

Structural Change with Long-run Income and Price Effects

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Abstract

We present a new multi-sector growth model that features nonhomothetic, constant-elasticity-of-substitution preferences, and accommodates long-run demand and supply drivers of structural change. The model generates a log-linear relationship between relative sectoral demand and real income, implying non-vanishing nonhomotheticities for all income levels. The model is consistent with the decline in agriculture, the hump-shaped evolution of manufacturing, and the rise of services over time. We estimate the demand system derived from the model using household-level data from the U.S. and India, as well as historical aggregate-level panel data for 39 countries during the postwar period. The estimated model parsimoniously accounts for the broad patterns of sectoral reallocation observed among rich, miracle and developing economies. Our estimates support the presence of strong nonhomotheticity that is stable across time, income levels, and countries. We find that income effects account for over 80% of the observed patterns of structural change.

Keywords: Structural Transformation, Nonhomothetic CES preferences, Implicitly Additively Separable Preferences.

JEL Classification: E2, O1, O4, O5.

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1 Introduction

Economies undergo large scale sectoral reallocations of employment and capital as they develop, in a process commonly known as structural change (Kuznets, 1973; Maddison, 1980; Herrendorf et al., 2014; Vries et al., 2014). These reallocations lead to a gradual fall in the relative size of the agricultural sector and a corresponding rise in manufacturing. As income continues to grow, services eventually emerge as the largest sector in the economy. Leading theories of structural change attempt to understand these sweeping transformations through mechanisms involving either supply or demand. Supply-side theories focus on differences across sectors in the rates of technological growth and capital intensities, which create trends in the composition of consumption through price (substitution) effects (Baumol, 1967; Ngai and Pissarides, 2007; Acemoglu and Guerrieri, 2008). Demand-side theories, in contrast, emphasize the role of heterogeneity in income elasticities of demand across sectors (nonhomotheticity in preferences) in driving the observed reallocations accompanying income growth (Kongsamut et al., 2001).

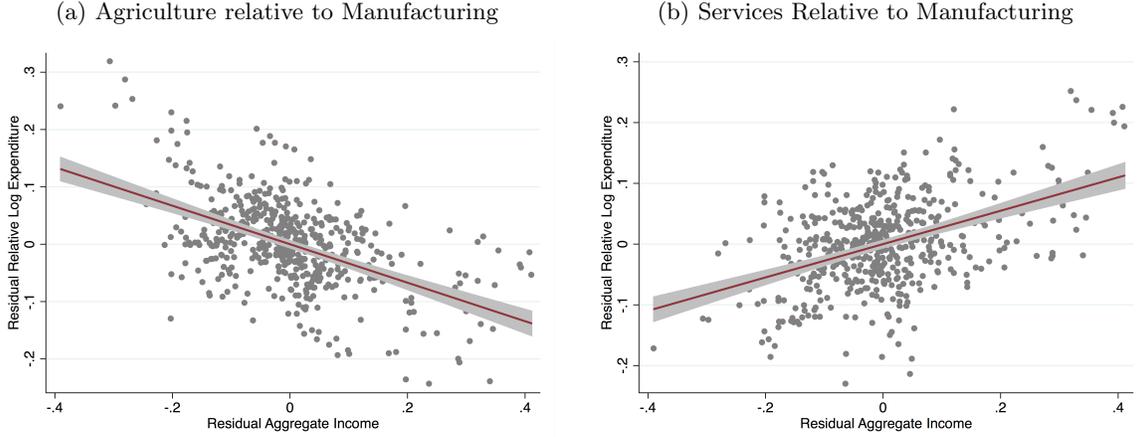
The shapes of sectoral Engel curves play a crucial role in determining the contribution of supply and demand channels to structural change.¹ If the differences in the slopes of Engel curves are large and persistent, demand channels can readily explain the reallocation of resources toward sectors with higher income elasticities. For instance, steep upward Engel curves for services, flat Engel curves for manufacturing, and steep downward Engel curves for agricultural products can give rise to sizable shifts of employment from agriculture toward services. However, demand-side theories have generally relied on specific classes of nonhomothetic preferences, e.g., generalized Stone-Geary preferences, that imply Engel curves that level off quickly as income grows. Because of this rapid flattening-out of the slopes of Engel curves across sectors, these specifications limit the explanatory power of the demand channel in the long-run.

The empirical evidence suggests that the differences in the slopes of sectoral Engel curves are stable at different income levels, and do not level off rapidly as income grows. At the macro level, log-linear relative Engel curves provide reasonable approximations to aggregate shares of sectoral consumption. Figure 1 plots the relationship between the residual (log) expenditure share in agriculture (Figure 1a) and services (Figure 1b) relative to manufacturing on the y-axis and residual (log) income on the x-axis after controlling for relative prices. The depicted log-linear fit shows that a constant slope captures a considerable part of the variation in the data.² Similarly, at the micro level, Engel curves have been shown to be well approximated by

¹We define Engel curves as the relationship between the logarithm of the shares of sectoral consumption and the logarithm of the aggregate real consumption, holding prices constant. Correspondingly, we refer to relative Engel curves when the relationship is between the logarithm of relative shares and the logarithm of real consumption.

²The partial R^2 of the regressions shown in Figure 1 are 27% and 20%, respectively. In fact, if we split the

Figure 1: Partial Correlations of Sectoral Expenditure and Aggregate Consumption



Notes: Data for OECD countries, 1970-2005. Each point corresponds to a country-year observation after partialling-out sectoral prices and country fixed effects. The red line depicts the OLS fit, the shaded regions, the 95% confidence interval. Residual Aggregate Income is constructed by taking the residuals of the following OLS regression: $\log Y_t^n = \alpha \log p_{at}^n + \beta \log p_{mt}^n + \gamma \log p_{st}^n + \xi^n + \nu_t^n$ where superscript n denotes country, and subscript t , time. Y_t^n , p_{at}^n , p_{mt}^n , and p_{st}^n denote aggregate income, the prices of agriculture, manufacturing, and services, respectively. ξ^n denotes a country fixed effect and ν_t^n the error term. Residual log-expenditures are constructed in an analogous manner using the log of relative sectoral expenditures as dependent variables. Table F.1 in the online appendix reports the estimates of the regression.

log-linear functions with constant slopes.³ As we discuss below, we complement this evidence, using data from the Consumption Expenditure survey (CEX) from the US and the National Sample Survey (NSS) from India, and show that the differences in the income elasticities of sectoral goods remain stable when estimated across different household-income brackets.

Motivated by this evidence, we develop a multi-sector model of structural change that accommodates nonhomotheticity of demand in the form of log-linear relative Engel curves, as

sample into observations below and above the median income in the sample and estimate the relative Engel curves separately, we cannot reject the hypothesis of identical slopes of the Engel curves. See Table F.1 in the online appendix. If we reported separately the Engel curves for agriculture, manufacturing and services, we would find a negative, zero and positive slope, respectively.

