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Aggregation methods on the basis of structural international prices

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Introduction

The inner comparability of macroeconomic data is permanently increasing due to harmonization work by national statistical offices and international organisations resulting in standard definitions, classifications, methods of valuation, etc. This circumstance allows to take more attention to the computational procedures within international comparisons. So, the paper focuses on the aggregation methods of multilateral comparisons.

Since the beginning of the United Nations International Comparison Project / Program (UN ICP) remarkable efforts have been made in order to provide for an appropriate statistical treatment of the heterogeneities of the participating countries, e.g. the principle of graduality, the request for sufficient representativity of product basket for all countries, the flexibility of methods applied to different fields of the comparison. Since phase IV (reference year 1980) the ICP has been organised on a regional basis with further generation of worldwide global figures by linking the results of regional comparisons. Regional sets of countries were considered as more homogeneous. It was, maybe, right in the past but the present situation is changed substantially in the 90th – even the OECD and the Eurostat comparisons comprise heterogeneous sets of the countries for the moment.

It is desired that the results of international comparisons (Purchasing Power Parities = PPPs / Volume indices) posses some properties important from the analytical point of view. Detailed analysis of these properties are given by W. Diewert (1986) (1996), B. Balk (1995) and R. Hill (1997). Therefore only two following major properties are mentioned here specially:

Characteristicity[1]. This property implies that the basket of products priced and weights (or common international prices) used in an international comparison are representative of all participating countries. This property is easier to satisfy in a bilateral comparison, especially if two comparing countries are similar, than in a multilateral comparison when a wider group of countries is involved.

Additivity[2] (for aggregation procedures only). This property, when satisfied, means that real values (comparable between all countries) for any country are directly comparable between categories or, in other words, countries' real values at any level of aggregation can be obtained as the sum of real values of lower level categories of a given aggregate. Additivity

[1] The term „Characteristicity“ was introduced by L.Drechsler (1973) and it meant initially that the „best“ bilateral index is obtained on the basis of input data of two countries in question only (without data of the 3rd countries). This term is used now very oft (incl. this paper) in some modified sense.

[2] The property of additivity is linked with the property of matrix consistency. The relationship between the additivity and matrix consistency is explained in detail in the paper: I.Sakuma, D.S.Prasada Rao, Y.Kurabayashi „*Additivity, Matrix Consistency and a new Method for International Comparisons of Real Income and Purchasing Power Parities*“, 26th General Conference of the IARIW (Cracow, Poland, 27.08 – 02.09.2000). The categorization of additive methods is given in the paper of J.R.Cuthbert „*Categorisation of additive PPPs*“ – The Review of Income and Wealth“, No.2 (June), 1999.

requires a method to compare all countries using a common vector of prices (a vector of international prices).

Several methods have been proposed for making multilateral comparisons of price and volumes but no method of calculation satisfies all the desired requirements. The importance attached to each of the properties may vary depending on the combination of comparing countries and the uses to which the results of the comparison are put. The comprehensive reviews of aggregation procedures using in multilateral comparisons are contained in the papers prepared by W. Diewert (1996), B. Balk (1996) and R. Hill (1997).

The ICP consists of the set of very heterogeneous countries. The calculations for different parts of ICP (EU, OECD, ESCAP comparison) show that the choice an aggregation procedure has a sufficient impact on the results.[3] The differences among the results calculated by different methods for several countries and aggregates are 10-20%. A question about the choice of the best multilateral method was widely discussed on two special meetings of Experts Group in a field of ICP-methodology organized by EUROSTAT, OECD and Statistical Office of UN together: EUROSTAT, Luxembourg, June, 1988 and OECD, Paris, June, 1989. Majority of experts inclines to an opinion that no uniform method is and various methods are necessary used for various aims. For example, new version SNA_(SNA'93, p.397) indicates: "...*the methods used to compile statistics must be influenced by the purposes for which they are to be used*".

The most popular aggregation procedures presently using in ICP are the Geary-Khamis (GK) method (the block approach – weighted averaging of national prices recalculated into a common currency by the PPPs) and the EKS method[4] (the averaging of bilateral results). Both of these methods are described in detail in the several reports[5].

The Geary-Khamis method provides additivity[6] which is very desirable if international comparisons are made at varying levels of aggregation (the comparisons of „ICP-type"). The main drawback of GK arises as a result of the fact that the GK common vector of international prices is obtained by taking a weighted average of the countries' price vectors.[7] Hence the

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[3] The recent investigations about the influence of the choice of multilateral method on the results of ICP are the following: Y. Dikhanov „*Sensitivity of PPP-Based Income Estimates to Choice of aggregation Procedures*" (IARIW Conference; St.Andrews, New Brunswick, Canada on 21-27 August 1994); R. Hill „*Comparing Price Levels and Living Standards across the ESCAP Countries Using Spanning Trees and other Aggregation Methods*" (Paper for a UN Seminar on comparison within the ESCAP, Bejing, 16 - 20.06.1997).

[4] Some historical research found that the EKS method was first proposed by C.Gini in 1924 and it was later rediscovered by three independent researchers in 1964 (Ö. **E**lteto, P. **K**öves and B. **S**zulc), therefore the name **GEKS** is used in the last publications.

[5] Detailed description of the GK and the EKS methods and the analysis of their advantages and disadvantages are given in many publications: Kravis I. a.o. *A System of International Comparisons of GrossProduct and Purchasing Power*. Baltimore, 1975**;** Kravis I. a.o. *World Product and Income. Intenational Comparisons of Real Gross Product*. Baltimore,1982**;** Hill P. Multilateral *Measuremerents of Purchasing Power and Real GDP*. SOEC, 1982**;** *Handbook of ICP*. UN, N.Y.,1992**;** „International Comparison of Gross Domestic Product in Europe,1993. Results of the European Comparison Programme". UN, Statistical Standarts and Studies - No.47. N.Y. and Geneva,1997**;** „*Comparison in real term of the aggregates of ESA. Results 1998*". Luxembourg, Eurostat, 2000**;** „*PPP and Real expenditures. Results 1999*", Paris, OECD, 2002. Short but a very strong description of multilateral methods is there also in the new SNA-1993, chapter XYI, part F.

[6] The GK provides additivity in terms of international average prices only but not in terms of prices of any country. For example, the original G-K PPPs to I$ were recalculated (and published) in the ICP 1975 as PPPs to USD. In effect, the obtained PPPs to USD did not satisfy with the „average test". For example, the following PPPs to USD were obtained for Japan: „Government-Total"- 323 Y/USD but both partial PPPs were lower: „Compensation"- 277 Y/USD and „Commodities" – 298 Y/USD.

[7] For some comparisons this feature of GK method can be *vice versa* an advantage but not a drawback.

vector of the international prices tends to be closer to the price vectors faced by large (or rich) countries than small (or poor) countries. It is well-known that the volume of a country tends to sink as the prices used in the comparison becomes relatively more closeness to its own national prices as compared with the prices of other countries, or, in other words, the more characteristic the common price vector is for a given country, the more its volume index will tend to be underestimated. This bias caused by unequal relative closeness of used prices is usually referred to in literature as the Engel-Gerschenkron effect[8]. The GK average prices calculated for a set of heterogeneous countries are not characteristic of outlying countries[9]. This effect may significantly distort the comparative real product levels (especially in the developing countries, which are more sensitive to the choice of used methods).

The EKS method attempts to guarantee equi-characteristicity of results. It ignores the differences in the size of countries compared and permits avoiding "Engel-Gerschenkron" type of distortions in the results. Also, EKS results have another attractive property, namely that relationships between countries are only marginally influenced by the composition of the group of countries compared due to minimization procedure applied. The main inconvenience of EKS is the lack of additivity. This means that the sum of real values obtained by EKS-type PPPs at the given aggregation level doesn't produce the EKS-type real value of higher level of aggregation. Consequently, the percentage distribution of these real values does not add up to 100 per cent. Therefore, the possibilities of structural analysis are limited. Moreover, the lack of additivity can lead to paradoxical results: the average index (or PPP) can be higher (or lower) as each of particular indices (this is the distortion so called „average test"). P.Hill gave the comments about multilateral methods that take the Fisher indices as the starting point (EKS and the like) in an aphoristic form: „*The construction of a multilateral set of measurements at a later stage has then to be regarded as a process whereby an initial set of perfectly good binary measures has to be distorted, rather in the manner practiced by Procrustes, in the interests of securing transitivity*" (see Kravis I. a.o., 1982, p.77). S.Khamis commented the use of the EKS-method within EUROSTAT-comparison with the following words: „*In the opinion of the author, the adoption of the EKS(F) results by EUROSTAT is a retrogressive step in comparison with their excellent earlier comparisons including those of 1975 based on the Gerardi UCW method*" (see, S. Khamis. *„On some aspects of the measurement of Purchasing Power Parities*". Reports of ISI Session, Florenz, August 1993).

To combine the advantages and to decrease the disadvantages of different methods, it was decided to use in the framework of EUROSTAT-OECD PPP Programme both methods parallelly: the EKS-method for volume comparisons (official results) and the Geary-Khamis method for structural analyses (for analytical purposes). Obviously, such mechanical combination is not very appropriate for the concrete analytical works. The use of different outcomes (EKS results and GK results) can lead to some irritations. These circumstances are the reasons for further investigations of multilateral methods.

The GK method provides additivity but avoids characteristicity, the EKS method - *vice versa.* Is it possible to combine advantages of both methods within an unique method? S. Ahmad (1994, p.2) expressed the following considerations about this problem: „*To reduce the Gerschenkron effect and at the same time retain matrix consistency (additivity) of the results, it is proposed that an unweighted or equal weighted) Geary - Khamis be used. Tests show that*

[8] Some authors believe that there is no a clear evidence that the GK method is influenced by this effect (or, at least, the EKS method is not „free" from this effect also) – see, for example, S.Khamis „Measurement of Real product: Some Index Number Aspects" (25th General conference of the IARIW, Cambridge, 23-29.08.1998).

[9] These outliers are usually the poor but sot sometimes the rich small countries involved in the comparison.

the results are very similar to EKS but with an added advantage of additivity. Should another aggregation method such as Ikle be used instead? ".

The following methods were proposed for this aim:

1) The **Ikle-method** (see, D. Ikle." A new Approach to the Index Number Problem". Quarterly Journal of Economics, No.2, 1972)[10].

2) The **„Implicit price" method** (IP method) developed by EUROSTAT (see, the materials of Paris - ICP meeting, June 1989);

3) The **Minimum Spanning Tree** method (see, R.J. Hill „International Comparisons using Spanning Trees", in „ International and Interarea Comparisons of Prices, Income and Output", NBER, 1997).

4**) Weighted EKS** method and **Generalized CPD** method (see, D.S.Prasada Rao „Aggregation Methods for International Comparison of PPPs and Real Income: Analytical Issues and Some Recent Developments"; ISI Session, Istanbul, 18-26 August 1997)

5) **Generalized GK** method (see, J.Cuthbert „Categorisation of additive PPPs" – The Review of Income and Wealth", No.2 / June, 1999).

6) **CKS method** (**C**ommensurable **K**urabayashi-**S**akuma method) – see I.Sakuma, D.S. Prasada Rao, Y. Kurabayashi „Additivity, Matrix Consistency and a new Method for International Comparisons of Real Income and Purchasing Power Parities", 26th General Conference of the IARIW (Cracow, Poland, 27.08 – 02.09.2000)

7) **Shared GK-Rao method** (see, D.S.Prasada Rao „ Expenditure Share Weighted Size-neutral Geary-Khamis Method for International Comparisons: Specification and Properties", 2000).

