Estimating Intergenerational Mobility in Developing Countries: New Methods and Evidence from India

Sam Asher, World Bank
Paul Novosad, Dartmouth College
Charlie Rafkin, MIT

DEC Policy Research Talk

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Outline

Preview

Motivation and Context

Estimating IGM in Developing Countries

Data

Results: Intergenerational Mobility in India
Understanding local growth and development

- Broader agenda examines determinants of growth and structural transformation in developing countries
- Explore within-country distribution of economic activity
- Assemble high spatial resolution administrative datasets covering every business and individual in India
Understanding local growth and development

- Broader agenda examines determinants of growth and structural transformation in developing countries
- Explore within-country distribution of economic activity
- Assemble high spatial resolution administrative datasets covering every business and individual in India
- Different papers testing the role of:
  - Natural resources
  - Local politics
  - Infrastructure (roads, canals)
  - Urban-rural spillovers
Motivation

- WDR 2006: Equity and Development starts with a comparison of expected life outcomes for two children in South Africa
  - Nthabiseng is black, poor parents, rural
  - Pieter is white, rich parents, urban
- Then only three pages dedicated to intergenerational mobility, due to constraints on data and methods
- Since then: much progress in rich countries, much less in developing countries
- Today’s talk:
  - Methods to generate estimates of mobility that are valid for comparisons across groups, time, geography
  - Application to India: Ram vs Rahim
Preview of Presentation

**Original Goal**: Estimate intergenerational mobility in India, across subgroups and across time (fathers and sons mostly, some new results from women’s data)

**Challenge**: Changing education distribution, coarse measurement of education

**Solution**: Partial Identification of expected child rank given parent rank

**Some Results**:
- No change in mobility from pre-liberalization to present
- Scheduled Castes and Tribes doing better, Muslims doing worse
- Substantial geographic variation

**Other Applications of These Methods**:
- Rising Mortality in the U.S.
- Fertility, Marriage, Bond Ratings, etc..
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Results: Intergenerational Mobility in India
Motivation

The last three decades have been an economic miracle in India.
Motivation

The last three decades have been an economic miracle in India.
Two stories about modern India:

1. Markets overturn old hierarchies
   - Caste identities less important in cities
   - Political mobilization of marginalized groups
   - Growing equality of opportunity?

The New York Times

*Scaling Caste Walls With Capitalism’s Ladders in India*

By LYDIA POLGREEN  DEC. 21, 2011

THE TIMES OF INDIA

The unexpected rise of dalit millionaires

July 31, 2011, 2:57 AM IST  SA Aiyar in Swaminomics | India | TOI
Two stories about modern India:

2. Persistence of Traditional Social Structure
   ▶ Punishment of norm violators, even in cities
   ▶ Identity-based politics
   ▶ Persistent inequality

The Economist explains

Why caste still matters in India

Its importance has diminished in the world of work, but persists in society, particularly where marriage is concerned

India's caste system: Outlawed, but still omnipresent

By Ravi Agrawal
© Updated 9:27 PM ET, Tue February 23, 2016
Research Agenda

Intergenerational Mobility: the dependence of children’s social status on their parents’ social status

- A measure of equality of opportunity
- Implications for income inequality, but can move independently
- A central political issue in many countries, including U.S., and esp. subgroup mobility
  - Affirmative action, tolerance of inequality and POUM (Benabou and Ok 2001), criminal politics in India (Vaishnav 2017)
Preview

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Results: Intergenerational Mobility in India
Standing on the Shoulders of Giants

World Development Report 2006
Equity and Development

Fair Progress?
Economic Mobility across Generations around the World

Ambar Narayan
Roy Van der Weide
Alexandra Coppsin
Christoph Lakner
Silva Redaelli
Daniel Guerin Mahler
Rakesh Gupta N. Ramasubbaiah
Stefan Thewissen

The World Bank Group
Measurement of Mobility (Chetty et al. 2014)

- Common measures:
  - Absolute Upward Mobility ($p_{25}$)
  - Absolute Downward Mobility ($p_{75}$)
  - Rank-Rank Gradient ($\beta$)
Matching parent-child income data is rarely available
- And almost never at the same age → life cycle bias

Matching parent-child income data is unreliable
- Key issue: measurement error → higher measured mobility
- Measuring rural income is hard
- How to attribute household income to coresident parents/children?