³Examples of log-linear specifications with constant slopes include, among others, the following. Aguiar and Bils (2015) use the U.S. Consumer Expenditure Survey (CEX) to estimate Engel curves for 20 different consumption categories. Their estimates for the income elasticities are different from unity and vary significantly across consumption categories. Young (2012) employs the Demographic and Health Survey (DHS) to infer the elasticity of real consumption of 26 goods and services with respect to income for 29 sub-Saharan and 27 other developing countries. He estimates the elasticity of consumption for the different categories with respect to the education of the household head and then uses the estimates of the return to education from Mincerian regressions to back out the income elasticity of consumption. Young also uses a log-linear Engel curve formulation and finds that the slopes of Engel curves greatly differ across consumption categories but appear stable over time. Olken (2010) discusses Young's exercise using Indonesia survey data and finds similar results for a small sample of three goods and services. Young (2013) also makes use of log-linear Engel curves to infer consumption inequality.

well as trends in relative prices. The model builds on the standard framework used in recent empirical work on structural transformation (e.g., [Herrendorf et al., 2013](#)). Our key departure from the standard framework is the introduction of a class of utility functions that generates nonhomothetic sectoral demands for all levels of income, including when income grows toward infinity. These preferences, which we will refer to as nonhomothetic Constant Elasticity of Substitution (CES) preferences, have been studied by [Gorman \(1965\)](#), [Hanoch \(1975\)](#), [Sato \(1975\)](#), and [Blackorby and Russell \(1981\)](#) in the context of static, partial-equilibrium models. Our theory embeds these preferences into a general equilibrium model of economic growth. We then estimate these preferences and use their structure to assess the contribution to structural change of income and price effects.

Nonhomothetic CES preferences present a number of unique features that make them suitable for the study of the drivers of structural transformation. In addition to their tractability, they allow for income elasticity parameters to be independent of the elasticity of substitution across goods. Since our framework does not impose functional relationships between income and substitution elasticities, it lends itself to the task of decomposing the contributions of the demand and supply channels to structural change. Our framework can accommodate an arbitrary number of goods with heterogeneous and independent income elasticities. As a result, they generate Engel curves for different goods that match the evidence discussed above: the logarithm of relative demand for the output of each sector has an approximately linear relationship with the logarithm of real income. More specifically, this relationship is characterized by a sector-specific income elasticity parameter. As part of our contributions in this paper, we also derive a strategy for structurally estimating these income elasticity parameters using both micro and macro data.

We characterize the equilibrium paths of our growth model in the long-run and derive the dynamics of the economy along the transition path. The equilibrium in our model asymptotically converges to a path of constant real consumption growth. The asymptotic growth rate of real consumption depends on parameters characterizing both the supply and demand channels; it is a function of the sectoral income elasticities as well as sectoral growth rates of TFP and sectoral factor intensities. In this respect, our model generalizes the results of [Ngai and Pissarides \(2007\)](#) and [Acemoglu and Guerrieri \(2008\)](#) to the case featuring nonhomothetic CES demand. Our theory can produce similar evolutions for nominal and real sectoral measures of economic activity, which is a robust feature of the data.⁴ This is a consequence of the role of income elasticities in generating sectoral reallocation patterns. Our framework can generate hump-shaped patterns for the evolution of manufacturing consumption shares, which is a well-documented feature in the data ([Buera and Kaboski, 2012a](#)).

In the empirical part of the paper, we first provide micro evidence in favor of the log-linear

⁴[Herrendorf et al., 2014](#) show that supply-side driven structural transformation cannot account for the similar evolution of nominal and real sectoral measures of activity.

relative demand functions implied by nonhomothetic CES preferences. We estimate the demand system using household-level data from the US and India, two countries at very different levels of economic development. In contrast to Figure 1 that studies changes in income, sectoral consumption expenditures, and prices over time, here we study the relationship between these variables in the cross-section of households. The data comes from the Consumption Expenditure survey (CEX) from the US and the National Sample Survey (NSS) from India. We group household expenditures into three broad categories of products: agriculture, manufacturing, and services. We show that the estimated income elasticity parameters are ranked such that the agriculture parameter is smaller than the manufacturing parameter, and the parameter for services is larger than that for manufacturing. We also show that the estimated income elasticity parameters are similar for households across different income brackets, time periods, and between the two countries. Furthermore, we present a micro-level counterpart to Figure 1: we show that, after controlling for prices and household characteristics, the residuals of (log) expenditure shares of consumption in agriculture and services relative to manufacturing appear to be close to a linear function of (log) real consumption.

We then empirically evaluate the implications of our growth model for structural transformation at the macro level. We use structural equations derived from our theory to estimate the elasticities that characterize our demand function using historical, cross-country sectoral data in a panel of 39 countries. The countries in our sample substantially vary in terms of their stages of development. We find that the estimated slopes of the Engel curves are stable across different measures of sectoral activity (employment and output) and country groupings (OECD and Non-OECD countries). This demonstrates that the patterns presented in Figure 1 not only characterize the Engel curves in the OECD but also apply more broadly to countries at other stages of development. We take this ability to parsimoniously account for structural change in a variety of contexts as evidence in favor of our model.

Armed with the estimated parameters of our model, we turn to the analysis of the drivers of structural change. We use our model to decompose structural change into income and price effects. We find that income effects are the main contributors to structural transformation. They account for over 80% of the sectoral reallocations of employment predicted by the estimated model. This finding contrasts with the previous studies (e.g., [Dennis and Iscan, 2009](#), [Boppart, 2014a](#)). A potential reason for this discrepancy is that in our framework income effects are not hard-wired to have only transitory effects on the structural transformation (as in Stone-Geary preferences) or to be correlated with price effects. Without these constraints on income effects, our estimates are consistent with a predominant role of income effects in accounting for the structural transformation during the postwar period in a large sample of countries at different stages of development.

We further investigate the predictive power of our model by running a horse-race with the two most prominent demand systems that are consistent with nonhomotheticity of prefer-

ences: the Stone-Geary (Kongsamut et al., 2001) and the price-independent generalized-linear (PIGL) preferences (Boppart, 2014a). We find that the nonhomothetic CES preferences provide a better account for the patterns of structural transformation across agriculture, manufacturing and services in our cross-country sample. Finally, we take advantage of the fact that nonhomothetic CES can accommodate an arbitrary number of goods. We extend our empirical analysis to a richer sectoral disaggregation and document substantial heterogeneity in income elasticity within manufacturing and services.

Our paper relates to a large literature that aims to quantify the role of nonhomotheticity of demand on growth and development (see, among others, Matsuyama (1992), Echevarria, 1997, Gollin et al., 2002, Duarte and Restuccia, 2010, Alvarez-Cuadrado and Poschke, 2011).⁵ Buera and Kaboski (2009) and Dennis and Iscan (2009) have noted the limits of the generalized Stone-Geary utility function to match long time series or cross-sections of countries with different income levels. More recently, Boppart (2014a) has studied the evolution of consumption of goods relative to services by introducing a sub-class of PIGL preferences that also yield non-vanishing income effects in the long-run. PIGL preferences also presuppose specific parametric correlations for the evolution of income and price elasticities over time (Gorman, 1965), and only accommodate two goods with distinct income elasticities. In contrast, our framework allows for an arbitrary number of goods.⁶ The differences between the two models are further reflected in their empirical implications. Whereas we find a larger contribution for demand nonhomotheticity in accounting for structural change, Boppart concludes that supply and demand make roughly similar contributions.⁷

The remainder of the paper is organized as follows. Section 2 introduces the properties of the nonhomothetic CES preferences and presents the model. Section 3 contains the estimation of the model using the household level and aggregate data. Section 4 uses the model estimates to investigate the relative importance of price and income effects for the patterns of structural

⁵An alternative formulation that can reconcile demand being asymptotically nonhomothetic with balanced growth path is given by hierarchical preferences (e.g., Foellmi and Zweimüller, 2006, 2008 and Foellmi et al., 2014). Swiecki (2017) estimates a demand system that features non-vanishing income effects in combination with subsistence levels à la Stone-Geary. However, this demand system also imposes a parametric relation between income and price effects. In subsequent work, Duernecker et al. (2017b) use a nested structure of nonhomothetic CES to study structural change within services. Sáenz (2017) extends our framework to time-varying capital intensities across production sectors and calibrates his model to South Korea. Matsuyama (2015, 2017) embeds nonhomothetic CES preferences in a monopolistic competition framework with international trade à la Krugman to study the patterns of structural change in a global economy and endogenizes the pattern of specialization of countries through the home market effect. Sposi (2016) incorporates nonhomothetic CES in a quantitative trade model of structural change.