These methods have some advantages contrary the EKS and the GK but they do not solve directly the task concerning the obtaining of simultaneously additive and characteristic (for all comparing countries) results of multilateral comparison. This means in the practice that a set of possible multilateral methods should be broader and new ideas and proposals are desirable.

Mainly two kinds of multilateral aggregation methods are used in the present time:

A) **Averaging of bilateral indices** (e.g. the EKS method, etc.)

B) **Use of average international prices** – an averaging of national prices recalculated by PPPs into a common currency (the Geary-Khamis, Van IYzeren, Ikle methods, etc)

The methods of type B) are based usually on the simultaneous calculation of the international prices and overall PPPs or overall quantities within a system of equations, i.e. international prices are some functions from global PPPs (eg. **Geary-Khamis, Van IYzeren, Rao** methods) or overall quantities (e.g. the **CKS** method – **C**ommensurable **K**urabyashi-**S**akuma) or even both (**Ikle** method) and simultaneously overall PPPs (overall quantities) are some functions from these international prices.

[10] This method was analyzed in detail by Y.Dichanov (1994).

A new kind of the aggregation procedures of type B) is proposed in this paper. It is based on the use of so called structural international prices. It is easy to show that volume index (IQ) for any pair of countries j and k can be calculated not only on the basis of a given set of prices (π) but also on the basis of their ratios (e.g. to a commodity M selected as a basis):

$$IQ^{j/k} = \frac{\sum_{i=1}^{M} \pi_i * q_{ij}}{\sum_{i=1}^{M} \pi_i * q_{ik}} = \frac{\sum_{i=1}^{M} (\pi_i / \pi_M) * q_{ij}}{\sum_{i=1}^{M} (\pi_i / \pi_M) * q_{ik}}$$

This circumstance allows to use an averaging of national price ratios instead of the averaging of national prices as it is done by the traditional methods of type B).

The structure of national prices for country j can be presented as the quadratic matrix of price ratios: P_{ij}/P_{lj} (i=1,2,...,M; l=1,2,...,M) or in a compressed form – as the vector of ratios "P_{ij}/P_{Mj}", where P_{Mj} is numeraire commodity M. The ratios "P_{ij}/P_{Mj}" are not depend on the national currencies and therefore are directly comparable between countries and they can be averaged without the use of PPPs. These average international price ratios can be named as **international structural prices**. The respective PPPs are derived indirectly as a ratio between value for a given aggregate in national prices (national currency) and value in international structural prices. In effect, the methods using structural prices are strictly additive.

It seems that this approach was not used yet directly in international comparisons. However is it easy to demonstrate that the well known Gerardi method (Unit-Country-Weight = UCW) can be presented in the form of structural international prices. Traditionally the Gerardi - UCW method (based on prices P_{ij} for N countries and M commodities) is presented in the form of average international prices (π) which are obtained as simple geometric averages from national prices without any PPPs:

$$\pi_i = (P_{i1} * P_{i2} \ldots * \ldots P_{ij} \ldots * \ldots P_{iN})^{1/N} \qquad i = 1,2,\ldots,M \quad j = 1,2,..,N$$

This form is not very understandable from an economic point of view and this feature was sometimes a point for the criticism of the G-UCW method. Nevertheless the Gerardi formula presented above can be transformed to the form:

$$\pi_i = \left[\frac{P_{i1}}{P_{M1}} * \frac{P_{i2}}{P_{M2}} * \ldots * \frac{P_{ij}}{P_{Mj}} * \ldots * \frac{P_{iN}}{P_{MN}} \right]^{1/N} \qquad i = 1,2,\ldots,M; \quad j = 1,2,..,N$$

This formula shows clearly that the Gerardi method can be presented as an averaging of the structural national prices.

The main aim of this paper is to introduce a new type of international prices – structural international prices – into the PPP calculations. To obtain the characteristic and simultaneously additive results over all countries involved in a multilateral comparison, it is necessary to use a vector of international prices which are maximally possible characteristic for all countries involved.

The EKS-method was selected as the official method for Eurostat/OECD comparison and for the European Comparison Program (ECP) since 1990 because the imaginary prices of the EKS method[11] were claimed by an expert group to be more neutral in relation to national price structures than those of the other alternative [OECD (1995), p.4]. However, to examine the hypothesis about this neutrality, it is necessary to have the method for measuring of similarity (dissimilarity) of price structures. An appropriate procedure is proposed below[12].

I. Countries'price structures and measure of their similarity

There are several well-known methods for measuring of similarity (dissimilarity) of national price structures:

1) So called, Laspeyres - Paasche spread = LPS[13] calculalted on the basis of the ratios of Laspeyres and Paasche indices or its modification proposed by R.Hill[14], so called, Hill's Distance, which is a version of L/P ratio in logarithmic terms - | Log(L/P) |. The LPS is an useful „tool" for analyzing of the similarity of price and volume structures but it seems that the LPS indicator is not enough sensitive to big differences in the price structures. The countries with very different price structures can have Laspeyres/Paasche ratio not very far from 1. One imaginary numeric example is given below for an illustration:

Table I.1

Calculation of Laspeyres/Paasche ratio

Country A				**Country B**			
Price	Quantity	Pa*Qa	Pb*Qa	Price	Quantity	Pb*Qb	Pa*Qb
1	10	10	30	3	5	15	5
2	15	30	30	2	10	20	20
3	20	60	20	1	15	15	45
Total Values		100	80	Total Values		50	70

Volume Index - Laspeyres 'A/B' =	1.6000
Volume Index - Paasche 'A/B' =	1.4286
Ratio 'L/P' = 1.1199	

The countries A and B have very different price structures although L/P ratio is close to 1 (LPS = 1.1199). This is possible to explain: the L/P ratio depends on both sets of input data:

[11] In reality, the EKS method does not use any vector of international average prices. Therefore it is more correct to say about imaginary hidden prices by the EKS calculation [see, Köves (1995)]. The recent OECD PPP publication [OECD (1999)] indicated on this feature (see page 25) but the titles of Tables 7-11 were not changed - „... at international prices").

[12] The main ideas of this method were suggested by S. Sergeev in the Ph. D. Dissertation „Multilateral Methods for International Comparisons". The Researcher Institute of Central Statistical Committee of Soviet Union, Moscow, 1982 (in Russian).

[13] See, for example, the following publications: P. Köves. *Index Theory and Economic Reality*. (§ 8.3.3). Budapest, 1983; M. Martini. *The multilateral comparison in the axiomatic approach*.; B. Zavanella. *Comparison of consumption among EEC countries*: Prices, Quantity and Values (Both latter reports were prepared for International Statistical Institut Session: Florenz, August 1993). Sometimes an inverse indicator „Paasche – Laspeyres spread" (PLS) is used.

[14] R.Hill. *Chained PPPs and Minimum Spanning Trees*. (Conference of Income and Wealth, March 1996, Washington).

differences in prices and differences in quantities and therefore the L/P ratio is not an appropriate tool for the measuring of „pure“ price similarities.

2) A more sophistical method was used in the earlier phases of ICP:

„…the measure of similarity between the vectors of national prices is their weighted raw-correlation coefficient between the ratios domestic price and international prices referring to any pair of countries“ (see, Kravis I. et.al., 1982; p.348).[15]

In our opinion the latter method is more preferable than the former. However, there are two problematic points: the use of this method is possible after the calculation the international prices only (a) and the use the international prices as a bridge-vector can bring the effect of the overestimation (so called ‘false’ correlation because of common factor - international prices) of the similarity (b).

3) The thoughtful method was proposed by R.Allen and W.Diewert (1981)[16]. The Allen-Diewert distance (D) is calculated as the dispersion of individual price ratios (PPPs) between countries (in logarithmic terms). The larger the value of D, the less similar the price structures in given countries. The Allen-Diewert measure lies between zero and infinity, i.e. it has a lower bound only. This is main disadvantage of the method.

4) J.Cuthbert (2000) proposed to calculate a measure of price structure in terms of the root-mean-square deviation from 1 of the ratios of actual prices recalculated by overall PPP into common currency to average international prices (IP). The similarity of price structure between the countries is calculated as a correlation coefficient. This indicator is a useful analytical tool but there are two drawbacks: the proposed indicator is not symmetric about **1** and this is too sensible for extreme values of IP. In effect, the values of log(IP) were actually used and values of IP below 0.333 and above 3 were excluded from the analysis. Obviously an arbitrary exclusion of some products from the analysis is a weakness of the method.

To avoid the drawbacks of the methods mentioned, a new measure of similarity (dissimilarity) of price structures can be proposed. As it was indicated in the section „Introduction“, the structure of national prices for country "j" can be presented as a quadratic matrix of price ratios: P_{ij}/P_{lj} (i=1,2,...,M; l=1,2,...,M) or in a compressed form – as a vector of scalars **"P_{ij}/P_{Mj}"**, where P_{Mj} is price of numeraire commodity M. If the price data of countries **j** and **k** satisfy the conditions for Hicks’s composite commodity theorem - $P_{ij} = \lambda * P_{ik}$ – then the price structures are identical. In effect, the coefficient of similarity of price structures can be concluded in a similar way with the deduction a coefficient of linear correlation as a calculation a geometric mean from the two regression coefficients)[17]. The minimization of the sum of squares of <u>relative differences</u> in the both directions is used to obtain the measure which is invariant to changes in the units of measurement. So, the coefficient of the price similarity can be calculated by the following steps:

a) First step: Let $P_{ij} = c1 * P_{ik}$ – the regression I.

$$\sum_i \{(P_{ij} - c1 * P_{ik})/P_{ij}\}^2 \rightarrow \min;$$

[15] This method is stil used by the OECD too.
[16] W.Diewert proposed in 2002 some further modifications.
[17] Coefficient of similarity of national quantity structures can be obtained in the same way. The formula will be the same as (II.1) but the price symbols must be replaced by quantity symbols.

$$c1 = \sum_i (P_{ik}/P_{ij}) / \sum_i (P_{ik}/P_{ij})^2. \qquad i= 1,2,...,M$$

b) Second step: Let $P_{ik} = c2 * P_{ij}$ – the regression II.

$$\sum_i \{(P_{ik} - c2* P_{ij})/P_{ik}\}^2 \rightarrow \min;$$

$$c2 = \sum_i (P_{ij}/P_{ik})/ \sum_i (P_{ij}/P_{ik})^2 \qquad i= 1,2,...,M$$

b) <u>Third step</u>: the calculation <u>of an average from both regression coefficients</u>

$$\tau_{jk} = (c1* c2)^{1/2};$$

The indicator τ_{jk} is a certain analogy (but not identical) with a coefficient of correlation[18] and has some analogous attractive properties:

$\tau_{jk} = \tau_{kj}$;

$0 < \tau_{jk} \leq 1$;

- if $\tau_{jk} = 1$ - price structures in the countries "j" and "k" are identical;
- if $\tau_{jk} \rightarrow 0$ - price structures are very different.

So, the level of similarity of national price structures between country **j** and country **k** can be measured by help of the following coefficient (indicator) τ_{jk}:

$$\text{(II.1)} \qquad \tau_{jk} = \left[\frac{\sum_i (P_{ij}/P_{ik}) * \sum_i (P_{ik}/P_{ij})}{\sum_i (P_{ij}/P_{ik})^2 * \sum_i (P_{ik}/P_{ij})^2} \right]^{1/2} \qquad j = 1,2,..,N; \quad k = 1,2,...,N \quad i = 1,2,...,M$$

An example (see imaginary data in Table I.1) is given below for the illustration:

Table I.2

Calculation of coefficient of similarity of price structures

Pa/Pb	(Pa/Pb)²	Pb/Pa	(Pb/Pa)²
0.3333	0.1111	3.0000	9.0000
1.0000	1.0000	1.0000	1.0000
3.0000	9.0000	0.3333	0.1111
4.3333	10.1111	4.3333	10.1111
TauCoeff =		**0.4286**	

As we can see the proposed indicator τ (TauCoeff = 0.4286) showed the significant differences in the price structures between countries A and B. The indicator τ measures the „pure“ similarity of price structures (without influence of differences in quantities) and it is a

[18] The coeffcient of the correlation itself is not very appropriate for the measuring of price similarities because it reflects the correlation in the form Y = c*X + b but we need the form Y = c*X.

preference relatively the Laspeyres/ Paasche ratio which reflects simultaneously the differences (and correlation)[19] between the price and quantity structures.

