1 Fisher index which was equal to 6.26177. Thus the relative regional consumption volumes for the Sergeev method are 1 and 6.17739 and when we convert these volumes into shares, we find that the Region 1 share of world consumption is 0.13933 and the Region 2 share is 0.86067.

The resulting Region 1 shares of "world" consumption that have been generated by the various methods discussed above are summarized in Table 14 below.

Table 14: The Share of World Output for Region 1 using Various Methods

	Region 1 Share of Consumption
Option 1 (Regional Unit Values Method)	0.13771 (3.4 %)
Option 3: Geometric Mean Average Prices in each Region	0.13933 (4.6 %)
GEKS	0.13815 (3.7 %)
GK	0.14243 (7.0 %)
IDB	0.14326 (7.6 %)
Spatial Linking	0.13316

While the variations in the Region 1 shares that the various methods generate are not huge, they are certainly not negligible. The percentage differences between the various estimated shares and the Spatial Linking share are listed in brackets in the above Table.[16]

More experimentation using the 2005 data needs to be done before we can come to a definitive decision on which method should be used to link the regions in ICP 2011.

7. Conclusion

Here are some points for discussion at the TAG meeting:

- The Option 2 method should be ruled out for ICP 2011 since as Sergeev has pointed out, it is not invariant to the choice of the numeraire countries in the regions.
- The Option 1 method should also be ruled out for ICP 2011 since it depends on market exchange rates which are not reliable and hence their use should be avoided if possible.
- The Option 3 method should also be ruled out because the method is inherently an additive method and additive methods are not able to give the most accurate volume comparisons, at least from the economic approach to index number theory.
- The use of GK or IDB should also be ruled out for "headline" estimates for country and interregional parities due to their inherent substitution biases. These methods however could be used to provide users with analytical tables where the users demand an additive method.

[16] Again, the author's preference is for the spatial linking method.

- GEKS remains a viable method for constructing regional shares in a consistent manner.
- Various forms of spatial linking should also be in the running but the use of this method should await more experimental results using the 2005 data base.[17]

Appendix: Can We Compare the Incomparable?

Angus Deaton (2009) in a communication to the author raised the issues associated with making comparisons between countries where the consumption bundles consumed in say two countries are entirely different. Under these conditions he asked whether it was possible to make any comparisons at all between these two countries. He suggested the following numerical example where we have only two commodities and three countries:

Table A1: Prices for 3 Countries and 2 Commodities

Good	Country 1	Country 2	Country 3
1	p_{11}	p_{21}	•
2	p_{12}	•	p_{32}

The price p_{cn} is the price of good n in country c, and a • indicates that the corresponding commodity is not consumed at all in the country and thus there is no price. Deaton further supposes for simplicity that in country 1, half the budget gets devoted to each of the two goods, so that we do not have to worry too much about weights for this example. Deaton then goes on to explain the problem associated with making comparisons in the above situation as follows:

"My starting point is that there really is no basis for comparing country 2 and country 3, since there is no overlap between them. Country 2 is Ethiopia, where everyone eats teff, and Country 3 is Sri Lanka, where everyone eats rice. So any solution is arbitrary, and we ought to be suspicious of methods that "solve" this problem."

"We could, of course, use transitivity (or circularity). The parity for 2 relative to 1 is p_{21}/p_{11} and the parity for 3 relative to 1, is p_{32}/p_{12}, so that we have a parity for 3 relative to 2 by chaining them, to get

(A1) $P^{32} \equiv [p_{32}/p_{12}]/[p_{21}/p_{11}]$.

And if my algebra is correct, this is what CPD would give applied to these data, also EKS. But it seems entirely arbitrary. We could have another country come along, and yield a quite different answer, and we can think up imaginary other countries that would give us any number we like."

"More generally, we know that we can't have both transitivity and independence of irrelevant data, but this seems like an extreme case, where everything depends on the irrelevant data, and nothing on the two countries being compared."
I am not sure I understand the minimum spanning methodology, but isn't its claim that it "solves" these insoluble problems by chaining? Angus Deaton (2009).

[17] Some discussion of the dissimilarity measure that should be used to measure the degree of relative price similarity across countries is also required. We should also discuss whether dissimilarity between the per capital quantity structures should also enter into the picture.

Deaton is correct in his claims in the above quotation. Using spatial linking, country 2 is most similar to country 1 and so country 2 would be linked to country 1 via the relative price of commodity 1. Similarly country 3 is also most similar in price structure to country 1 so country 3 would be linked to country 1 via the relative price of commodity 2. Thus the price level of country 1 will be set equal to:

(A2) $P^{11} = 1 \equiv \alpha_1$.

The price level of country 2 relative to country 1 will be set equal to:

(A3) $P^{21} \equiv p_{21}/p_{11} \equiv \alpha_2$.