⁶One can extend PIGL preferences to more than two goods by nesting other functions as composites within the two-good utility function (Boppart, 2014a), e.g., CES aggregators (this is how we proceed to estimate them in our empirical analysis). However, the resulting utility function does not allow for heterogeneity in income elasticity among the goods within each nested composite.

⁷In terms of the scope of the empirical exercise, while Boppart (2014a) estimates his model with U.S. data and considers two goods, the empirical evaluation of our model includes, in addition to the U.S., a wide range of other rich and developing countries and more than two goods. The variable elasticity implied by PIGL is also quantitatively important in accounting for the difference in the decomposition results (see Section 4).

transformation observed in our sample. Section 5 compares the fit of our model with those constructed based on the Stone-Geary and PIGL demand systems, and presents a number of further extensions of the empirical analysis and some robustness checks. Section 6 presents a calibration exercise where we investigate the transitional dynamics of the model, and Section 7 concludes. Appendix A presents some general properties of nonhomothetic CES. All proofs are in Appendix B.

2 Theory

In this section, we present a class of preferences that rationalize the empirical regularities on relative sectoral consumption expenditures discussed in the Introduction. These preferences are part of a class of demand systems that we call nonhomothetic CES preferences. We begin this section providing results that show how nonhomothetic CES can be used to analyze household consumption expenditure data. We then incorporate these preferences in a multi-sector growth model and show how we can use them to account for the patterns of structural transformation across countries. The growth model closely follows workhorse models of structural transformation (e.g., Buera and Kaboski, 2009; Herrendorf et al., 2013, 2014). The only difference with these is that we replace the standard aggregators of sectoral consumption goods with a nonhomothetic CES aggregator. This single departure from the standard workhorse model delivers the main theoretical results of the paper and the demand system later used in the estimation.

2.1 Nonhomothetic CES Preferences

Consider preferences over a bundle of goods $\mathbf{C} \equiv (C_1, C_2, \dots, C_I)$ such that C , an index of real income measuring consumer utility, is implicitly defined through the constraint

$$\sum_{i=1}^I (\Omega_i C^{\epsilon_i})^{\frac{1}{\sigma}} C_i^{\frac{\sigma-1}{\sigma}} = 1. \quad (1)$$

We impose the parametric restrictions that 1) $\sigma > 0$ and $\sigma \neq 1$, 2) $\Omega_i > 0$ for all $i \in \mathcal{I} \equiv \{1, \dots, I\}$, and 3) if $\sigma < 1$ (if $\sigma > 1$), then $\epsilon_i > 0$ ($\epsilon_i < 0$) for all $i \in \mathcal{I}$.⁸ Each sectoral good i is identified with a parameter ϵ_i that controls the income elasticity of demand for that good. Intuitively, as the index C rises, the weight given to the consumption of good i varies

⁸We can show that under these parameter restrictions the aggregator C introduced in equation (1) is globally monotonically increasing and quasi-concave, yielding a well-defined utility function over the bundle of goods \mathbf{C} , see Hanoch (1975). The additional restriction $\epsilon_i \geq 1 - \sigma$ for $\sigma < 1$ ($\epsilon_i \leq 1 - \sigma$ for $\sigma > 1$) ensures strict concavity, which simplifies the analysis of the dynamics in Section 2.2.1 below. In the case of $\sigma = 1$, the only globally well-defined CES preferences are homothetic and correspond to the Cobb-Douglas preferences (Blackorby and Russell, 1981).

at a rate controlled by parameter ϵ_i . As a result, the demand for sectoral good i features a constant elasticity in terms of the index of real consumption C .

Consider the expenditure minimization problem with prices $\mathbf{p} \equiv (p_1, p_2, \dots, p_I)$ and preferences defined as in equation (1). The nonhomothetic CES Hicksian demand function is given by

$$C_i = \Omega_i \left(\frac{p_i}{E} \right)^{-\sigma} C^{\epsilon_i}, \quad \forall i \in \mathcal{I}, \quad (2)$$

where we have defined E as the expenditure function

$$E(C; \mathbf{p}) \equiv \left[\sum_{i=1}^I \Omega_i C^{\epsilon_i} p_i^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad (3)$$

that gives the cost $E = \sum_{i=1}^I p_i C_i$ of achieving real consumption C . Alternatively, we can define an index for the average cost of real consumption $P(C; \mathbf{p}) \equiv E(C; \mathbf{p})/C$, rewriting (2) as

$$C_i = \Omega_i \left(\frac{p_i}{P} \right)^{-\sigma} C^{\epsilon_i + \sigma}, \quad \forall i \in \mathcal{I}. \quad (4)$$

We retrieve the standard CES preferences as the specific case of $\epsilon_i = 1 - \sigma$ for all $i \in \mathcal{I}$. In this case, the expenditure function becomes linear in the index of real consumption C , and the average cost of real consumption corresponds to the CES price index, $P = \left[\sum_{i=1}^I \Omega_i p_i^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$. In general, however, when ϵ_i 's vary across goods, the expenditure function varies nonlinearly in the index of real consumption, and the average and marginal costs of real consumption C deviate from one another.

Two unique features of nonhomothetic CES Hicksian demand function make these preferences a natural choice for capturing the patterns discussed in the Introduction:

1. The elasticity of the relative demand for two different goods with respect to aggregate consumption is constant, i.e.,

$$\frac{\partial \log(C_i/C_j)}{\partial \log C} = \epsilon_i - \epsilon_j, \quad \forall i, j \in \mathcal{I}. \quad (5)$$

2. The elasticity of substitution between goods of different sectors is uniquely defined and constant⁹

$$\frac{\partial \log(C_i/C_j)}{\partial \log(p_j/p_i)} = \sigma, \quad \forall i, j \in \mathcal{I}. \quad (6)$$

⁹Note that for preferences defined over I goods when $I > 2$, alternative definitions for elasticity of substitution do not necessarily coincide. In particular, equation (6) defines the so-called Morishima elasticity of substitution, which in general is not symmetric. This definition may be contrasted from the Allen (or Allen-Uzawa) elasticity of substitution defined as $\frac{E \cdot \partial C_i / \partial P_j}{C_i C_j}$, where E is the corresponding value of expenditure. Blackorby and Russell (1981) prove that the only preferences for which the Morishima elasticities of substitution between any two goods are symmetric, constant, and identical to Allen-Uzawa elasticities have the form of equation (1), albeit with a more general dependence of weights on C .