Indicator τ has some similarity with the Allen-Diewert distance. But τ coefficient has an important preference because it has a lower bound as well as an upper bound (it lies between zero and 1), while the Allan-Diewert measure lies between 0 (zero) and infinity, i.e. it has only a lower bound but not upper bound.

The products have different shares in the GDP (or in an other related aggregate). Additionally, in actual comparisons, input data are not quite those originally envisioned: prices and quantities for individual commodities. Usually PPPs ('National currency/Numeraire currency') for primary groups (basic headings = BH) are used as „notional" (fictitious) prices and a set of „notional" (fictitious) quantities, each obtained as ratio of nominal value (in national currency) to corresponding PPP. The PPP for concrete basic heading represents a set of products. Therefore it is preferable to use the weighting in this situation.[20]

A weighted version of indicator τ can be proposed in the following form[21]:

$$\text{(I.2)}\quad \tau_{jk} = \left[\frac{\sum_i \{(P_{ij}/P_{ik}) * d_i^{jk}\} * \sum_i \{(P_{ik}/P_{ij}) * d_i^{jk}\}}{\sum_i \{(P_{ij}/P_{ik})^2 * d_i^{jk}\} * \sum_i \{(P_{ik}/P_{ij})^2 * d_i^{jk}\}} \right]^{1/2} \qquad \begin{array}{l} j = 1,2,...,N \\ k = 1,2,...,N \\ i = 1,2,...,M \end{array}$$

where

$d_i^{jk} = (d_i^j + d_i^k)/2$ - average weight for basic heading i between countries j and k,

$d_i^j = w_{ij} / W_j$; $d_i^k = w_{ik} / W_k$ - weights for basic heading i in country j and country k,

$w_{ij} = p_{ij} * |q_{ij}|$ - absolute[22] nominal value for ith basic heading in the jth country (in national currency);

$W_j = \sum_{i=1}^{M} w_{ij}$ - total absolute value of the aggregate in question for country j at national prices (w_{ik} and W_k are defined analogously).

The calculations carried out on the basis of information from the Eurostat 2002 comparisons (see Section IV) showed a good efficiency of proposed method for the measure of similarity of price structures.

There are also several possible collateral uses of the indicator τ:

- <u>analysis of dynamics of price structures</u>

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[19] The negative correlation exists usually between cross-country price quantity ratios.

[20] The use of unweighted version leads to the distortion of the overall results even if one product (basic heading) only has an extreme anomalous value (see, eg, J.Cuthbert (2000), page 433 – the respective indicator for New Zeland in the OECD 1993 comparison was fully distorted due to an anomal price for dried vegetables only).

[21] This measure is similar with the measure proposed by of van Ark, Monnikhov, Timmer (see „*Prices, Quantity and Productivity ...*" in R.Lipsey and A.Heston (eds), „International and Interarea Comparisons of Prices, Income and Output", NBER, 1999) but the measure (I.2) is much simpler and understandable.

[22] The reason of using of absolute values is the necessity of the obtaining a „correct" reliable structure by the presence of BH with negative expenditure, i.e. the individual shares must lay in the limits [0;1].

for example, the indicator τ can be used as one from the characteristics of transition process in the former CPE countries or as an impact of the EURO introduction.

- measure of reliability of bilateral comparisons

The L/P ratio or its modification – Hill‘s distance [see R.Hill, (1999)] are mainly used for this purpose. The comparisons with smaller differences between Laspeyres and Paasche indices are regarded as more reliable because these are based on data of the countries with more similar structures. In general, this postulate can be accepted but there is a circumstance which should be taken into account. So, the Hill's distance is a symmetrical measure, i.e. the situations L/P = 1.25 and L/P = 0.8 (=1/1.25) bring the same distances between countries. However, if the situations with L/P > 1 can be considered as a usual "normal" case then all situations with L/P < 1 should be considered as problematic cases. The ratio L/P <1 (P/L > 1) shows that the comparing countries have very different price and quantity structures. The L/P ratios < 1 are very rare cases and they are obtained by very unusual circumstances (distorted structures, etc.)[23]. Obviously, the countries with different structures are less comparable and in effect the bilateral results are less reliable[24]. Therefore the measure of reliability on the basis of the L/P ratios should take into account the direction of the L/P ratios: the L/P ratios are lower than 1 should be considered as less reliable and should have lower weights. It can believe that the coefficient of similarity of price structures between a pair of given countries will show the lower similarity for the countries with L/P ratios < 1 than for the countries with the same deviation of L and P indices but with L/P > 1. Additionally, the Hill's measure is too sensitive for the L/P ratios which are very close to 1. The reliability is inversely proportional to the Hill’s distance. The pairs of countries with the L/P ratios close to 1 receive extraordinary high weights. The indicator τ is applied in a direct form - the larger the value of this measure, the greater the similarity (weight) and the greater the reliability of bilateral indices – in effect, the problem of sensitivity is avoided.

II. Method of "Maximal Possible Characteristic Prices" (MPCP - method)

An analysis of the results for the different phases and the parts of the ICP showed that the significant differences are kept in the price structures between the countries. Therefore the elaboration of the multilateral methods which could take into account (with neutralization of respective negative effect) the existing differences in the price structures, is an actual task. A new aggregation multilateral method producing additive and simultaneously the most characteristic results was elaborated on the basis of the use of the similarities of price structures described in the previous section. This method was named as the **MPCP**-method – the method of „**M**aximal **P**ossible **C**haracteristic **P**rices“.

Suppose it is necessary to carry out a multilateral comparison among N countries of an aggregate which contains M commodities (or basic headings). The matrix of national prices $\mathbf{P_{ij}}$ (i=1,2,...,M; j=1, 2,...,N) and the matrix of corresponding quantities $\mathbf{q_{ij}}$ (i=1,2,...,M; j=1,2,...,N) are used as input data. The aim is to obtain transitive, additive and most possible characteristic (for each country) results.

[23] R.Hill found (Hill, 1999) that only **5** pairs of countries from **870** binary comparisons of 30 countries for two benchmark ICP years (1980 and 1985) had L/P ratios smaller than 1.

[24] A selective method is used sometimes in the the EKS method: original direct F-PPPs with L/P ratios greater than some limit (usually 1.5) and lower than 1 are replaced by indirect PPPs obtained via 3rd countries.

Suppose there is a positive vector of international prices π_i (i=1,2,...,M; $\pi_i > 0$). The level of similarity between the national price structure of each country and the structure of international prices can be measured analogously (I.1):

$$\text{(II.1)} \qquad \tau_j = \left[\frac{\sum_i (P_{ij}/\pi_i) * \sum_i (\pi_i/P_{ij})}{\sum_i (P_{ij}/\pi_i)^2 * \sum_i (\pi_i/P_{ij})^2} \right]^{1/2} \qquad j = 1,2,...,N; \; i = 1,2,...,M$$

The term τ_j can be considered as the degree of characteristicity of a given vector of international prices π for country j.

As it has been mentioned already in the section I, it is preferable to use the weighting in actual comparisons. Weighted version of (II.1) can be proposed in the following form:

$$\text{(II.2)} \qquad \tau_j = \left[\frac{\sum_i \{(P_{ij}/\pi_i)*d_i\} * \sum_i \{(\pi_i/P_{ij}) *d_i\}}{\sum_i \{(P_{ij}/\pi_i)^2*d_i\} * \sum_i \{(\pi_i/P_{ij})^2*d_i\}} \right]^{1/2} \qquad j = 1, 2,...,N; \; i = 1, 2,...,M$$

where

$d_i = \sum^N_{k=1} d_i^k / N$ - average weight for basic heading i across all countries (d_i^k are defined earlier in the section I).

Let the indicator τ_{min} is the minimal value among τ_j (j=1,N):

$$\text{(II.3)} \qquad \tau_{min} = \min (\tau_1, \tau_2, \tau_3, \ldots, \tau_N).$$

The indicator τ_{min} can be considered as the degree of general characteristicity of a given international prices π for the set of all comparing countries (j=1,N).

The MPCP method consists of searching for the vector of scalars π in such way that the indicator τ_{min} must have maximal (highest) possible value (τ_{opt}), which can be obtained by primary data:

$$\text{(II.4)} \qquad \tau_{opt} = \max_{J=1,2,...,N}(\min_{J=1,2,...,N} \tau_j) = \max \left\{ \min \left[\frac{\sum_i (P_{ij}/\pi_i)*d_i * \sum_i (\pi_i/P_{ij})*d_i}{\sum_i (P_{ij}/\pi_i)^2*d_i * \sum_i (\pi_i/P_{ij})^2 *d_i} \right]^{1/2} \right\}$$

$$\pi_i > 0 \; (i = 1, 2, 3, ..., M)$$

Obviously, if a vector π is a solution of (II.4) then each vector $\lambda*\pi$ is also a solution. Therefore we can find the prices with accuracy up to scalar. The structural (relative) prices are very appropriate for this purpose: a base product (e.g. product B) is selected, its price is set as 1 (π_B=1) and all other prices are measured to the relation to the price of base product.

The term (II.4) is a non-linear function. The method of configurations (R.Hooke's - T.Jeeves's method) or J.Nelder's - R.Mead's method can be used for solving of (II.4). Latter method is more preferable for practical calculations with significant number of unknown variables π_i.

The values π_i are searched by help of an iterative procedure starting from an initial vector. Several tests showed that the best approximation brings the following initial vector π:

(II.5) $\pi_i = [\min_j (P_{ij}/P_{Bj}) * \max_j (P_{ij}/P_{Bj})]^{1/2}$; i=1, 2,..., M; $\pi_B=1$

where

P_{Bj} - the price for basic commodity B ($1 \le B \le M$) in 'j'th country (the indicators like τ are invariant to the commodity selected as basis).

International prices π_i obtained from (II.4) have the highest possible degree of characteristicity for all comparing countries (j=1,N). The results of volume comparisons on the basis of vector πi from (II.4) are transitive, additive and they have the highest possible degree of characteristicity for all countries.

Two main new ideas of proposed method are the following:

1) Using the indicator of similarity of price structures instead of using a distance between values of different kinds of indices. For example, „IP" method uses the distance between IP-indices and Fisher's (or EKS) indices; EKS method uses the distance between EKS indices and Fisher's indices (or other type of bilateral indices).

2) Using the principle of **maximin** of similarity of price structures for searching of international price π_i. The indicators used earlier like (a) the minimization of the total sum of distances (for all countries in question) between different kinds of indices (as it is made in the EKS or the IP methods) or (b) the maximization of the total sum of coefficients τ_j (for all countries in question), i.e. $\sum\tau_j \rightarrow \max$ are not fully appropriate.

The latter feature 2) is very important because there are the situations when total sum of similarity coefficients is high but some countries have very small τ_j . <u>For example</u>: let us have two variants for 3 countries:

<u>1st variant</u>: $\tau_1 = 0.90$; $\tau_2 = 0.95$; $\tau_3 = 0.40$.
min $(\tau_j)= 0.40$; $\sum\tau_j = 2.25$.

<u>2nd variant</u>: $\tau_1= 0.70$; $\tau_2 = 0.80$; $\tau_3 = 0.60$.
min $(\tau_j) = 0.60$; $\sum\tau_j = 2.10$.