Most studies in developing countries use education as $Y$
- Arguably the best available proxy for lifetime income
Limitations of Conventional Method for IM Estimation

Standard approach:

- Linear estimation of child education (rank) on father education (rank)
- High coefficient $\rightarrow$ Low mobility

Some weaknesses of this measure:

1. Pools information from top and bottom of rank distribution
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2. Not useful for subgroup analysis
Gradient Not Useful for Subgroup Analysis

Group 2 has a lower rank-rank gradient $\rightarrow$ more mobile?
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Standard approach:

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Some weaknesses of this measure:

1. Pools information from top and bottom of rank distribution
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3. Education is observed coarsely
   - 57% of fathers of 1960s birth cohort in India have $< 2$ years education
   - Internationally comparable datasets (e.g. IPUMS) use $\leq 5$ ed bins
Parent-Child Education Rank Distribution, India

Mean Son Rank by Father Rank (1960s birth cohorts, India)
Comparing CEFs across time: India 1960s vs. 1980s

How should we compare the 1980 and 1960 birth cohorts?
Other Approaches in the Recent Literature

- Card et al. (2018) on educational mobility (IEM) in the 1920s
  - Definition: the 9th grade completion rate of children whose parents have 5–8 years of school
    - This is approximately $E(y > 50|x = 50)$
  - Compare this in 1980s with Chetty measure $E(y|x = 25)$

- Alesina et al. (2019) on IEM in Sub-Saharan Africa
  - Definition: Probability that a child completes primary school conditional on a parent who didn't
  - This is $E(y > 52|x \in [0,76])$ in Mozambique...
  - ... and $E(y > 18|x \in [0,42])$ in South Africa

- Our goal: calculate $E(y|x \in [a,b])$ for any $a$ and $b$
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▶ Our goal: calculate $E(y|x \in [a, b])$ for any $a$ and $b$
Our Strategy: A Partial Identification Approach

- **Key Idea:** Under minimal assumptions, we can *bound* the set of feasible mobility functions

- **Goal:** Conditional expectation function of child rank given parent *education percentile rank*
  - Call this $E(y|x = i)$
  - From this function, we can calculate $p_{25}$, $p_{75}$, $\beta$, and other measures of mobility

- **Problem:** Education rank $X$ is interval censored — only observed in coarse bins

- **Solution:** Use techniques from partial identification to bound child rank (Manski and Tamer 2002; Asher, Novosad, and Rafkin 2018)
Two Candidate Father-Son CEFs: India (1960s birth cohort)

Key question: What can we say about the latent conditional expectation function?

- Both of these CEFs $Y(i)$ fit the data with zero MSE
Overview of Methods

- Assumption 1: There exists a latent education rank, observed in coarse intervals
Overview of Methods

▶ Assumption 1: There exists a latent education rank, observed in coarse intervals

▶ Arises out of the most standard human capital investment model (e.g. Card 1999, Card et al. 2018)
  ▶ Schooling choice determined by heterogeneous cost and benefit shifters
  ▶ Model suggests a continuous optimal level of schooling $E$ for each individual
    ▶ Individuals complete the last year with positive expected value
  ▶ High ranked individuals within bin would advance to next level if marginal cost/benefit shifted only a little

▶ Note: this is a descriptive exercise
  ▶ We are not trying to estimate causal effects of parent education
Overview of Methods

Assume:
1. There exists a latent education rank, observed in coarse intervals
2. Monotonicity: Expected child rank is weakly increasing in parent rank \(^{(Dardanoni 2012)}\)
3. Child CEF has discrete jumps or kinks at major education boundaries only (if at all)
4. Child rank directly observed (loosened in paper)
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- **Manski and Tamer (MT 2002):**
  - Bound a CEF $E(y|x)$ with interval-censored $x$
  - But bounds are too wide to be meaningful in this case
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Manski and Tamer (MT 2002)
- Bound a CEF $E(y|x)$ with interval-censored $x$
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Three extensions to MT can get us useful estimates:
1. Use knowledge of the distribution (ranks are uniform)
2. Constrain curvature of CEF
3. Estimate $E(y|x \in (a, b))$ instead of $E(y|x = i)$
An Example for Intuition

- Assume $E(y|x \in (0, 60)) = 40$, $E(y|x \in (60, 80)) = 55$.
- We want to know $z = E(y|x \in (0, 50))$. 