The first property ensures that the nonhomothetic features of these preferences do not systematically vary as income grows. As discussed in the Introduction, available data on sectoral consumption, both at the macro and micro levels, suggest stable and heterogeneous income elasticities across sectors. Therefore, we propose to specify preferences that do not result in systematically vanishing patterns of nonhomotheticity, as, for instance, would be implied by the choice of Stone-Geary preferences. The second property ensures that different goods have a constant elasticity of substitution and price elasticity regardless of the level of income.¹⁰ It is because of this property that we refer to these preferences as nonhomothetic CES.¹¹

The demand system implied by nonhomothetic CES for the *relative* consumption expenditures of goods transparently summarizes the two properties above. Let ω_i denote the expenditure share in sector i , that is, $\omega_i \equiv p_i C_i / E$. The Hicksian demand for any pair $i, j \in \mathcal{I}$ of goods satisfies

$$\log \left(\frac{\omega_i}{\omega_j} \right) = (1 - \sigma) \log \left(\frac{p_i}{p_j} \right) + (\epsilon_i - \epsilon_j) \log C + \log \left(\frac{\Omega_i}{\Omega_j} \right), \quad (7)$$

where C is the index of real consumption. Equation (15) highlights the key features of the nonhomothetic CES demand system: (i) the relative demand for different goods is log-linear in relative prices and real consumption, and (ii) there is a separation of the price and the income effects. The first term on the right hand side shows the price effects characterized by a constant elasticity of substitution σ . The second term on the right hand side shows the change in relative sectoral demand as consumers move across indifference curves. Equation (15) exhibits a log-linear form for relative demand of different goods in terms of prices and the nonhomothetic CES index of real consumption. This log-linear specification will be the basis for our empirical analysis in Section 3.

The expenditure elasticity of demand for sectoral good i is given by

$$\eta_i \equiv \frac{\partial \log C_i}{\partial \log E} = \sigma + (1 - \sigma) \frac{\epsilon_i}{\bar{\epsilon}}, \quad (8)$$

where E is the consumer's total consumption expenditure, and we have defined the expenditure-weighted average of income elasticity parameters, $\bar{\epsilon} \equiv \sum_{i=1}^I \omega_i \epsilon_i$ with ω_i denoting the expendi-

¹⁰Nonhomothetic CES preferences inherit this property because they belong to the class of implicitly additively separable preferences (Hanoch, 1975). In contrast, any preferences that are explicitly additively separable in sectoral goods imply parametric links between income and substitution elasticities. This result is known as Pigou's Law (Snow and Warren, 2015). For a discussion of specific examples, see Appendix A.

¹¹Alternatively, if we assume that consumer preferences satisfy two properties (5) and (6) for given parameter values $(\sigma, \epsilon_1, \dots, \epsilon_I)$, the preferences will correspond to the nonhomothetic CES preferences given by equation (1). More specifically, imposing condition (6) defines a *general* class of nonhomothetic CES preferences, defined in equation (A.1) in the appendix. Further imposing condition (5), together with the additional restriction that we should recover homothetic CES preferences in the case of $\epsilon_i \equiv 1 - \sigma$, yields the definition in equation (1). See Appendix A for more details.

ture share in sector i as defined above.¹² As Engel aggregation requires, the income elasticities average to 1 when sectoral weights are given by expenditure shares, $\sum_{i=1}^I \omega_i \eta_i = 1$. If good i has an income elasticity parameter ϵ_i that exceeds (is less than) the consumer's average elasticity parameter $\bar{\epsilon}$, then good i is a luxury (necessity) good, in the sense that it has an expenditure elasticity greater (smaller) than 1 at that point in time. This implies that being a luxury or necessity good is not an intrinsic characteristic of a good, but rather depends on the consumer's current composition of consumption expenditures and ultimately income.

Equation (8) implies that the predictions of the model for observables remain invariant to any scaling of all income elasticity parameters ϵ_i 's by a constant factor.¹³ Therefore, without loss of generality, we can normalize the values of ϵ_i 's such that the income elasticity parameter for a specific base good m equals an arbitrary given value ϵ_m . Furthermore, as with the case of homothetic CES, we can also normalize $\Omega_m = 1$ without loss of generality. With these two normalizations, we can use the expression for the demand of the base good in Equation (2) to write the real consumption index in terms of the price and expenditure of this good, as well as the total consumption expenditure:

$$\log C = \frac{1 - \sigma}{\epsilon_m} \left[\log \left(\frac{E}{p_m} \right) + \frac{1}{1 - \sigma} \log \omega_m \right]. \quad (9)$$

We can then substitute the expression for the real consumption index (9) for the consumption expenditure shares of goods $i \in \mathcal{I}_{-m} \equiv \mathcal{I} \setminus \{m\}$ relative to the base good m to find

$$\log(\omega_i) = (1 - \sigma) \log \left(\frac{p_i}{p_m} \right) + (1 - \sigma) \left(\frac{\epsilon_i}{\epsilon_m} - 1 \right) \log \left(\frac{E}{p_m} \right) + \frac{\epsilon_i}{\epsilon_m} \log(\omega_m) + \log \Omega_i. \quad (10)$$

For a given a base good m and a given value for the elasticity parameter ϵ_m , Equation (10) provides an expression for the consumption shares of all other goods in terms of observables.

2.2 Multi-sector Growth with Nonhomothetic CES

We now present a growth model where we integrate the nonhomothetic CES preferences in a general-equilibrium growth model to study the effect of the demand forces documented in the Introduction on shaping the long-run patterns of structural change. On the supply side, the model combines two distinct potential drivers of sectoral reallocation previously highlighted in the literature: heterogeneous rates of technological growth (Ngai and Pissarides, 2007) and

¹²The expenditure elasticity of relative demand is $\partial \log(C_i/C_j) / \partial \log E = (1 - \sigma)(\epsilon_i - \epsilon_j) / \bar{\epsilon}$. Note the difference with equation (5) that expresses the elasticity instead in terms of the nonhomothetic CES index of real consumption.

¹³To see why, note that the scaling of parameters ϵ_i by constant $\zeta > 0$ in Equation (1) simply maps the index C into a new index $\tilde{C} \equiv C^{1/\zeta}$. However, the index C is only defined as a monotonically increasing function of consumer utility and is not an observable. Therefore, the transformation implied by the scaling of the power by ζ maintains the validity of C as a real consumption index.

heterogeneous capital-intensity across sectors (Acemoglu and Guerrieri, 2008).

Households A unit mass of homogenous households has the following intertemporal preferences

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\theta} - 1}{1-\theta} \right), \quad (11)$$

where $\beta \in (0, 1)$ is the discount factor, θ is the parameter controlling the elasticity of intertemporal substitution¹⁴, and aggregate consumption, C_t , combines a bundle of I sectoral goods, \mathbf{C}_t , according to the nonhomothetic CES function defined by equation (1). Henceforth, we focus on the empirically relevant case $\sigma \in (0, 1)$, where broad categories of goods are gross complements. To complete the characterization of the household behavior, we assume that each household inelastically supplies one unit of perfectly divisible labor, and starts at period 0 with a homogeneous initial endowment \mathcal{A}_0 of assets.

Firms Firms in each consumption sector produce sectoral output under perfect competition. In addition, firms in a perfectly competitive investment sector produce investment good, Y_{0t} , that is used in the process of capital accumulation. We assume constant-returns-to-scale Cobb-Douglas production functions with time-varying Hicks-neutral sector-specific productivities,

$$Y_{it} = A_{it} K_{it}^{\alpha_i} L_{it}^{1-\alpha_i}, \quad i \in \{0\} \cup \mathcal{I},$$

where K_{it} and L_{it} are capital and labor used in the production of output Y_{it} in sector i at time t (we have identified the sector producing investment good as $i = 0$) and $\alpha_i \in (0, 1)$ denotes sector-specific capital intensity. The aggregate capital stock of the economy, K_t , accumulates using investment goods and depreciates at rate δ , $Y_{0t} = K_{t+1} - (1 - \delta) K_t$.