It can believe that the 2nd variant is preferable from the point of view of obtaining the maximal possible characteristic results for all countries.

A practical example, from the Eurostat 2002 comparison (see Table IV.2): average coefficient of similarity between international prices by the GK method and national prices is some

higher (= 0.8241) than by the MPCP method (= 0.8081) but minimal value obtained by the MPCP is much higher (= 0.6799, CH) than by the GK method (= 0.4840, Bulgaria).

The principal features of the proposed method can be illustrated by help one simple imaginary example. Let us have 3 countries (A, B, C) and an aggregate which consists from two commodities only:

- for country A : $P_{1A} = 2$, $P_{2A} = 1$;
- for country B : $P_{1B} = 8$, $P_{2B} = 1$;
- for country C : $P_{1C} = 10$, $P_{2C} = 1$.

The unweighted version (II.1) is used for the simplicity. If there are only two products then the formula (II.1) can be transformed to a more easy form:

(II.6) $$\tau_j = \sqrt{P_{1j} * \pi_1} * \frac{P_{1j} + \pi_1}{P_{1j}^{\,2} + \pi_1^{\,2}} \quad ; \qquad j = A, B, C$$

because there is only one unknown variable π_1 ($\pi_2 = 1$).

The functions τ_j (II.6) for countries A, B, C are shown on the Chart 1. The function "min τ_j" with its maximal value is shown on the Chart 2. The max(min τ_j) for given input data is 0.8065 ($\tau_A = 0.8065$; $\tau_B = 0.8881$; $\tau_C = 0.8065$) which is obtained with international price $\pi_1 = 4.47$ ($\pi_2 = 1$). To the point, in case of only two products the initial approximation for π_1 calculated by help (II.5) is exactly equal of optimal solution: $4.47 = (2*10)^{1/2}$.

Chart 1

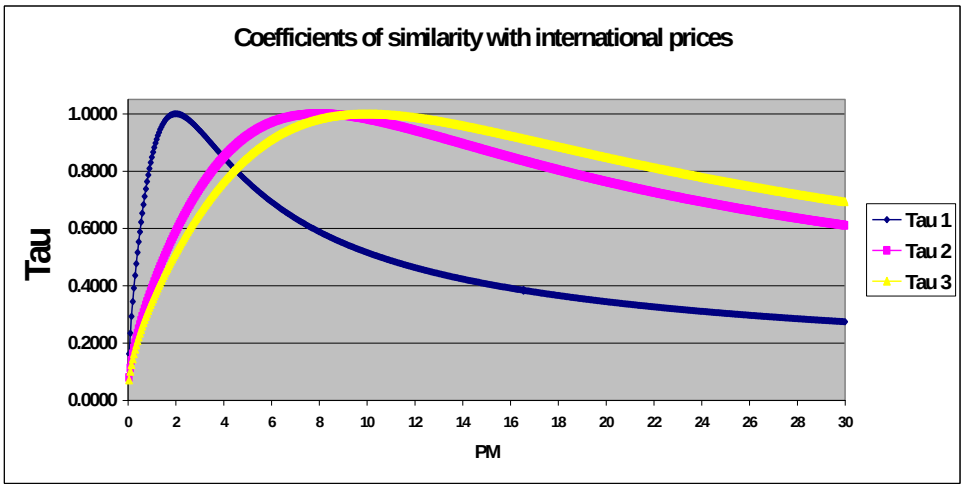
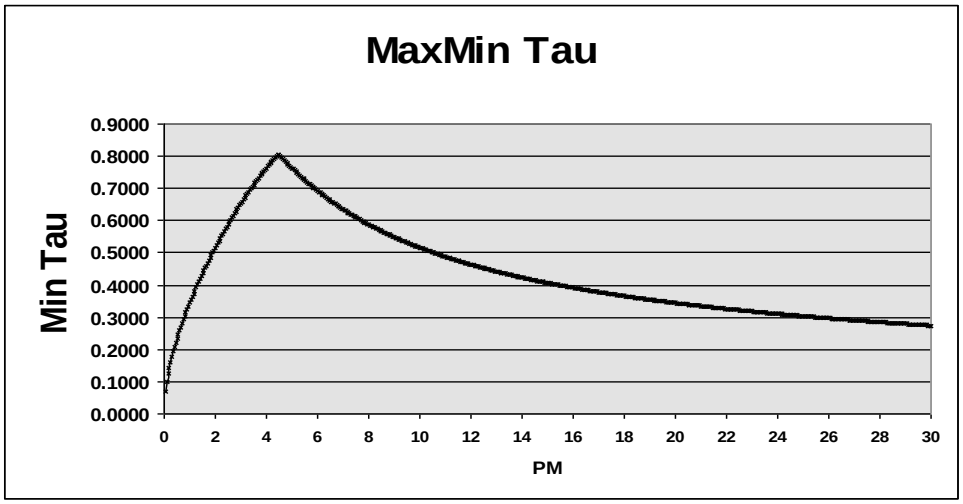


Chart 2



The practical application of the MPCP method in actual comparisons is not easy from the computional side when a comparing aggregate includes very great number of products (about 200-300 items / basic headings). The searching of vector π from (II.4) is a difficult computational task for the calculations in this case. To do this task more easy, the following procedure can be proposed:

(II.7) $\pi_i = [\min_j (P_{ij} / PPP_j) * \max_j (P_{ij} / PPP_j)]^{1/2}$; i=1, 2,..., M;

where

PPP_j - global PPP for a given aggregate between the currency of country 'j' (national prices) and implicit common currency (international prices) which should be obtained from (II.4).

In this case we have only (N-1) unknown variables PPP_j ($PPP_N = 1$) which are used in function (II.4) instead of unknown variables π_i. The values PPP_j obtained from (II.4) by help (II.7) can be used for the calculation of the international prices.

The experiments showed that the „best" initial values for the variables PPP_j are the following:

a) calculation of initial values of standardized international prices

$$\pi_i = [\min_j (P_{ij}/P_{Mj}) * \max_j (P_{ij}/P_{Mj})]^{1/2} ; \qquad i=1, 2,..., M .$$

b) calculation of initial values for PPP_j as ratio between total real value in international prices and total nominal value in the national currency.

It is possible to use also a combination of the GK method with the proposed method: to present initially the international prices in accordance with the GK method as quantity-weighted average of the PPPs-adjusted national prices:

(II.8) $$\pi_i = \sum_{j=1}^{N} (P_{ij} / PPP_j) * q_{ij} / \sum_{j=1}^{N} q_{ij} \; ; \qquad i = 1,2,...,M$$

and to introduce these prices in the (II.4) for the further transformation. The PPP_j obtained from (II.4) are a basis for calculation of the international prices. Latter modification of searching of international prices is a compromise between depending of international prices by weighting procedure (averaging the national prices) which means their gravitation to the prices of large (rich) countries and their characteristicity for all countries. Due to this modification there is a possibility to use only one set of results of multilateral comparison instead of two sets of results as in EUROSTAT - OECD PPP Programme (EKS - for volume comparisons / the request of characteristicity has the preference and GK - for structural comparisons / the request of addivity has the preference).

The MPCP method produces additive as well as highest possible results. Hence the MPCP results can be used for structural analysis as well as for volume comparison (due to elimination of Engel-Gerschenkron effect).

The MPCP method described above can be used also for linking results of several comparisons (e.g. to make up overall ICP results from ICP-Group results or world comparison from a set regional comparisons). Suppose we have a set of results from 'L' regional comparisons. Each regional comparison is based on its own vector of regional international prices π_k (k=1,L). The term (II.4) can be solved for these L regional price sets.

In effect it is possible to obtain the set of world international prices which are characteristic for each region in a maximally possible degree.

III. Method of Standardized Structure (SS-method)

The obtaining of meaningful international prices by the MPCP method is enough difficult. Some considerations of J.Cuthbert concerning the IP algorithm [see Cuthbert (1997)][25] - the absence of uniqueness and possible instability - are valid, in principle, for the MPCP method too. Both methods (IP and MPCP) are not very operational: the search of an optimum of a complicated function in multidimensional space is very difficult task from a computational point of view and, respectively, the obtaining of meaningful international price vector by an iterative method depends partly on the selection of a good initial approximation.

Therefore an attempt is done in this paper to elaborate a new aggregation multilateral method producing additive and simultaneously characteristic results on the basis of using some non-traditional elements of standard elementary indices. This method was named by the author as the method of **Standardized Structure** (**SS** method).

The application of an average structure has been used by some methods, e.g. the Tornkqvist-method or the Walsh-method. These methods use the averaging of individual indices between countries. To obtain the invariant results the geometric mean is used by these methods. However this leads simultaneously to the non-additivity of the results because geometric mean is additive in logarithmic terms but not in usual linear terms. To use the average structure with the obtaining of additive results, it is useful to look on the standard quantity indices of Laspeyres and Paasche from an another point of view than it does usually.

III.1 Non-traditional presentation of Laspeyres and Paasche indices

Let us have two countries A and B. Input data for an aggregate are sets of national prices (p_A and p_B) and quantities (q_A and q_B).

The Paasche Quantity index ($P_q^{A/B}$) – an index on the basis of prices of country A can be presented in the following form (subsripts of individual items are omitted for the simplicity):

$$\text{(III.1)}\quad P_q^{A/B} = \frac{\Sigma\, p_A * q_A}{\Sigma\, p_A * q_B} = \frac{\Sigma\, [q_A / (q_A+q_B)] * [p_A * (q_A+q_B)]}{\Sigma\, [q_B / (q_A+q_B)] * [p_A * (q_A+q_B)]} = \frac{\Sigma\, dQ_A * wT_A}{\Sigma\, dQ_B * wT_A} =$$

$$= \frac{\Sigma\, dQ_A * [wT_A / \Sigma\, wT_A]}{\Sigma\, dQ_B * [wT_A / \Sigma\, wT_A]} = \frac{\Sigma\, dQ_A * dT_A}{\Sigma\, dQ_B * dT_A}$$

$Q = (q_A+q_B)$ – total quantity for a concrete item for countries A and B

$dQ_A = q_A/Q$; $dQ_B = q_B/Q$ – the shares of countries A and B in the total quantity for an item;

[25] See also a paper to the Asia-Pacific Seminar on the use of PPP prepared by I.Sakuma together with Y.Kurabayashi; pages 166-168; 26-30.11.90, Niigata.

$wT_A = p_A * Q$ – total value for a concrete item at the prices of country A

$dT_A = wT_A / \Sigma wT_A$ – the share of a concrete item in the total value ($\sum dT_A = 1$)

The Laspeyres Quantity index ($L_q^{A/B}$) – an index on the basis of prices of country B can be presented in a similar form:

$$\text{(III.2)} \quad L_q^{A/B} = \frac{\Sigma\, p_B * q_A}{\Sigma\, p_B * q_B} = \frac{\Sigma\, [q_A / (q_A+q_B)] * [p_B * (q_A+q_B)]}{\Sigma\, [q_B / (q_A+q_B)] * [p_B * (q_A+q_B)]} = \frac{\Sigma\, dQ_A * wT_B}{\Sigma\, dQ_B * wT_B} =$$

$$= \frac{\Sigma\, dQ_A * [wT_B / \Sigma\, wT_B]}{\Sigma\, dQ_B * [wT_B / \Sigma\, wT_B]} = \frac{\Sigma\, dQ_A * dT_B}{\Sigma\, dQ_B * dT_B}$$

$Q = (q_A+q_B)$ - – total quantity for a concrete item for countries A and B

$dQ_A, = q_A/Q$; $dQ_B = q_B/Q$ - – the shares of countries A and B in the total quantity for a concrete item;

$wT_B = p_B * Q$ – total value for a concrete item at the prices of country B;

$dT_B = wT_B / \Sigma\, wT_B$ - the share of a concrete item in the total value ($\sum dT_B = 1$)

So, two principally new features are included in the considerations relatively the traditional presentation of Laspeyres and Paasche indices:

1) the use of the shares of quantities (dQ) instead of the quantity indices,

2) the use of general weights dT (calculated on the basis of set of Quantities – Total) instead of the use on national structures.