An Example for Intuition
An Example for Intuition
An Example for Intuition

Monotonicity within bin $\rightarrow z \leq 40$
Monotonicity with next bin $\rightarrow z \geq 37$ — but suggests desirability of curvature constraint.
Constraining Curvature of the CEF

- Edges of CEFs imply large discrete changes at arbitrary rank intervals
  - These are unlikely to occur in real life
- Add assumption $Y''(x) \leq \bar{C}$
- Numeric rather than analytic solution (currently)
  - Matlab solver calculates bounds on discrete approximation to CEF
  - CEF approximation takes the form of 100 values for 100 integer ranks
- Permit (but do not impose) sheepskin effects by allowing discrete jumps at bin thresholds (e.g. where people attain degrees)
- Most extreme curvature constraint $\bar{C} = 0$ gives us the rank-rank gradient
  - Curvature-constrained estimation generalizes the standard approach
Choosing a Curvature Restriction

How to choose \( \bar{C} \)?

- We found parent-child income rank CEFs for USA, Denmark, Sweden, Norway \( \text{Chetty et al. (2016), Bratberg et al. (2017)} \)
- Fit splines to each of these and calculate max 2nd derivative

*Example Spline*

- Conservative \( \bar{C} \): Double the maximum value of 1.6 to get \(~ \bar{C} = 3~\)
- All results go through without curvature constraint
CEF Bounds under Interval Data: Manski-Tamer 2002

MT Bounds under an Unknown Distribution Bin Means
CEF Bounds under Interval Data: Uniform X

Bounds under Uniform Distribution Bin Means

Parent rank

Child rank

Bounds under Uniform Distribution

Bin Means
CEF Bounds under Interval Data: $\bar{C} = 20$
CEF Bounds under Interval Data: $\overline{C} = 10$
CEF Bounds under Interval Data: $\overline{C} = 3$
CEF Bounds under Interval Data: $\overline{C} = 0$
Some Additional Details

- Validate this approach using fully supported income rank CEF from Denmark (Bratberg et al. 2017)
  - Interval censor the data using education bin boundaries, and recover bounds on the original CEF

- Can assume curvature constraint and loosen monotonicity assumption
  - Allows estimation of interval-censored function even without monotonicity assumption

- Get standard errors through bootstrap
CEF Bounds With Bootstrap Errors: $\bar{C} = 3$
Introducing $\mu_a^b$

- Same structure allows us to bound any function of the CEF
- We focus on one function of interest $\mu_a^b = E(Y|x \in (a, b))$
  - e.g., $\mu_0^{25}$: expected child rank, given a parent in the bottom 25%

- $a$ and $b$ can be arbitrary and unrelated to actual bin boundaries

- Bounds on $\mu_a^b$:
  - Trivially point estimated if $a$ and $b$ are boundaries in the data
  - Tightest when bin boundaries are close to $a$, $b$

- We will use $\mu_0^{50}$ and $\mu_{50}^{100}$
  - $\mu_0^{50}$ is a measure of upward mobility analogous to $p_{25}$ from Chetty et al.
    - $p_{25}$ is the expected rank of a child born to the median parent in the bottom half
    - $\mu_0^{50}$ is the expected rank of a child born to any parent in the bottom half
  - If the CEF is linear, $\mu_0^{50} = p_{25}$
Mobility Statistics: 1960s birth cohort

Bounds on key mobility statistics $\overline{C} = 3$:

- Gradient $\beta$: $[0.45, 0.63]$
- Abs. Mobility $p_{25}$: $[31.0, 46.0]$
- Interval Mobility $\mu_0^{50}$: $[36.5, 38.5]$

Why are $\mu_0^{50}$ bounds so much tighter?

- Bin 1 has fathers in ranks 1-57:
  - $\mu_0^{57}$ is point identified – we observe it in the data
  - $\mu_0^X$ is mean of $\mu_0^1, \mu_0^2, \ldots, \mu_0^{X-1}$
  - Given $\mu_0^{57}$ and monotonicity, narrow set of possible values of $\mu_0^{50}$.
- In paper: proof of analytical bounds for $\mu_a^b$ given interval data
Comparison with Other Approaches

Other approaches to dealing with coarse data

- Focus on groups for whom education has not changed very much
  - Useful, but limiting
Comparison with Other Approaches

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- Assume linearity of CEF
  - Canonical approach in intergenerational educational mobility
  - But many fully supported CEFs are concave at bottom
  - Doesn’t distinguish change at top from change at bottom
  - Identical to our approach with $\bar{C} = 0$
Comparison with Other Approaches

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- Randomly reassign people across bins to get same bin sizes
  - Common in the mortality / education literature
  - Used in Fair Progress (2018)
    - Concludes Ethiopia has almost perfect upward mobility
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Results: Intergenerational Mobility in India
Data