2.2.1 Competitive Equilibrium

We focus on the features of the competitive equilibrium of this economy that motivate our empirical specifications.¹⁵ Households take the sequence of wages, real interest rates, and prices of goods and services $\{w_t, r_t, \mathbf{p}_t\}_{t=0}^{\infty}$ as given, and choose a sequence of asset stocks $\{\mathcal{A}_t\}_{t=1}^{\infty}$ and aggregate consumption $\{C_t\}_{t=0}^{\infty}$ to maximize their utility defined in equations

¹⁴As we will explain in Section 2.2.3, the elasticity of intertemporal substitution is not constant in this model.

¹⁵Given an initial stock of capital K_0 and a sequence of sectoral productivities $\{(A_{it})_{i=0}^I\}_{t \geq 0}$, a competitive equilibrium is defined as a sequence of allocations $\{C_t, K_{t+1}, Y_{0t}, L_{0t}, K_{0t}, (Y_{it}, C_{it}, K_{it}, L_{it})_{i=0}^I\}_{t \geq 0}$ and a sequence of prices $\{w_t, R_t, (p_{it})_{i=0}^I\}_{t \geq 0}$ such that (i) agents maximize the present discounted value of their utility given their budget constraint, (ii) firms maximize profits and (iii) markets clear.

(1) and equations (11), subject to the per-period budget constraint

$$\mathcal{A}_{t+1} + \sum_{i=1}^I p_{it} C_{it} \leq w_t + (1 + r_t) \mathcal{A}_t, \quad (12)$$

where we have normalized the price of assets to 1. The next lemma characterizes the solution to the household problem.

Lemma 1. (*Household Behavior*) Consider a household with preferences as described by equations (11) and (1) with $\sigma \in (0, 1)$ and $\epsilon_i \geq 1 - \sigma$ for all $i \in \mathcal{I}$,¹⁶ budget constraint (12), and the No-Ponzi condition $\lim_{t \rightarrow \infty} \mathcal{A}_t \left(\prod_{t'=1}^{t-1} \frac{1}{1+r_{t'}} \right) = 0$. Given a sequence of prices $\{w_t, r_t, \mathbf{p}_t\}_{t=0}^{\infty}$ and an initial stock of assets \mathcal{A}_0 , the problem has a unique solution, fully characterized by the following conditions.

1. The intratemporal allocation of consumption goods satisfies $C_{it} = \Omega_i (p_{it}/E_t)^{-\sigma} C_t^{\epsilon_i}$ where consumption expenditure E_t at time t satisfies $E_t = \sum_{i=1}^I p_{it} C_{it} = E(C_t; \mathbf{p}_t)$ for the expenditure function defined by Equation (3).
2. The intertemporal allocation of real aggregate consumption satisfies the Euler equation

$$\left(\frac{C_{t+1}}{C_t} \right)^{1-\theta} = \frac{1}{\beta(1+r_t)} \frac{\bar{\epsilon}_{t+1}}{\bar{\epsilon}_t} \frac{E_{t+1}}{E_t}, \quad (13)$$

and the transversality condition

$$\lim_{t \rightarrow \infty} \beta^t (1+r_t) \frac{\mathcal{A}_t}{\bar{\epsilon}_t E_t / C_t^{1-\theta}} = 0, \quad (14)$$

The key insight from Lemma 1 is that the household problem can be decomposed into two sub-problems: one involving the allocation of consumption and savings over time, and one involving the allocation of consumption across sectors. The first part of the household problem involves the *intratemporal* problem of allocating consumption across different goods based on the sectoral demand implied by the nonhomothetic CES aggregator. Therefore the sequence of sectoral prices $\{\mathbf{p}_t\}_{t=1}^{\infty}$, consumption expenditure shares $\{\boldsymbol{\omega}_t\}_{t=1}^{\infty}$, and total consumption expenditures $\{E_t\}_{t=1}^{\infty}$ satisfy

$$\log \left(\frac{\omega_{it}}{\omega_{mt}} \right) = (1 - \sigma) \log \left(\frac{p_{it}}{p_{mt}} \right) + (\epsilon_i - \epsilon_m) \log C_t + \log \left(\frac{\Omega_i}{\Omega_m} \right), \quad i \in \mathcal{I}_{-m} \quad (15)$$

for a base sector m .

¹⁶Note that we can impose the parametric constraint $\epsilon_i \geq 1 - \sigma$ without loss of generality. As we discussed in Section 2.1, we have one degree of freedom in scaling all parameters ϵ_i 's by a constant factor without changing the underlying preferences of households. To satisfy the constraint, it is sufficient to choose the scaling of ϵ_i parameters large enough so that $\epsilon_{min} + \sigma \geq 1$.

The second part is the *intertemporal* consumption-savings problem. The household solves for the sequence of $\{\mathcal{A}_{t+1}, C_t\}_{t=0}^{\infty}$ that maximizes utility (11) subject to the constraint

$$\mathcal{A}_{t+1} + E(C_t; \mathbf{p}_t) \leq w_t + \mathcal{A}_t(1 + r_t), \quad (16)$$

where $E(C_t; \mathbf{p}_t)$ is the total expenditure function for the nonhomothetic CES preferences, defined in equation (3). Because of nonhomotheticity, consumption expenditure is a nonlinear function of real aggregate consumption, reflecting changes in the sectoral composition of consumption as income grows. The household incorporates this relationship in its Euler equation (13), where we see a wedge between the marginal cost of real consumption and the average cost $P_t = E_t/C_t$. The size of this wedge, given by $\bar{\epsilon}_t/(1 - \sigma)$, depends on the average income elasticities across sectors, $\bar{\epsilon}_t = \sum_{i=1}^I \omega_{it}\epsilon_i$, and varies over time. In the case of homothetic CES where $\epsilon_i \equiv 1 - \sigma$, this wedge disappears.

Firm profit maximization and equalization of the prices of labor and capital across sectors pin down prices of sectoral consumption goods,

$$p_{it} = \frac{p_{it}}{p_{0t}} = \frac{\alpha_0^{\alpha_0} (1 - \alpha_0)^{1 - \alpha_0}}{\alpha_i^{\alpha_i} (1 - \alpha_i)^{1 - \alpha_i}} \left(\frac{w_t}{R_t} \right)^{\alpha_0 - \alpha_i} \frac{A_{0t}}{A_{it}}, \quad (17)$$

where, since the units of investment good and capital are the same, we normalize the price of investment good, $p_{0t} \equiv 1$. Equation (17) shows that price effects capture both supply-side drivers of sectoral reallocation: heterogeneity in productivity growth rates and heterogeneity in capital intensities.

Goods market clearing ensures that household sectoral consumption expenditure equals the value of sectoral production output, $\omega_{it}E_t = P_{it}Y_{it}$.¹⁷ Competitive goods markets and profit maximization together imply that a constant share of sectoral output is spent on the wage bill,

$$L_{it} = (1 - \alpha_i)\omega_{it}\frac{E_t}{w_t}, \quad (18)$$

where ω_{it} is the share of sector i in household consumption expenditure.

The main prediction of the theory that we take to the data in the next section is equation (15). It provides a log-linear relationship between relative sectoral demand, relative sectoral prices, and the nonhomothetic CES index of real consumption. Our growth model in Section 2.2 shows that this relationship can generate strong demand-side forces that shape the structural transformations of a growing economy in the long-run. In particular, from the market-clearing equation (18) note that

$$\frac{L_{it}}{L_{jt}} = \frac{1 - \alpha_i}{1 - \alpha_j} \frac{\omega_{it}}{\omega_{jt}}, \quad i, j \in \mathcal{I}. \quad (19)$$

¹⁷In our empirical applications, we account for sectoral trade flows.