And finally, the price level of country 3 relative to country 1 will be set equal to:

(A4) $P^{31} \equiv p_{32}/p_{12} \equiv \alpha_3$.

Thus the price level of country 3 relative to country 2 will be

(A5) $P^{32} \equiv P^{31}/P^{21} = [p_{32}/p_{12}]/[p_{21}/p_{11}]$

which is the answer as (A1) above. Thus in Deaton's view, the minimum spanning tree methodology simply does not work (since nothing seems to work in this situation!).

However, there are possible responses to the above Deaton critique. I will present three ways that could be used to justify the rather indirect methodology that emerges from the spatial linking methodology.

Attempt 1:

Suppose final demanders in all 3 countries *had the same linear indifference curve preferences*. Then the above solution would in fact give us exactly the right answer in terms of welfare! It could be argued that this is an extreme assumption but perhaps it gives us an adequate approximation to the "truth".

Attempt 2:

We can justify the above solution as being the same solution that the *country product dummy method* generates, which is a purely "statistical" model. The model works as follows: we set the country c price for a product n, p_{cn} equal to the product of a *country general price level* α_c times a *product effect* β_n. Thus in the present situation, we can fit the data exactly and the estimating equations are as follows:

(A6) $p_{11} = \alpha_1\beta_1$;
(A7) $p_{12} = \alpha_1\beta_2$;
(A8) $p_{21} = \alpha_2\beta_1$;

(A9) $p_{32} = \alpha_3\beta_2$.

We also require a normalization on the country general price levels so we choose country 1 as the numeraire country and set

(A10) $\alpha_1 = 1$.

We have 5 equations in 5 unknowns and *the solution for the country parities is the same as the Deaton or Hill solution*; i.e., we have in addition to (A10), the following solution for equations (A6)-(A10) as the CPD parities:

(A11) $\alpha_2 = p_{21}/p_{11}$; (7) $\alpha_3 = p_{32}/p_{12}$; (8) $\beta_1 = p_{11}$; (9) $\beta_2 = p_{12}$.

Now the CPD method was originally justified as a purely statistical method. But the CPD parities can be given a deeper meaning in terms of *imputed prices for the missing items* in countries 2 and 3. In this particularly simple case, it can be seen that if a final demander in country 2 wanted to purchase a unit of commodity 2, then he or she could purchase a unit of commodity 2 in country 1 at the country 2 price (in terms of units of commodity 1 foregone) of $\alpha_2\beta_2 = [p_{21}/p_{11}]p_{12}$. Similarly, if a final demander in country 3 wanted to purchase a unit of commodity 1, then he or she could purchase a unit of commodity 1 in country 1 at the country 3 price (in terms of units of commodity 2 foregone) of $\alpha_3\beta_1 = [p_{32}/p_{12}]p_{11}$. In fact, this is why the CPD method was invented in the first place by Summers (1973): as a way of filling in missing prices.

Attempt 3:

The World Bank and many academic and government economists want to have estimates of relative output quantities and relative price levels across countries. Thus it seems to me that we should do our best to meet this demand! In other words, even though the methods are not perfect, we need to give the World Bank our best advice in meeting this demand. Thus, we have to pick the "best" method for making international comparisons out of the galaxy of possible methods. There is substantial overlap across most countries for most commodities. Thus linking countries in a chain of bilateral comparisons where relative prices and per capita quantities match up as much as possible seems to me to be the "best" method that we have available to us since it tries to make comparisons when they are possible and match items and categories according to the similarity of price and (per capita) quantity structures across countries.[18]

[18] Deaton (2010; 33-34) raises a related problem associated with the use of superlative indexes to make bilateral comparisons between countries who have very different structures in their quantity vectors: "But Table 7 identifies a different issue that is probably more important, which is that the PPP of a country can be strongly affected by the prices of an item that has very little consumption in that item. Air travel accounts for between 0.28 (Kenya) and 0.89 (Cameroon) percent of total consumption in these four countries, and somewhat more in the part of consumption covered by the ring prices but superlative indexes use weights from both base and comparison countries so that, when these countries are compared with Britain, the high relative prices of air travel—in Cameroon, more than 11 times higher than the average Cameroon to British price ratio—is weighted by the *average* of the British and Cameroon share. In consequence, air travel raises the overall (pairwise Törnqvist) price level by 4.2 percent. With Törnqvist

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indexes, this can happen even when the NAS report no consumption on the item—see catering services in Zambia in Table 7. The Fisher index is not well defined in this case, but the same general phenomenon occurs, with the budget share from the comparison rich country powering up the price level of a rarely consumed commodity in the poor country." A way of controlling for this problem would be to use spatial linking of countries but the dissimilarity measure should not only measure the dissimilarity of the relative price structures in the two countries but also measure the dissimilarity in the structures of (absolute) per capita quantity vectors.

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