The shares dT_A reflect the structure of prices of country A, the shares dT_B – the structure of prices of country B (the vector of quantities is the same in both cases). An arithmetic average share of the shares dT_A and dT_B is equi-characteristic for both countries (the distances between the average structure and the structures on the basis of national prices is the same)[26]:

$$\text{(III.3)} \qquad \overline{dT} = (dT_A + dT_B)/2 \qquad\qquad (\sum \overline{dT} = 1)$$

In effect, the quantity index between countries A and B by the method of standardized structure ($IQ_{SS}^{A/B}$) can be obtained as:

$$\text{(III.4)} \qquad IQ_{SS}^{A/B} = \frac{\Sigma\, dQ_A * \overline{dT}}{\Sigma\, dQ_B * \overline{dT}}$$

It is easy to show that taking into account that $L_q^{A/B} = 1 / P_q^{B/A}$ the term (III.4) can be presented as :

[26] Therefore this method was named as the method of **S**tandardized **S**tructure = **SS method**

$$\text{(III.5)} \qquad IQ_{SS}^{A/B} = \frac{\Sigma\, dQ_A * \overline{dT}}{\Sigma\, dQ_B * \overline{dT}} = \frac{\Sigma\, dQ_A * (dT_A + dT_B)/2}{\Sigma\, dQ_B * (dT_A + dT_B)/2} =$$

$$= \frac{\{\Sigma p_A q_A * [\Sigma p_B*(q_A+q_B)] + \Sigma p_B q_A * [\Sigma p_A*(q_A+q_B)]\} / (\Sigma p_A q_A * \Sigma p_B q_B)}{\{\Sigma p_A q_B * [\Sigma p_B*(q_A+q_B)] + \Sigma p_B q_B * [\Sigma p_A*(q_A+q_B)]\} / (\Sigma p_A q_A * \Sigma\, p_B q_B)} =$$

$$= \frac{(1 + L_q^{A/B}) + L_q^{A/B} + L_q^{A/B} * L_q^{B/A}}{L_q^{A/B} * L_q^{B/A} + L_q^{B/A} + (1 + L_q^{B/A})} = \frac{L_q^{A/B} *(1 + P_q^{A/B}) + P_q^{A/B} *(1+ L_q^{A/B})}{(1 + P_q^{A/B}) + (1 + L_q^{A/B})}$$

So, both volume indices: $F_q^{A/B}$ and $IQ_{SS}^{A/B}$ are some averages from the L- and P-indices. In effect, it can believe that the $IQ_{SS}^{A/B}$ index will be in the most cases (if differences between L- and P-indices are note very high) very close to the Fisher's index.

The SS method has some similar features with the CKS method[27]:

a) the PPPs by both methods are not defined directly from the method, rather indirectly (dividing the expenditure ratio by the quantity ratio)

b) both methods use the total values of national quantities as a basis for the standardization (normalization)

These common features have a general character, the concrete algorithms are different. Some comparative analysis is given below.

The CKS quantity index ($IQ_{CKS}^{A/B}$) can be presented as follows:

$$\text{(III.6)} \qquad IQ_{CKS}^{A/B} = L_q^{A/B} * \frac{1 + P_q^{A/B}}{1 + L_q^{A/B}}$$

The Edgeworth-Marshall quantity index[28] ($IQ_{EM}^{A/B}$) can be presented in a very similar form:

$$\text{(III.7)} \qquad IQ_{EM}^{A/B} = P_q^{A/B} * \frac{1 + L_q^{A/B}}{1 + P_q^{A/B}}$$

The product of CKS and EM indices (it is related to the price indices as well as to the quantity indices) is Fisher index in square, i.e. Fisher's index ($F_q^{A/B}$) can be presented as a geometric unweighted average from the CKS and EM indices:

$$F_q^{A/B} = (IQ_{CKS}^{A/B} * IQ_{EM}^{A/B})^{1/2}.$$

It is easy to show that the $IQ_{SS}^{A/B}$ index is an arithmetic average from $IQ_{CKS}^{A/B}$ and $IQ_{EM}^{A/B}$ indices with the specific weighs $(1 + L_q^{A/B})$ and $(1+ P_q^{A/B})$:

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[27] **C**ommensurable **K**urabayashi-**S**akuma method – see I.Sakuma, D.S.Prasada Rao, Y.Kurabayashi (2000)

[28] The Edgeworth-Marshall method focuses on the PPP calculation. Here the Edgeworth-Marshall quantity index is the index obtained as value index divided by EM-PPP.

$$IQ_{SS}^{A/B} = \frac{IQ_{CKS}^{A/B} *(1 + L_q^{A/B}) + IQ_{EM}^{A/B} *(1+ P_q^{A/B})}{(1 + L_q^{A/B}) + (1 + P_q^{A/B})}$$

In effect, one can believe that the $IQ_{SS}^{A/B}$ index will be better to approximate the F-index in the most cases. Numerous imitations on the basis of simple numerical examples confirmed this hypothesis. The bilateral versions of the following methods were tested: Edgeworth-Marshall (EM), Geary-Khamis (GK), Gerardi (G), Tornqvist (T), Commensurable Kurabayshi-Sakuma (CKS) and Standardized Structure (SS)[29]. The SS method produced in the most cases the closest results to the Fisher index. The Fisher's index is regarded usually as an „ideal" („best") index in a bilateral case. The $IQ_{SS}^{A/B}$ index possesses all properties of the Fisher's index and simultaneously this is an additive index.

III.2 Multilateral version of the SS method

A bilateral case was considered above. The proposed method can be applied in a multilateral case also[30]. Let us have a multilateral comparison with N countries for an aggregate with M primary groups (basic headings = BH). The standard sets (matrices of size M x N) of input data are the following:

- p_{ij} is actual or „notional" price of ith item in the jth country (expressed in the units of national currency or as PPP for primary group i);

- q_{ij} is actual (physical units) or „notional" quantity (values in a common currency) for ith item in the jth country.

The general computational algorithm of the multilateral SS method is the same as in a bilateral case described above. The following intermediate indicators should be calculated (i = 1,2, ...,M; j = 1, 2, ...,N):

$Q_i = \sum_{j=1}^{N} q_{ij}$ - total quantity for all countries of the ith basic heading,

---

[29] It is interesting to note that the bilateral Fisher, EM, CKS and SS indices lay strictly between Laspeyres and Paasche indices. The bilateral Tornqvist, Gerardi and Geary-Khamis indices do not possess this property in general case.

[30] The main idea of the SS method was proposed firstly by S. Sergeev in „*Methods for the multilateral comparisons*". - Ph.D.Dissertation. Research Institut of Central Statistical Office of SU. Moscow, 1982 (in Russian). This method was described also in some later publications of the author of this paper (in Russian):
„*The making of the indices for international comparisons*". - Journal "Economic and mathematical methods", vol.XIX, No.6, 1983. USSR Academy of Sciences, Central Economic Mathematical Institute. (in co-autorship).
„An improvement of the methods of making of inter-space indices." - Journal of the Central State Statistical Commitee of USSR "Westnik statistiki" (Herold of Statistics), Moscow, N 2, 1986.
„An improvement of international comparisons of main macroeconomic indicators of CMEA-countries." - In the book "Statistical cooperation the CMEA-countries", Vol.5. Moscow, CMEA, 1989. (in co-autorship).
„International comparisons of macroeconomic economic indicators." - Journal "Economic cooperation of the CMEA-countries", CMEA, N 11, 1989. (in co-autorship).
„Modern tendencies in the cross-country comparison of the macroeconomic indicators." - Journal "Economic and mathematical methods", vol.XXYI, No.4, 1990. USSR Academy of Sciences, Central Economic Mathematical Institute. (in co-autorship).

$dQ_{ij} = q_{ij} / \sum_{j=1}^{N} q_{ij}$ - share of country j in total Quantity of BH i

$dT_{ij} = (p_{ij} * Q_i) / \sum_{l=1}^{M} (p_{lj} * Q_l)$ - share of BH i in total Value at prices of country j.

The average standardized structure is calculated on the basis of the sets of standardized structures at the national prices of all participating countries:

(III.8) $$\overline{dT_i} = \sum_{j=1}^{N} dT_{ij}/N \qquad i = 1, 2, ..., M$$

The quantity (volume) index between each pair of countries j and k for the aggregate in question can be obtained as the following:

(III.9) $$IQ^{j/k} = \frac{\sum_{i=1}^{M} dQ_{ij} * \overline{dT_i}}{\sum_{i=1}^{M} dQ_{ik} * \overline{dT_i}}$$

The formula (III.9) allows to calculate the volume indices for each sub-aggregate but it is much more appropriate to calculate the detailed results (the results at the lower aggregate levels) by help of the respective common set of international prices which can be obtained in the way described below.

The SS method is based on a set of shares ($\overline{dT_i}$) and a set of respective quantities (Q_i). Therefore, a set of respective prices (we named these prices as structural prices or the prices of standardized structure) can be obtained from the following system:

(III.10) $$\overline{dT_i} = \frac{\pi_i * Q_i}{\sum_{l=1}^{M} (\pi_l * Q_l)} \qquad i = 1, 2, ..., M$$

where

π_i is „International structural price“ of the ith item.

The system of linear equations (III.10) consists of M linear equations in M unknowns, one is redundant. This system is homogeneous. Obviously, it is sufficient for the comparison to have the prices with accuracy up to scalar, i.e. relative prices. We can select a base product (e.g. product M), set its price as 1 (π_M) and to measure all other prices to the relation to the price of base product. By dropping one equation (e.g. last equation) and setting $\pi_M = 1$ we become modified system which is no longer homogeneous because everything is now standardized on the product M.

There is no problem to solve the system of linear equations (III.10) by the standard methods of linear algebra. However it is possible to obtain an explicit term for the prices of

standardized structure by help of an elementary procedure taking into account that the ratio of two shares is equal to product of price and quantity ratios:

$$\text{(III.11)} \qquad \overline{dT_i} : \overline{dT_M} = \frac{\pi_i * Q_i}{\sum_{l=1}^{M}(\pi_l * Q_l)} : \frac{\pi_M * Q_M}{\sum_{l=1}^{M}(\pi_l * Q_l)} = (\pi_i : \pi_M) * (Q_i : Q_M)$$

Selecting the product (basic heading) M as a basis we represent all other prices in the relation to the price of the selected product up to a factor of proportionality:

$$\text{(III.12)} \qquad \pi_i = (\overline{dT_i} : \overline{dT_M}) / (Q_i : Q_M); \qquad i = 1, 2, ..., M \qquad \pi_M = 1$$

The volume index for each pair of countries (j and k) and for each level of aggregation (for each sub-aggregate - assume that this sub-aggregate include basic headings with the sequences No. from l1 to l2: $1 \le l1 < l2 \le M$) can be obtained on the basis of the prices (III.12) of standardized structure (SS-prices) in an usual way:

$$\text{(III.13)} \qquad IQ^{j/k} = \sum_{i=l1}^{l2} (\pi_i * q_{ij}) / \sum_{i=l1}^{l2} (\pi_i * q_{ik})$$

The purchasing power parity of given aggregate (GDP, etc.) for the jth country (PPP_j - national currency to common currency) can be obtained in an indirect way as by the following formula:

$$\text{(III.14)} \qquad PPP_j = \sum_{i=l1}^{l2}(p_{ij} * q_{ij}) / \sum_{i=l1}^{l2}(\pi_i * q_{ij}); \qquad j = 1,2,...,N$$

The general approach was described above. However it should be indicated here one particular problem which is important for practical calculations for all aggregation procedures. There are so-called „Problematic balancing categories“, i.e. primary groups that sometimes have negative nominal values (and correspondingly- negative „notional“ quantities) – „Changes in inventories“, „Net export“, etc. A direct use of negative values can lead to meaningless results - as, for example, a negative international price for primary group or even a negative global PPP for GDP. Problematic categories are usually excluded from the aggregation calculations and for these categories are made some special calculations after the main aggregation procedure (sometimes very complicated).