This implies that relative sectoral employment is proportional to relative expenditure shares. Thus, relative sectoral employment also follows the same log-linear relationship with relative prices and the index of real income. Equation (17) suggests that relative prices capture the effect of supply-side forces in the form of differential rates of productivity growth and heterogeneous capital intensities in the presence of capital deepening. Therefore, equation (15) also offers an intuitive way to separate out the impact of demand and supply-side forces in shaping long-run patterns of structural change.

Finally, we note that equation (15) also shows how our model can generate a positive correlation between relative sectoral consumption in real and nominal terms, as it is observed in the data (Herrendorf et al., 2014). The combination of the price effect and gross complementarity ($\sigma < 1$) implies that relative real sectoral consumption should negatively correlate with relative sectoral prices, as in the case of homothetic CES aggregators.¹⁸ However, our demand system has an additional force: income effects. The nonhomothetic effect of aggregate consumption affect both series in the same way and thus is a force to make both time series co-move. Thus, if income effects are sufficiently strong, both time series can be positively correlated. We revisit this result in Sections 3 and 6, where we show that this is indeed the case empirically.

2.2.2 Constant Growth Path

We characterize the asymptotic dynamics of the economy when sectoral total factor productivities grow at heterogeneous but constant rates. In particular, let us assume that sectoral productivity growth is given by

$$\frac{A_{it+1}}{A_{it}} = 1 + \gamma_i, \quad i \in \{0\} \cup \mathcal{I}. \quad (20)$$

Under this assumption, the competitive equilibrium of the economy converges to a path of constant per-capita consumption growth. Along this path, nominal consumption, investment, and the stock of capital all grow at a rate dictated by the rate of growth of the investment sector γ_0 . Denoting the rate of growth of real consumption by γ^* , the share of each sector i in consumption expenditure also exhibits constant growth along a constant growth path, characterized by constants

$$1 + \xi_i \equiv \lim_{t \rightarrow \infty} \frac{\omega_{it+1}}{\omega_{it}} = \frac{(1 + \gamma^*)^{\epsilon_i}}{\left[(1 + \gamma_0)^{\frac{\alpha_i}{1-\alpha_0}} (1 + \gamma_i) \right]^{1-\sigma}}. \quad (21)$$

¹⁸To see why, note that relative real consumption is decreasing in relative prices with an elasticity of $-\sigma$, while relative nominal expenditure is increasing with an elasticity of $1 - \sigma$. Thus, with CES aggregators and gross complementarity, real and nominal variables are negatively correlated—a counterfactual prediction.

Given the fact that expenditures shares have to be positive and sum to 1, equation (21) allows us to find the rate of growth of real consumption as a function of sectoral income elasticity, factor intensity, and the rates of technical growth. The next proposition presents these results that characterize the asymptotic dynamics of the competitive equilibrium.

Proposition 1. *Let γ^* be defined as*

$$\gamma^* = \min_{i \in \mathcal{I}} \left[(1 + \gamma_0)^{\frac{\alpha_i}{1-\alpha_0}} (1 + \gamma_i) \right]^{\frac{1-\sigma}{\epsilon_i}} - 1. \quad (22)$$

Assume that γ^ satisfies the following condition*

$$(1 + \gamma_0)^{-\frac{\alpha_0}{1-\alpha_0}} < \beta (1 + \gamma^*)^{1-\theta} < \min \left\{ \frac{(1 + \gamma_0)^{-\frac{\alpha_0}{1-\alpha_0}}}{\alpha_0 + (1 - \alpha_0) (1 + \gamma_0)^{-\frac{1}{1-\alpha_0}} (1 - \delta)}, 1 \right\}. \quad (23)$$

Then, for any initial level of capital stock, K_0 , there exists a unique competitive equilibrium along which consumption asymptotically grows at rate γ^ ,¹⁹*

$$\lim_{t \rightarrow \infty} \frac{C_{t+1}}{C_t} = 1 + \gamma^*. \quad (24)$$

Along this constant growth path, (i) the real interest rate is constant, $r^ \equiv (1 + \gamma_0)^{1/(1-\alpha_0)} / \beta (1 + \gamma^*)^{1-\theta} - 1$, (ii) nominal expenditure, total nominal output, and the stock of capital grow at rate $(1 + \gamma_0)^{\frac{1}{1-\alpha_0}}$, and (iii) only the subset of sectors \mathcal{I}^* that achieve the minimum in equation (22) employ a non-negligible fraction of workers.*

Equation (22) shows how the long-run growth rate of consumption is affected by income elasticities, ϵ_i , rates of technological progress, γ_i , and sectoral capital intensities, α_i . To build intuition, consider the case in which all sectors have the same capital intensity, and preferences are homothetic. Then, since $\sigma \in (0, 1)$, equation (22) implies that the long-run growth rate of real consumption is pinned down by the sectors with the lowest technological progress, as in Ngai and Pissarides (2007). Consider now the case in which there is also heterogeneity in income elasticities. In this case, sectors with higher income elasticity and faster technological progress can co-exist in the long-run with sectors with low income elasticity and slow technological progress. The intuition is that the agents shift their consumption expenditure toward income-elastic good, as they become richer, and away from goods that are becoming cheaper due to technical progress. Finally, the role of heterogeneity in capital shares in shaping the long-run rate of consumption growth is analogous to the role of technological progress, as they both ultimately shape the evolution of prices.

¹⁹Here we follow the terminology of Acemoglu and Guerrieri (2008) in referring to our equilibrium path as a constant growth path. Kongsamut et al. (2001) refer to this concept as generalized balanced growth path. As these papers, we normalize the investment sector price. See Duernecker et al. (2017a) for a discussion on the connection between this price normalization and chained-price indexing of real consumption.

Which sectors survive in the long-run? At all points in time, all sectors produce a positive amount of goods, and its production grows over time. In relative terms, however, only the subset of sectors \mathcal{I}^* satisfying equation (22) will comprise a non-negligible share of total consumption expenditure in the long-run. Indeed, if the initial number of sectors is finite, generically only one sector survives in the long-run.

2.2.3 Transitional Dynamics

To study the transitional dynamics of the economy, we focus on the special case where all sectors have a common capital intensity $\alpha \equiv \alpha_i$ for all i .²⁰ Let us normalize each of the aggregate variables by their respective rates of growth, introducing normalized consumption expenditure $\tilde{E}_t \equiv (1 + \gamma_0)^{-\frac{t}{1-\alpha}} E_t$, per-capita stock of capital $\tilde{k}_t \equiv (1 + \gamma_0)^{-\frac{t}{1-\alpha}} K_t$, and real per-capita consumption $\tilde{C}_t \equiv (1 + \gamma^*)^{-t} C_t$. Using the assets market clearing condition, we can translate equations (13) and (16) into equations that characterize the evolution of the normalized aggregate variables

$$\tilde{k}_{t+1} = (1 + \gamma_0)^{-\frac{1}{1-\alpha}} \left[\tilde{k}_t^\alpha + \tilde{k}_t (1 - \delta) - \tilde{E}_t \right], \quad (25)$$

$$\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{\theta-1} \frac{\bar{\epsilon}_{t+1}}{\bar{\epsilon}_t} \frac{\tilde{E}_{t+1}}{\tilde{E}_t} = \frac{1 + \alpha \tilde{k}_t^{\alpha-1} - \delta}{1 + r^*}, \quad (26)$$

where the normalized consumption expenditure \tilde{E}_t is a function of \tilde{C}_t and the two functions of the growth in \tilde{C}_t , that is, $\tilde{C}_{t+1}/\tilde{C}_t$, as

$$\left(\frac{\tilde{E}_{t+1}}{\tilde{E}_t} \right)^{1-\sigma} = \sum_{i=1}^I \omega_{it} \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{\epsilon_i} (1 + \xi_i)^t, \quad (27)$$

$$\frac{\bar{\epsilon}_{t+1}}{\bar{\epsilon}_t} = \left(\frac{\tilde{E}_t}{\tilde{E}_{t+1}} \right)^{1-\sigma} \sum_{i=1}^I \omega_{it} \left(\frac{\epsilon_i}{\bar{\epsilon}_t} \right) \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{\epsilon_i} (1 + \xi_i)^t. \quad (28)$$