Two Data Sources

1. India Human Development Survey
   - Household survey, men are asked about fathers’ education
   - Because education is static, records mobility going back 30 years
   - Subgroup characteristics: caste and religion
   - Mothers/Daughters — data limitations, some preliminary results

2. Socioeconomic and Caste Census (2012)
   - Complete enumeration of household demographics and assets
   - One of many administrative data sources used internally by government
   - 31 million sons aged 20-23 who coreside with fathers
   - Limited subgroup characteristics—but has names
The SHRUG

The Socioeconomic High-res Rural-Urban Geographic panel for India (SHRUG)

- High resolution socioeconomic aggregation of remote sensing and administrative data
- 600,000 villages and 5,000 towns from 1990 to the present
- Version 1.0 release is imminent
Motivation and Context

Estimating IGM in Developing Countries

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Results: Intergenerational Mobility in India
Comparing 1950s to 1980s birth cohort (raw data)
Comparing 1950s to 1980s birth cohort (preferred estimate)
Comparing 1950s to 1980s birth cohort (naive estimate)

- Naive estimates suggest small but decisive mobility improvement.
- Closes 40% of gap with USA, and 17% of gap with Denmark.
- Nonparametric graphs show this is driven by decrease in persistence at the top.
Upward and Downward Mobility over Time: All India

![Graph showing upward and downward mobility over time. The x-axis represents birth cohorts from 1950 to 1990, and the y-axis represents expected son rank. The graph shows that downward mobility decreases over time, while upward mobility increases.]
Upward Mobility: By Subgroup

- Forward / Others
- Muslims
- Scheduled Castes
- Scheduled Tribes

Expected Son Rank

Birth Cohort:
- 1950
- 1960
- 1970
- 1980
- 1990
Downward Mobility: By Subgroup

Expected Son Rank

- Forward / Others
- Scheduled Castes
- Scheduled Tribes
- Muslims

Birth Cohort

1960 1970 1980
High School ($\mu_{0}^{50}$, i.e. given Parent in Bottom Half)
Gender and Mobility

- Thus far all results for fathers and sons
- Breaking out results by gender tells a richer story:
  - Relative decline in Muslim education does not hold for women
  - Same-sex parents matter most for children’s outcomes
  - Mother’s education cannot explain decline in Muslim mobility
Educational Attainment by Group and Cohort (Full Sample)

- All groups gaining education
- Persistent gap between forward castes and others
- Muslim men but not women falling behind SCs
Mobility by Gender

- Lower values mean that parent’s low rank matters more for child outcomes
- Different intervals allow us to make different comparisons
Mobility by Gender and Group

Upward Mobility by Parent/Child Gender (µ−0−50) group

Upward Mobility by Parent/Child Gender (µ−0−80) group

Forwards Muslims
Scheduled Castes
Scheduled Tribes

Upward Mobility by Parent/Child Gender (µ−0−50) group

Upward Mobility by Parent/Child Gender (µ−0−80) group

Forwards Muslims
Scheduled Castes
Scheduled Tribes
Mother’s Education Does Not Explain Muslim Decline

Fertility differences also unable to explain the gap
How Much Does Geography Explain?

Two extreme hypotheses:

1. The mobility curve exists locally:
   - Within any location, boys with low-ranked fathers have little opportunity to advance
   - Within any location, mobility is highest for forwards and lowest for Muslims

2. Group differences are entirely based on geography:
   - e.g. Maybe everyone does well in AP, and poorly in Bihar
   - In places where Muslims live, everyone has low mobility
   - If we control for place fully, group differences will disappear
Mobility Differences Across Groups

Upward Mobility By Group, Unadjusted

Unadjusted

Forwards/Others
Muslims
Scheduled Castes
Scheduled Tribes

Forwards/Others: 41.2
Muslims: 28.8
Scheduled Castes: 36.8
Scheduled Tribes: 33.0
Transform These to Mobility Gaps to Forward/Others

Upward Mobility Gaps to Forward/Others, By Group

Unadjusted | Within-State Mobility | Within-District Mobility

Muslims | Scheduled Castes | Scheduled Tribes

-12.4 | -4.4 | -8.2
-15 | -10 | -5
0
Mobility Gaps Controlling for State and District

Upward Mobility Gaps to Forward/Others, By Group

Unadjusted

Within-State Mobility

Within-District Mobility

Muslims

Scheduled Castes

Scheduled Tribes

-12.4
-4.4
-8.2
-10.2
-3.6
-6.3
-11.3
-3.8
-3.4
The Local Geography of Intergenerational Mobility