A simple method was elaborated by the author to include all categories in general calculations. The „balancing Items“ (Net exports“, „Changes in inventories“, etc.) are included by the SS aggregation procedure by the use of absolute quantities for the calculation of common structural prices (III.12)[31]. This modified method guarantees the obtaining of positive meaningful structural prices. Of course, the absolute quantities are used for the calculations of structural prices only. For calculations of volume indices (III.13) the actual quantities (values) based upon fact (with sign) have to be used.

[31] This modification is applicable to other aggregation procedures (GK, EKS, CKS, etc.) also. See S.Sergeev „*Treatment of basic headings with negative nominal values within the aggregation procedures*“ - Eurostat, Meeting of the Working Group on Purchasing Power Parities (LUX, 23th - 25th May 2000).

IV. Empirical illustrations on the basis of data from actual comparisons

To examine the efficiency of the new methods and the numerical differences with the results obtained by the traditional methods within the actual comparisons, some experimental calculation were made on the basis of input data from official Eurostat comparison for 2002 which included 282 primary groups (basic headings) for 31 countries.

The comparative results (Volume indices per capita) obtained by different methods are placed in the Table IV.1. Table IV.2 contains the detailed matrices of the coefficients of price similarity between the countries as well as between the national prices and different international prices. To obtain the full comparability with the official results, the two-stage procedure was applied by all methods to hold the fixity of the results for the 15 EU Members[32] (a standard principle used within the Eurostat comparisons due to political reasons). The following methods were used:

- official EKS
- Gerardi (Unit-Country-Weight)
- Geary-Khamis
- CPD-Rao (geometric share weighted GK)
- Arithmetic share weighted GK-Rao
- Ikle (harmonic share weighted GK)
- MPCP method (**M**aximal **P**ossible **C**haracteristic **P**rices)
- method of Standardized Structure

Tables IV.1 and IV.2 shows that the SS method works enough efficiently within the actual comparisons on the basis of detailed data. The SS method can be recommended for the practical applications due to its simplicity. In fact the SS results are quite close to the official Eurostat results obtained by the EKS method, i.e. these SS results posess the property of characteristicity and additionally the SS method produced additive results.

[32] It means that the methods were applied fistly to the 15 EU countries, secondly – to all 31 participating countries and the results (Volume indices) for 15 EU Members from the 1st calculation were integrated in the results obtained during the 2nd calculation. The answer on the question: What should be a object for the fixity by the methods like the EKS – Volume indices or PPPs? is not unambigious. This problem is described in the S.Sergeev „Calculation of fixed results within the EU comparison: some considerations“ - Paper for a special meeting of PPP compiliers (Eurostat, OECD, OeSTAT) -LUX, 20.11.98).

Table IV.1

GDP Volume indices per capita (EU15 = 100):
Eurostat comparison 2002 for 31 countries with fixity for EU15

	EKS official	Stand.structure	MPCP	Gerardi-UCW	GK	CPD-Rao	ShGK-Rao	Ikle	Max	Min	Max/Min
D	99.6	99.2	99.3	99.1	99.3	99.3	99.3	99.3	99.6	99.1	**1.004**
B	106.5	105.4	107.1	105.5	105.7	105.7	105.6	105.7	107.1	105.4	**1.016**
DK	112.7	113.1	114.9	113.2	113.5	113.4	113.4	113.6	114.9	112.7	**1.020**
EL	70.9	73.2	72.6	73.1	73.0	72.2	72.5	72.1	73.2	70.9	**1.033**
E	86.1	86.2	85.7	86.2	86.1	85.7	85.8	85.7	86.2	85.7	**1.006**
F	104.6	104.1	105.0	104.1	104.4	104.6	104.6	104.5	105.0	104.1	**1.009**
IRL	125.4	127.6	128.7	127.9	129.1	128.1	128.2	128.3	129.1	125.4	**1.029**
I	98.2	97.5	96.7	97.5	97.3	97.0	97.0	97.0	98.2	96.7	**1.016**
L	188.9	196.3	199.0	196.1	197.0	195.1	195.6	195.0	199.0	188.9	**1.053**
NL	111.5	111.3	112.8	111.4	111.6	111.3	111.2	111.4	112.8	111.2	**1.014**
A	110.9	109.8	110.2	109.9	110.2	110.1	110.0	110.2	110.9	109.8	**1.010**
P	70.7	73.5	72.3	73.4	73.6	73.1	73.3	73.0	73.6	70.7	**1.040**
FIN	101.7	102.4	103.1	102.5	103.5	102.7	102.6	102.9	103.5	101.7	**1.017**
S	104.6	105.8	107.8	106.0	106.2	106.2	106.0	106.4	107.8	104.6	**1.031**
UK	107.5	108.2	107.1	108.2	107.5	108.4	108.3	108.4	108.4	107.1	**1.012**
IS	109.0	109.4	110.2	109.2	111.4	108.8	108.7	108.9	111.4	108.7	**1.025**
NO	136.3	146.1	154.5	145.9	141.4	144.9	146.6	144.5	154.5	136.3	**1.133**
CH	114.1	119.1	122.1	118.7	116.9	119.2	120.3	118.7	122.1	114.1	**1.070**
BG	26.4	30.9	26.9	29.9	32.8	29.9	30.9	29.0	32.8	26.4	**1.241**
CY	76.1	76.5	73.5	77.3	80.9	78.5	77.4	79.5	80.9	73.5	**1.101**
CZ	61.9	61.3	59.1	61.2	63.9	61.9	62.0	61.9	63.9	59.1	**1.081**
EE	40.1	42.3	38.4	41.6	44.8	42.1	42.8	41.5	44.8	38.4	**1.166**
HU	53.4	53.8	51.0	53.1	55.9	54.0	54.6	53.4	55.9	51.0	**1.095**
LV	34.8	37.7	34.2	36.9	40.0	37.3	38.1	36.7	40.0	34.2	**1.168**
LT	39.1	42.3	39.0	41.3	44.9	42.0	43.0	41.1	44.9	39.0	**1.152**
MT	69.1	69.0	65.3	68.8	71.8	69.4	69.2	69.2	71.8	65.3	**1.100**
PL	41.2	41.8	39.5	41.2	43.7	41.9	42.4	41.4	43.7	39.5	**1.107**
RO	26.5	29.3	26.9	28.6	30.7	28.7	29.4	28.1	30.7	26.5	**1.157**
SK	47.0	49.0	45.8	48.6	51.7	49.1	49.4	48.8	51.7	45.8	**1.129**
SI	69.0	68.5	68.3	68.5	69.5	68.8	69.1	68.8	69.5	68.3	**1.017**
TU	24.6	27.1	24.7	26.5	28.6	26.7	27.3	26.4	28.6	24.6	**1.163**
EU15	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**			

Table IV.2

Coefficients of similarity of price structures: Eurostat 2002 comparison

		D	B	*DK*	EL	E	F	IRL	I	L	NL	A	P	FIN	*S*	*UK*
1	**D**	**1.0000**	0.9633	0.9339	0.8253	0.8774	0.9310	0.8996	0.9117	0.9239	0.9503	0.9548	0.7571	0.8907	0.9201	0.8973
2	**B**	0.9633	**1.0000**	0.9498	0.8873	0.9296	0.9608	0.9260	0.9420	0.9001	0.9637	0.9686	0.8175	0.9217	0.9297	0.9105
3	***DK***	0.9339	0.9498	**1.0000**	0.8195	0.8608	0.9255	0.9108	0.9132	0.8402	0.9468	0.9554	0.7961	0.9108	0.9385	0.9038
4	**EL**	0.8253	0.8873	0.8195	**1.0000**	0.9194	0.8935	0.8440	0.8784	0.7588	0.8628	0.8623	0.8562	0.8440	0.8248	0.8066
5	**E**	0.8774	0.9296	0.8608	0.9194	**1.0000**	0.9231	0.8487	0.9216	0.8510	0.9068	0.9068	0.8477	0.8423	0.8948	0.8738
6	**F**	0.9310	0.9608	0.9255	0.8935	0.9231	**1.0000**	0.9164	0.9141	0.8493	0.9549	0.9431	0.7793	0.9051	0.9251	0.9040
7	**IRL**	0.8996	0.9260	0.9108	0.8440	0.8487	0.9164	**1.0000**	0.8906	0.8295	0.9071	0.9061	0.7296	0.9443	0.9080	0.9087
8	**I**	0.9117	0.9420	0.9132	0.8784	0.9216	0.9141	0.8906	**1.0000**	0.8483	0.9080	0.9291	0.8521	0.8989	0.8997	0.8412
9	**L**	0.9239	0.9001	0.8402	0.7588	0.8510	0.8493	0.8295	0.8483	**1.0000**	0.8825	0.8595	0.7050	0.8103	0.8611	0.7860
10	**NL**	0.9503	0.9637	0.9468	0.8628	0.9068	0.9549	0.9071	0.9080	0.8825	**1.0000**	0.9469	0.7798	0.8800	0.9344	0.9152
11	**A**	0.9548	0.9686	0.9554	0.8623	0.9068	0.9431	0.9061	0.9291	0.8595	0.9469	**1.0000**	0.8428	0.9021	0.9350	0.9233
12	**P**	0.7571	0.8175	0.7961	0.8562	0.8477	0.7793	0.7296	0.8521	0.7050	0.7798	0.8428	**1.0000**	0.7445	0.7413	0.8094
13	**FIN**	0.8907	0.9217	0.9108	0.8440	0.8423	0.9051	0.9443	0.8989	0.8103	0.8800	0.9021	0.7445	**1.0000**	0.9202	0.8665
14	***S***	0.9201	0.9297	0.9385	0.8248	0.8948	0.9251	0.9080	0.8997	0.8611	0.9344	0.9350	0.7413	0.9202	**1.0000**	0.9087
15	***UK***	0.8973	0.9105	0.9038	0.8066	0.8738	0.9040	0.9087	0.8412	0.7860	0.9152	0.9233	0.8094	0.8665	0.9087	**1.0000**
16	**IS**	0.8249	0.8636	0.8803	0.8452	0.8505	0.8747	0.8890	0.8761	0.7856	0.8391	0.8669	0.7939	0.9242	0.9052	0.8749
17	**NO**	0.8730	0.8896	0.9189	0.7968	0.8387	0.8967	0.9027	0.8805	0.7894	0.8869	0.9120	0.7859	0.9143	0.9302	0.9286
18	**CH**	0.9343	0.9148	0.8842	0.7533	0.8324	0.8991	0.8844	0.8399	0.9227	0.9131	0.8999	0.6500	0.8781	0.9097	0.8526
19	**BG**	0.3707	0.3998	0.3750	0.5638	0.4867	0.4290	0.4040	0.4196	0.2826	0.4183	0.4259	0.4586	0.3962	0.3539	0.4019
20	**CY**	0.7197	0.7562	0.7771	0.8196	0.8258	0.7417	0.7024	0.8712	0.6704	0.7566	0.7948	0.8653	0.7247	0.7304	0.7793
21	**CZ**	0.6557	0.7041	0.6680	0.8349	0.7792	0.7171	0.6506	0.7283	0.5720	0.7132	0.7225	0.7918	0.6429	0.6434	0.6861
22	**EE**	0.5450	0.5871	0.5601	0.7566	0.6756	0.6273	0.5696	0.5924	0.4482	0.6182	0.6110	0.6298	0.5621	0.5577	0.5979
23	**HU**	0.6409	0.6923	0.6578	0.8266	0.7739	0.7135	0.6431	0.7048	0.5437	0.7154	0.7151	0.7726	0.6301	0.6478	0.6978
24	**LV**	0.4448	0.4895	0.4707	0.6152	0.5413	0.4886	0.4297	0.4966	0.3751	0.4956	0.5291	0.6094	0.4430	0.4451	0.5037
25	**LT**	0.4620	0.5070	0.4727	0.6561	0.5599	0.5186	0.4661	0.5133	0.3818	0.5138	0.5326	0.5998	0.4670	0.4541	0.5140
26	**MT**	0.6908	0.7592	0.7396	0.8350	0.7449	0.7461	0.7226	0.7640	0.6254	0.7423	0.7705	0.8601	0.7201	0.6861	0.7362
27	**PL**	0.6185	0.6734	0.6426	0.8097	0.7367	0.6841	0.6240	0.6864	0.5210	0.6705	0.6980	0.7697	0.6467	0.6161	0.6715
28	**RO**	0.4043	0.4420	0.4018	0.6192	0.5311	0.4788	0.4433	0.4570	0.3189	0.4592	0.4527	0.4913	0.4316	0.3987	0.4403
29	**SK**	0.5417	0.5847	0.5390	0.7391	0.6586	0.6062	0.5493	0.5857	0.4515	0.5945	0.6029	0.6697	0.5359	0.5211	0.5950
30	**SI**	0.8436	0.8853	0.8560	0.8928	0.8955	0.8682	0.8305	0.8982	0.7738	0.8555	0.8791	0.8564	0.8454	0.8346	0.8235
31	**TU**	0.4626	0.5129	0.4950	0.6632	0.6129	0.5351	0.4700	0.5646	0.3863	0.5185	0.5369	0.6471	0.4744	0.4569	0.5222
Average - Total		**0.7556**	**0.7877**	**0.7648**	**0.7970**	**0.7916**	**0.7850**	**0.7517**	**0.7809**	**0.6851**	**0.7817**	**0.7929**	**0.7436**	**0.7506**	**0.7544**	**0.7627**
Average - EU15		**0.9026**	**0.9265**	**0.9004**	**0.8488**	**0.8860**	**0.9089**	**0.8835**	**0.8964**	**0.8361**	**0.9100**	**0.9168**	**0.7899**	**0.8772**	**0.8958**	**0.8754**
Average - Euro12		**0.8986**	**0.9255**	**0.8969**	**0.8575**	**0.8886**	**0.9064**	**0.8766**	**0.8995**	**0.8380**	**0.9039**	**0.9111**	**0.7920**	**0.8712**	**0.8912**	**0.8702**
	Max	***0.9633***	***0.9686***	***0.9554***	***0.9194***	***0.9296***	***0.9608***	***0.9443***	***0.9420***	***0.9239***	***0.9637***	***0.9686***	***0.8653***	***0.9443***	***0.9385***	***0.9286***
	Min	***0.3707***	***0.3998***	***0.3750***	***0.5638***	***0.4867***	***0.4290***	***0.4040***	***0.4196***	***0.2826***	***0.4183***	***0.4259***	***0.4586***	***0.3962***	***0.3539***	***0.4019***