Starting from any initial levels of normalized per-capita consumption \tilde{C}_0 and stock of capital \tilde{k}_0 , we can find that period's allocation of expenditure shares ω_t using equations (2) and (3), and compute the normalized per-capita consumption and stock of capital of the next period using equations (25) and (26). The proposition establishes that the equilibrium path exists, is unique, and is therefore fully characterized by the dynamic equations above.

At the aggregate level, the transitional dynamics of this economy deviates from that of the standard neoclassical growth model because the household's elasticity of intertemporal substitution (EIS) varies with income. Goods with lower income elasticity are less intertem-

²⁰The online appendix characterizes the dynamics along an equilibrium path in the more general case with heterogeneous capital intensities α_i in a continuous-time rendition of the current model.

porally substitutable. Since the relative shares of high and low income-elastic goods in the consumption expenditure of households vary over time, the effective elasticity of intertemporal substitution of households correspondingly adjusts. Typically, as income rises, low income-elastic goods constitute a smaller share of the households' expenditure and therefore the effective elasticity of intertemporal substitution rises over time. When the economy begins with a normalized stock of capital \tilde{k}_t below its long-run level \tilde{k}^* , the interest rate along the transitional path exceeds its long-run level. With a rising elasticity of intertemporal substitution, households respond increasingly more strongly to these high interest rates. Therefore, the accumulation of capital and the fall in the interest rate both accelerate over time.²¹

In general, the transitional dynamics of the economy can generate a rich set of different patterns of structural transformation depending on relative income elasticity parameters and the rates of productivity growth of different sectors $\{\epsilon_i, \gamma_i\}_{i=1}^I$. In Section 3 we will estimate the demand-side parameters of the model using both micro and macro level data. We will then use these parameters to calibrate the model in Section 6 and study the implications for the evolution of sectoral shares as well as the paths of interest rate and savings.

3 Model Estimation

In this section, we bring our model to the data with two goals in mind. Our first goal is to show that nonhomothetic CES captures salient features of the data. In particular, we want to demonstrate that (i) the estimated income elasticity parameters differ systematically across sectors (specifically, $\epsilon_a < \epsilon_m < \epsilon_s$), (ii) that the relative sectoral demand is well-captured by a log-linear function of relative prices and aggregate consumption and (iii) that the income elasticities are stable across countries and income levels. Second, we aim to estimate the structural parameters of the model (i.e., income elasticities, ϵ_i 's, and the elasticity of substitution, σ) to calibrate our model and study its transitional dynamics in Section 6.

We use household-level and aggregate data to estimate sectoral demands (see Section 3.1 for a description) under alternative econometric specifications discussed in Section 3.2.²² We first analyze data on final-good household expenditure from two countries at different

²¹The mechanism operates with all preferences that feature nonhomotheticity (e.g., King and Rebelo (1993) discuss it in the context of a neoclassical growth model with Stone-Geary preferences). In the current model, if the rate of productivity growth in high income-elastic sectors is large enough, the share of these sectors may in fact fall over time, and the effective elasticity of intertemporal substitution of households may correspondingly fall. However, as we will see in the calibration of the model in Section 6, the empirically relevant case is one in which the share of more income-elastic goods rises as the economy grows.

²²The growth model developed in Section 2.2 abstracts from within-country dispersion of income and assumes all households are identical. In Section B of the online appendix, we derive approximate expressions for aggregate sectoral demand in an environment featuring within-country heterogeneity in income. In particular, we show that the equations characterizing household-level and aggregate-level allocation of expenditure are identical up to the first order of approximation in the standard deviation of the logarithm of consumption expenditure, if the latter has a symmetric distribution such as the log-normal distribution (see Battistin et al., 2009, for evidence for the log-normality of the distribution of total consumption expenditure across households.).

stages of development: the U.S. and India (see Section 3.3).²³ We then estimate the model using macro data in Section 3.4 using a panel of 39 countries over the post-war period. The sample of countries covers a wide range of growth experiences, including developing countries (e.g., Botswana and India), miracle economies (e.g., South Korea and Taiwan) and developed economies (e.g., the U.S. and Japan).

3.1 Data Description

We use the U.S. Consumer Expenditure Survey (CEX) and the India National Sample Survey (NSS) for studying relative sectoral demand at the household level. These datasets report the composition of household consumption expenditures on different final goods. For the aggregate data, we combine Groningen’s 10-Sector Database with aggregate consumption measures from the ninth version of the Penn World Table (PWT). The aggregate data contains sectoral employment, value-added output, sectoral prices and consumption per capita. We briefly discuss each of these datasets below in this section. We present more details on the data sources in Section D of the online appendix.

3.1.1 U.S. Household Expenditure Data

We use U.S. household quarterly consumption data for the period 1999-2010 from the Consumption Expenditure Survey (CEX). Each household is interviewed about their expenditures for up to four consecutive quarters. Our data construction is based on [Aguiar and Bils \(2015\)](#), who in turn follow very closely [Heathcote et al. \(2010\)](#) and [Krueger and Perri \(2006\)](#). As these authors, we focus on a sample of urban households with a present household head aged between 25 and 64. We also use the same total income measure (net of taxes) and household controls as [Aguiar and Bils \(2015\)](#). These controls are demographic dummies based on age range of the household head (25-37, 38-50, 51-64), household size dummies (≤ 2 , 3-4, 5+) and dummies for the number of household earners (1, 2+).

The key difference from [Aguiar and Bils \(2015\)](#) is that we construct our consumption categories to match expenditure in agriculture, manufacturing and services. We follow [Herrendorf et al. \(2013\)](#) to construct these three categories. The agricultural sector is composed by food-at-home expenditures. The main expenditure categories for the manufacturing sector are vehicles, housing equipment, other durables, clothing, shoes and personal care items. For services, these are housing, utilities, health, food away from home, television subscriptions and other entertainment fees.

We combine the CEX data with disaggregated regional quarterly price series from the BLS’s urban CPI (CPI-U). Similar to [Hobijn and Lagakos \(2005\)](#) and [Hobijn et al. \(2009\)](#), we

²³In Appendix E, we report the estimates from U.S. household data when utility is specified over value added (rather than final good expenditure). For the purpose of comparison, we also provide the estimates using aggregate US value-added data.

construct the price for each sector faced by a household by taking the expenditure-weighted average of the price of each of the expenditure categories belonging to the sector. Since expenditure weights are household-specific, this allows us to, albeit imperfectly, account for the fact that the effective price for each sector may be different across households.

3.1.2 Indian Household Expenditure Data

We use data from rounds 64, 66 and 68 of the India National Sample Survey (NSS), which span the years 2007 to 2012. The NSS is a representative survey of household expenditure that collects repeated cross-sections of expenditures incurred by households in goods and services. We construct total expenditure in agriculture, manufactures and services following the same classification as for the CEX data. Household total income is constructed from an earnings measure that averages (potential) different sources of income within the household from different occupations, including received benefits (net of taxes).