What characteristics of places predict high and low mobility?
Mobility is Higher in Cities

![Graph showing mobility comparison between Rural and Urban areas]
The Effect of Cities Depends on Group Identity

![Upward Mobility Gap Bar Chart]

<table>
<thead>
<tr>
<th></th>
<th>Rural</th>
<th>Urban</th>
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<td>-12.3</td>
<td>-14.9</td>
</tr>
<tr>
<td>Scheduled Castes</td>
<td>-4.2</td>
<td>-4.6</td>
</tr>
<tr>
<td>Scheduled Tribes</td>
<td>-7.3</td>
<td>-7.1</td>
</tr>
</tbody>
</table>

Note: The negative values indicate a gap in upward mobility behind Forward / Others.
Regional Variation in Mobility is Substantial
Local Variation in Mobility is Also Substantial
Correlates of Rural Mobility

Expected Upward Mobility Rank Gain from 1 SD Change

- Share Scheduled Caste
- SC/ST Segregation
- Land Inequality
- Consumption Inequality
- Manufacturing Jobs Per Capita
- Log Rural Consumption
- Average Years Education
- Average Remoteness
- Primary Schools per Capita
- High Schools per Capita
- Share Villages with Paved Roads
- Share Villages with Power
Correlates of Urban Mobility

Expected Upward Mobility Rank Gain from 1 SD Change

- Log Population
- Population Growth 2001–2011
- Share Scheduled Caste
- SC/ST Segregation
- Consumption Inequality
- Manufacturing Jobs Per Capita
- Log Urban Consumption
- Average Years Education
- Primary Schools per Capita
- High Schools per Capita
Conclusions on Mobility in India

- Our method for calculating mobility is valid for comparison across subgroups, countries, and time when only educational data available for older generation.
- Intergenerational mobility gains and growth are not inextricably linked.
- Access to opportunity is improving for policy-targeted Scheduled Castes, but Muslims are losing ground.
  - Contrasting political talking points on “Muslim appeasement.”
- Substantial regional and local mobility variation.
- Ongoing work on mechanisms: discrimination, affirmative action.
- Long run: understanding barriers to migration, especially rural to urban.
Appendix Figures
Choosing a Curvature Restriction

Panel A: U.S.A.

Cubic spline, 4 knots

2nd derivative in $[-.009,.017]$.

Panel B: Denmark

Cubic spline, 4 knots

2nd derivative in $[-.026,.028]$.

Panel C: Sweden

Cubic spline, 4 knots

2nd derivative in $[-.015,.066]$.

Panel D: Norway

Cubic spline, 4 knots

2nd derivative in $[-.011,.046]$.
Test Case: Simulate Interval Censoring in Danish Data

- We know the full CEF in Denmark
- Create parent bins on intervals from Indian data, and apply interval censoring
  - All we will observe is mean child rank in each parent bin
- Calculate bounds using our methodology and compare to original distribution
Test Case: Raw Danish Data

Cubic spline, 4 knots

2nd derivative in $[-0.026, 0.028]$
Test Case: Simulated Interval Censoring in Danish Data

![Graph showing the relationship between child rank and parent rank. The graph displays a trend line with scattered data points.](image-url)
Test Case: Recovering Bounds from Censored Data

Panel A

Panel B

Panel C

Panel D
Implementing the Curvature Constraint

- Numerically calculate set of feasible curvature-constrained CEFs
- Minimize MSE with respect to data, i.e. match mean outcome in each coarse bin
- Calculate the min and maximum of the CEF at a given point that satisfies this minimum MSE
- $\text{MSE} = 0$ is possible
Numerical Optimization Problem

We seek

$$Y_{\min}(i) = \min\{Y(i)\} \quad (1)$$

such that

$$Y(i) \text{ is weakly increasing in } i \quad \text{ (Monotonicity)}$$

$$|Y''(i)| \leq C, \quad \text{ (Curvature)}$$

$$\left[ \sum_{k=1}^{K} \frac{i_{k+1} - i_k}{100} \left( \left( \frac{1}{i_{k+1} - i_k} \int_{i_k}^{i_{k+1}} Y(i)di \right) - r_k \right)^2 \right] = \text{MSE}$$

(MSE Minimization)

- **MSE**: Lowest attainable $MSE$ under monotonicity and curvature constraints
  - Could use any desired loss function

Back