Coefficients of similarity of price structures: Eurostat 2002 comparison (cont.)

		IS	NO	CH	BG	CY	CZ	EE	HU	LV	LT	MT	PL	RO	SK	SI	TU
1	**D**	0.8249	0.8730	0.9343	0.3707	0.7197	0.6557	0.5450	0.6409	0.4448	0.4620	0.6908	0.6185	0.4043	0.5417	0.8436	0.4626
2	**B**	0.8636	0.8896	0.9148	0.3998	0.7562	0.7041	0.5871	0.6923	0.4895	0.5070	0.7592	0.6734	0.4420	0.5847	0.8853	0.5129
3	***DK***	0.8803	0.9189	0.8842	0.3750	0.7771	0.6680	0.5601	0.6578	0.4707	0.4727	0.7396	0.6426	0.4018	0.5390	0.8560	0.4950
4	**EL**	0.8452	0.7968	0.7533	0.5638	0.8196	0.8349	0.7566	0.8266	0.6152	0.6561	0.8350	0.8097	0.6192	0.7391	0.8928	0.6632
5	**E**	0.8505	0.8387	0.8324	0.4867	0.8258	0.7792	0.6756	0.7739	0.5413	0.5599	0.7449	0.7367	0.5311	0.6586	0.8955	0.6129
6	**F**	0.8747	0.8967	0.8991	0.4290	0.7417	0.7171	0.6273	0.7135	0.4886	0.5186	0.7461	0.6841	0.4788	0.6062	0.8682	0.5351
7	**IRL**	0.8890	0.9027	0.8844	0.4040	0.7024	0.6506	0.5696	0.6431	0.4297	0.4661	0.7226	0.6240	0.4433	0.5493	0.8305	0.4700
8	**I**	0.8761	0.8805	0.8399	0.4196	0.8712	0.7283	0.5924	0.7048	0.4966	0.5133	0.7640	0.6864	0.4570	0.5857	0.8982	0.5646
9	**L**	0.7856	0.7894	0.9227	0.2826	0.6704	0.5720	0.4482	0.5437	0.3751	0.3818	0.6254	0.5210	0.3189	0.4515	0.7738	0.3863
10	**NL**	0.8391	0.8869	0.9131	0.4183	0.7566	0.7132	0.6182	0.7154	0.4956	0.5138	0.7423	0.6705	0.4592	0.5945	0.8555	0.5185
11	**A**	0.8669	0.9120	0.8999	0.4259	0.7948	0.7225	0.6110	0.7151	0.5291	0.5326	0.7705	0.6980	0.4527	0.6029	0.8791	0.5369
12	**P**	0.7939	0.7859	0.6500	0.4586	0.8653	0.7918	0.6298	0.7726	0.6094	0.5998	0.8601	0.7697	0.4913	0.6697	0.8564	0.6471
13	**FIN**	0.9242	0.9143	0.8781	0.3962	0.7247	0.6429	0.5621	0.6301	0.4430	0.4670	0.7201	0.6467	0.4316	0.5359	0.8454	0.4744
14	***S***	0.9052	0.9302	0.9097	0.3539	0.7304	0.6434	0.5577	0.6478	0.4451	0.4541	0.6861	0.6161	0.3987	0.5211	0.8346	0.4569
15	***UK***	0.8749	0.9286	0.8526	0.4019	0.7793	0.6861	0.5979	0.6978	0.5037	0.5140	0.7362	0.6715	0.4403	0.5950	0.8235	0.5222
16	**IS**	**1.0000**	0.9252	0.8278	0.4187	0.8479	0.6608	0.6064	0.6612	0.4941	0.5023	0.7486	0.6561	0.4634	0.5559	0.8464	0.5299
17	**NO**	0.9252	**1.0000**	0.8659	0.3422	0.8058	0.6334	0.5383	0.6363	0.4567	0.4620	0.7193	0.6164	0.3749	0.5130	0.8344	0.4903
18	**CH**	0.8278	0.8659	**1.0000**	0.3127	0.6307	0.5638	0.4749	0.5470	0.3663	0.3914	0.6019	0.5280	0.3517	0.4569	0.7642	0.3939
19	**BG**	0.4187	0.3422	0.3127	**1.0000**	0.4500	0.7726	0.8104	0.7400	0.7671	0.8057	0.5911	0.7434	0.8533	0.8543	0.5004	0.7966
20	**CY**	0.8479	0.8058	0.6307	0.4500	**1.0000**	0.7383	0.6338	0.7229	0.5589	0.5577	0.8328	0.7226	0.4885	0.6294	0.8700	0.5868
21	**CZ**	0.6608	0.6334	0.5638	0.7726	0.7383	**1.0000**	0.8741	0.9576	0.8175	0.8669	0.8351	0.9048	0.7868	0.9214	0.8131	0.8481
22	**EE**	0.6064	0.5383	0.4749	0.8104	0.6338	0.8741	**1.0000**	0.9016	0.8982	0.9053	0.7592	0.8832	0.8070	0.8860	0.6637	0.8097
23	**HU**	0.6612	0.6363	0.5470	0.7400	0.7229	0.9576	0.9016	**1.0000**	0.8452	0.8596	0.8178	0.9114	0.7435	0.9180	0.7876	0.8361
24	**LV**	0.4941	0.4567	0.3663	0.7671	0.5589	0.8175	0.8982	0.8452	**1.0000**	0.9538	0.7171	0.8720	0.7068	0.8731	0.5408	0.8137
25	**LT**	0.5023	0.4620	0.3914	0.8057	0.5577	0.8669	0.9053	0.8596	0.9538	**1.0000**	0.7078	0.8973	0.7746	0.8968	0.5753	0.8595
26	**MT**	0.7486	0.7193	0.6019	0.5911	0.8328	0.8351	0.7592	0.8178	0.7171	0.7078	**1.0000**	0.8574	0.5749	0.7420	0.7931	0.7400
27	**PL**	0.6561	0.6164	0.5280	0.7434	0.7226	0.9048	0.8832	0.9114	0.8720	0.8973	0.8574	**1.0000**	0.7098	0.8720	0.7339	0.8210
28	**RO**	0.4634	0.3749	0.3517	0.8533	0.4885	0.7868	0.8070	0.7435	0.7068	0.7746	0.5749	0.7098	**1.0000**	0.8723	0.5442	0.7725
29	**SK**	0.5559	0.5130	0.4569	0.8543	0.6294	0.9214	0.8860	0.9180	0.8731	0.8968	0.7420	0.8720	0.8723	**1.0000**	0.6709	0.8627
30	**SI**	0.8464	0.8344	0.7642	0.5004	0.8700	0.8131	0.6637	0.7876	0.5408	0.5753	0.7931	0.7339	0.5442	0.6709	**1.0000**	0.6257
31	**TU**	0.5299	0.4903	0.3939	0.7966	0.5868	0.8481	0.8097	0.8361	0.8137	0.8595	0.7400	0.8210	0.7725	0.8627	0.6257	**1.0000**
Average - Total		**0.7546**	**0.7453**	**0.7015**	**0.5315**	**0.7204**	**0.7501**	**0.6797**	**0.7420**	**0.6020**	**0.6212**	**0.7394**	**0.7266**	**0.5531**	**0.6767**	**0.7801**	**0.6215**
Average - EU15		**0.8596**	**0.8763**	**0.8646**	**0.4124**	**0.7690**	**0.7007**	**0.5959**	**0.6917**	**0.4918**	**0.5079**	**0.7429**	**0.6713**	**0.4513**	**0.5850**	**0.8559**	**0.5239**
Average -		**0.8528**	**0.8639**	**0.8602**	**0.4213**	**0.7707**	**0.7094**	**0.6019**	**0.6977**	**0.4965**	**0.5148**	**0.7484**	**0.6782**	**0.4608**	**0.5933**	**0.8603**	**0.5320**
	Max	***0.9252***	***0.9302***	***0.9343***	***0.8543***	***0.8712***	***0.9576***	***0.9053***	***0.9576***	***0.9538***	***0.9538***	***0.8601***	***0.9114***	***0.8723***	***0.9214***	***0.8982***	***0.8627***
	Min	***0.4187***	***0.3422***	***0.3127***	***0.2826***	***0.4500***	***0.5638***	***0.4482***	***0.5437***	***0.3663***	***0.3818***	***0.5749***	***0.5210***	***0.3189***	***0.4515***	***0.5004***	***0.3863***

Coefficients of similarity of price structures: Eurostat 2002 comparison (cont.)

		SS prices		MPCP prices		GUCW prices		GK prices		CPD-Rao prices		ShGK-Rao prices		Ikle prices	
		EUR31	EU15	EUR31	EU15	EUR31	EU15	EUR31	EU15	EUR31	EU15	EUR31	EU15	EUR31	EU15
1	**D**	0.8754	0.9633	0.7618	0.9326	0.8541	0.9630	0.9294	0.9654	0.8650	0.9631	0.8690	0.9625	0.8400	0.9623
2	**B**	0.9247	0.9863	0.8179	0.9571	0.9073	0.9864	0.9650	0.9833	0.9180	0.9857	0.9205	0.9851	0.8965	0.9855
3	***DK***	0.8905	0.9615	0.7794	0.9338	0.8780	0.9610	0.9323	0.9529	0.8822	0.9571	0.8786	0.9578	0.8663	0.9556
4	**EL**	0.9542	0.9087	0.9131	0.8850	0.9530	0.9097	0.9282	0.9014	0.9510	0.9071	0.9504	0.9062	0.9445	0.9083
5	**E**	0.9439	0.9486	0.8768	0.9337	0.9361	0.9511	0.9586	0.9526	0.9392	0.9495	0.9368	0.9475	0.9273	0.9517
6	**F**	0.9289	0.9726	0.8322	0.9253	0.9162	0.9715	0.9664	0.9734	0.9240	0.9689	0.9242	0.9693	0.9084	0.9675
7	**IRL**	0.8787	0.9426	0.7631	0.8930	0.8597	0.9401	0.9187	0.9361	0.8761	0.9401	0.8785	0.9412	0.8567	0.9376
8	**I**	0.9299	0.9604	0.8433	0.9547	0.9135	0.9617	0.9511	0.9579	0.9221	0.9587	0.9251	0.9576	0.9010	0.9595
9	**L**	0.7958	0.8958	0.6801	**0.8846**	0.7682	0.8977	0.8523	0.8953	0.7817	0.9003	0.7905	0.8979	0.7487	0.9016
10	**NL**	0.9119	0.9688	0.8083	0.9260	0.8997	0.9683	0.9548	0.9683	0.9062	0.9671	0.9035	0.9671	0.8913	0.9662
11	**A**	0.9275	0.9773	0.8304	0.9574	0.9148	0.9780	0.9612	0.9750	0.9185	0.9740	0.9175	0.9736	0.9011	0.9739
12	**P**	0.8984	**0.8266**	0.8717	0.8963	0.8966	**0.8320**	0.8562	**0.8199**	0.8858	**0.8191**	0.8875	**0.8169**	0.8765	**0.8240**
13	**FIN**	0.8832	0.9421	0.7696	0.9010	0.8646	0.9398	0.9073	0.9249	0.8743	0.9400	0.8782	0.9417	0.8541	0.9370
14	***S***	0.8691	0.9499	0.7579	0.9085	0.8574	0.9492	0.9185	0.9452	0.8589	0.9463	0.8556	0.9473	0.8425	0.9448
15	***UK***	0.8941	0.9429	0.7864	0.9322	0.8848	0.9432	0.9317	0.9449	0.8866	0.9354	0.8847	0.9361	0.8735	0.9351
16	**IS**	0.8611	0.8899	0.7765	0.8802	0.8576	0.8907	0.8776	0.8785	0.8584	0.8893	0.8514	0.8898	0.8523	0.8889
17	**NO**	0.8672	0.9266	0.7537	0.9151	0.8563	0.9256	0.9005	0.9135	0.8551	0.9179	0.8525	0.9199	0.8399	0.9158
18	**CH**	0.7976	0.9098	**0.6799**	0.8545	0.7778	0.9089	0.8664	0.9096	0.7949	0.9107	0.7965	0.9097	0.7721	0.9097
19	**BG**	**0.5523**	0.4194	0.6927	0.3919	**0.5864**	0.4205	**0.4840**	0.4207	**0.5634**	0.4190	**0.5498**	0.4179	**0.5945**	0.4202
20	**CY**	0.8496	0.7999	0.8007	0.8487	0.8476	0.8051	0.8183	0.7967	0.8320	0.7906	0.8295	0.7881	0.8236	0.7953
21	**CZ**	0.8626	0.7222	0.9255	0.7197	0.8800	0.7251	0.7828	0.7224	0.8576	0.7202	0.8543	0.7182	0.8680	0.7233
22	**EE**	0.7593	0.6194	0.8667	0.5853	0.7950	0.6212	0.6907	0.6206	0.7679	0.6186	0.7518	0.6174	0.7977	0.6204
23	**HU**	0.8605	0.7212	0.9309	0.7161	0.8823	0.7240	0.7852	0.7238	0.8533	0.7188	0.8475	0.7172	0.8672	0.7215
24	**LV**	0.6320	0.4968	0.7461	0.5121	0.6643	0.5003	0.5540	0.4959	0.6310	0.4934	0.6201	0.4921	0.6560	0.4965
25	**LT**	0.6657	0.5217	0.7752	0.5210	0.6979	0.5245	0.5856	0.5230	0.6662	0.5193	0.6560	0.5177	0.6915	0.5220
26	**MT**	0.8674	0.7637	0.8770	0.7868	0.8754	0.7664	0.8019	0.7555	0.8603	0.7598	0.8598	0.7592	0.8606	0.7618
27	**PL**	0.8421	0.6960	0.9030	0.6926	0.8636	0.6978	0.7572	0.6923	0.8375	0.6933	0.8330	0.6929	0.8515	0.6950
28	**RO**	0.5923	0.4527	0.7156	0.4256	0.6239	0.4542	0.5202	0.4545	0.6064	0.4529	0.5951	0.4513	0.6340	0.4547
29	**SK**	0.7434	0.5947	0.8504	0.5832	0.7709	0.5976	0.6627	0.5969	0.7465	0.5934	0.7393	0.5915	0.7668	0.5963
30	**SI**	0.9395	0.9031	0.8848	0.9025	0.9317	0.9049	0.9184	0.8982	0.9258	0.8991	0.9292	0.8976	0.9150	0.9010
31	**TU**	0.6895	0.5433	0.7816	0.5470	0.7156	0.5457	0.6099	0.5425	0.6894	0.5379	0.6835	0.5365	0.7085	0.5405
Average - Total		**0.8351**	**0.8106**	**0.8081**	**0.7969**	**0.8365**	**0.8118**	**0.8241**	**0.8078**	**0.8302**	**0.8080**	**0.8274**	**0.8073**	**0.8267**	**0.8088**
Average - EU15		**0.9004**	**0.9431**	**0.8061**	**0.9214**	**0.8869**	**0.9435**	**0.9288**	**0.9398**	**0.8926**	**0.9408**	**0.8934**	**0.9405**	**0.8752**	**0.9407**
Average - Euro12		**0.9044**	**0.9411**	**0.8140**	**0.9206**	**0.8903**	**0.9416**	**0.9291**	**0.9378**	**0.8968**	**0.9395**	**0.8985**	**0.9389**	**0.8788**	**0.9396**
	Max	***0.9542***	***0.9863***	***0.9309***	***0.9574***	***0.9530***	***0.9864***	***0.9664***	***0.9833***	***0.9510***	***0.9857***	***0.9504***	***0.9851***	***0.9445***	***0.9855***
	Min	***0.5523***	***0.4194***	***0.6799***	***0.3919***	***0.5864***	***0.4205***	***0.4840***	***0.4207***	***0.5634***	***0.4190***	***0.5498***	***0.4179***	***0.5945***	***0.4202***

V. Conclusions

The multilateral methods are used within the international comparisons many years. The experiences showed that there is no an uniform method and various methods should be used for various aims. It is desirable to have the methods which combine the advantages and eliminate the disadvantages of different methods. This conclusion means in the practice that a set of possible multilateral methods should be broader and new ideas are necessary.

Mainly two kinds of multilateral aggregation methods are used in the present time:

A) **Averaging of bilateral indices** (e.g. the EKS method, etc.)

B) **Use of average international prices** – an averaging of national prices recalculated by PPPs into a common currency (the Geary-Khamis, Van IYzeren, Ikle methods, etc)

The methods of type **B** are based on the simultaneous calculation of the international prices and global PPPs (or global quantities) within a system of equations, ie. international prices are some functions from global PPPs (eg. **Geary-Khamis, Van IYzeren, Rao** methods) or global quantities (eg. the **CKS** method) or even both (**Ikle** method) and simultaneously global PPPs (global quantities) are some functions from these international prices.

A new kind of the aggregation procedures based on the use of, so called, structural international prices was proposed. It was shown that a volume index (IQ) for any pair of countries can be calculated not only on the basis of a set of common prices but also on the basis of their ratios (eg. to the price of a commodity M selected as a basis / numeraire). This circumstance allows to use an averaging of national price ratios instead of the averaging of national prices as it is done by the methods of type **B**.

The structure of national prices for a country can be presented as the quadratic matrix of price ratios or in a compressed form – as a vector of price ratios to price of basic commodity M. These internal (within a country) price ratios are not depend on the national currencies, ie. they are directly comparable between countries and can be averaged without the use of PPPs. These average international price ratios can be named as **international structural prices**. The respective PPPs are derived indirectly as ratio between value for a given aggregate in national prices (national currency) and value measured by known international prices. In effect, the methods using structural prices are strictly additive.

Two new methods of the basis of international structural prices were proposed.

The **MPCP method** (method of **M**aximal **P**ossible **C**haracteristic **P**rices) is an additive method which allows simultaneously to reflect in maximal possible degree the price structures of all participating countries, i.e. it allows to obtain the maximaly possible characteristic results. Therefore this method can be fruitfully used for volume comparisons as well as for structural comparisons for heterogeneous set of countries. It is very important for the overall worldwide ICP.

An attempt was done to elaborate a new aggregation multilateral method producing additive and simultaneously characteristic results on the basis of the use of some non-traditional elements of standard elementary indices like Lasypeyres and Paasche. The standard quantity Indices of Laspeyres and Paasche are presented from an another point of view than it did

usually. Two principially new features were included in the consideration relatively the traditional presentation of Laspeyres and Paasche indices:

2) the use of the shares of quantities (dQ) instead of the quantity indices

2) the use of general weights dT (calculated on the basis of set of Quantities –Total) instead of the use on national structures

Taking into account these new features in the bilateral case, a new multilateral method was proposed on the basis of the use of an average **S**tandardized **S**tructure for all participating countries (therefore this method was named as the **SS method** – method of **S**tandardized **S**tructure). It was demonstarted that a set of respective common international prices (these prices were named as the prices of standardized structure) can be obtained in a very simple way on the basis of the standardized structure and a set of respective common quantities.

Several empirical illustrations of the application of new methods were provided. The detailed experiments on the basis of detailed input data from the Eurostat comparison for 2002 (31 countries; 282 primary groups) were done to examine the efficiency of new methods. The experiments showed that the methods on the basis of international structural prices work efficiently within the actual comparisons on the basis of detailed data.

The SS method can be recommended for the practical applications due to its simplicity. In fact, the SS results were quite close to the official Eurostat/OECD results obtained by the EKS method for several exercises, ie. the SS results posess the property of characteristicity and additionally the SS method produced additive results.

Further investigations and comparative analysis of advantages and disadvantages of different methods allows to improve aggregation multilateral methods for the ICP-purposes.

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