We construct the controls in an analogous way to the U.S., with the only difference that the requirement of a prime age household is between ages of 18 and 60. In contrast to the U.S., we do not discard rural population as it represents more than half of the Indian population (around 55%). We instead show results for the entire sample and the subsample of urban households. The composition of expenditure in India is vastly different from the CEX. The average expenditure share in agricultural products in the sample is 52% (versus 12% in the CEX). Expenditure shares in manufactures and services in the NSS represent, on average, 27% and 21% of total expenditure (versus 27% and 61% in the CEX).

Finally, one limitation we face when estimating our demand system for India is the difficulty to find disaggregated price indices. Unlike the U.S., we have not found price series at the commodity-regional level. This makes it impossible to construct regional price indices for the sectoral categories to estimate the price elasticity. We discuss in Section 3.2 below how we address this shortcoming in our empirical analysis.

3.1.3 Macro Data

Our aggregate data comes from two sources. The sectoral data comes from Groningen's 10-Sector Database (Vries et al., 2014). The 10-Sector Database provides a long-run internationally comparable dataset on sectoral measures for 10 countries in Asia, 9 in Europe, 9 in Latin America, 10 in Africa and the United States. The variables covered in the data set are annual series of production value added (nominal and real) and employment for 10 broad sectors starting in 1947. In our baseline exercise, we aggregate the ten sectors into agriculture, manufacturing and services following Herrendorf et al. (2013). In Section 5, we estimate our model for 10 sectors. Our consumption per capita data (nominal and real) comes from the ninth version of the Penn World Tables, (Feenstra et al., 2015). Combining these

two datasets gives us a final panel of 39 countries with an average number of observations of 42 years per country. As we have discussed, the countries in our sample span very different growth trajectories. For example, the ratio of the 90th to the 10th percentile of consumption per capita in year 2000 is 18.2.²⁴

3.2 Econometric Specifications

We employ the structure implied by nonhomothetic CES preferences to discipline our empirical analysis of the micro and macro level data. We take a number of different approaches that all leverage the log-linear form of the relative sectoral demand functions found by solving the intratemporal consumption allocation problem in equation (15).

Let n denote the unit of observation, which can be either a household, in applying our framework to micro data, or aggregate outcomes of a country, in a macro setting. We consider the following empirical counterpart to equation (15) that characterizes the intratemporal allocation of consumption across different goods $i \in \mathcal{I}_{-m}$

$$\log \left(\frac{\omega_{it}^n}{\omega_{mt}^n} \right) = (1 - \sigma) \log \left(\frac{p_{it}^n}{p_{mt}^n} \right) + (\epsilon_i - \epsilon_m) \log C_t^n + \zeta_i^n + \nu_{it}^n, \quad (29)$$

where ω_{it}^n and p_{it}^n stand for the share of consumption expenditure and the price of sector- i goods in unit n at time t , C_t^n stands for the index of real consumption for the unit at time t , and ζ_i^n stands for the taste parameter of the good, which potentially varies across units, and it may also include additional controls (as discussed below). The key challenge in using this specification lies in the mapping between observed data and the index of real consumption C_t implied by nonhomothetic CES preferences. Below, we discuss two different strategies for dealing with this problem.

Baseline (Reduced-form) Specification The first strategy relies on a reduced-form approach that follows a long tradition of log-linear demand specifications (e.g., [Deaton and Muellbauer, 1980b](#)), and uses the observed consumption expenditure, deflated by a standard price index such as the Laspeyres, Törnqvist, or Stone index, as a proxy for the nonhomothetic CES index of real consumption.²⁵ Accordingly, we estimate the following system of linear

²⁴The countries in our sample are Argentina, Bolivia, Botswana, Brazil, Chile, Colombia, Costa Rica, Denmark Ethiopia, France, Germany, Ghana, Hong Kong, India, Indonesia, Italy, Japan, Kenya, Korea, Malawi, Malaysia, Mauritius, Mexico, Netherlands, Nigeria, Perú, Philippines, Senegal, Singapore, South Africa, Spain, Sweden, Taiwan, Tanzania, Thailand, United Kingdom, United States of America, Venezuela and Zambia. In the previous version of this paper ([Comin et al., 2015](#)), we used the Barro-Ursua dataset instead of the PWTv9.0. The main advantage of the PWTv9.0 is that it allows us to estimate our model for all countries in the 10-sector database (39) relative to Barro-Ursua, for which we only had data for 25 countries. We find very similar results using both data sets.

²⁵A similar issue also arises in the estimation of other (nonhomothetic) demand systems, most notably the Almost Ideal Demand System ([Deaton and Muellbauer, 1980a](#)). Deaton and Muellbauer also follow the strategy of using a readily constructed price index and use the Stone price index. More recently, [Aguilar and](#)

equations for $i \in \mathcal{I}_{-m}$

$$\log \left(\frac{\omega_{it}^n}{\omega_{mt}^n} \right) = (1 - \sigma) \log \left(\frac{p_{it}^n}{p_{mt}^n} \right) + (\epsilon_i - \epsilon_m) \log \left(\frac{E_t^n}{\mathcal{P}_t^n} \right) + \zeta_i^n + \nu_{it}^n, \quad (30)$$

where E_t^n and \mathcal{P}_t^n denote total household consumption expenditure and a corresponding household-specific price index, respectively, for unit n at time t . In Appendix C, we provide conditions under which this reduced form specification can in fact uncover the structural parameters of the demand system, in particular the income elasticity parameters $\epsilon_i - \epsilon_m$'s, up to an unknown scaling factor ϵ_m . Therein, we provide further evidence that these conditions appear to approximately hold in our data. These results make a connection between our demand specification with the prior empirical work on demand estimation: they show why this simple and intuitive strategy can offer insights about the relative income elasticity of different goods if demand follows nonhomothetic CES preferences.

Structural Specification The second strategy estimates the price and income elasticity parameters based on the full set of structural constraints imposed by the model. In this case, we rely on the expression (9) and estimate the following system of equations

$$\log \left(\frac{\omega_{it}^n}{\omega_{mt}^n} \right) = (1 - \sigma) \log \left(\frac{p_{it}^n}{p_{mt}^n} \right) + (1 - \sigma) \left(\frac{\epsilon_i}{\epsilon_m} - 1 \right) \log \left(\frac{E_t^n}{p_{mt}^n} \right) + \left(\frac{\epsilon_i}{\epsilon_m} - 1 \right) \log (\omega_{mt}^n) + \zeta_i^n + \nu_{it}^n, \quad (31)$$

where we use the price and expenditure shares of manufacturing goods m as covariates. This estimation identifies σ and ϵ_i/ϵ_m for all $i \in \mathcal{I}_{-m}$, which, as we discussed in Section 2.1, together fully characterize the underlying preferences. Note that unlike the reduced-form system of equations (30), the system (31) imposes *nonlinear* constraints on the coefficients and therefore cannot be estimated using a linear estimation strategy.²⁶

Bils (2015) have argued that a log-linear specification such as the one employed here have crucial advantages in dealing with the types of measurement errors commonly encountered in consumption data.

²⁶We have derived two alternative strategies for a structural estimation of the income elasticity parameters above. The first approach applies to the cases with unit fixed-effects, ζ_i^n . It relies on the following result connecting the changes in the nonhomothetic CES index of real income to the changes in standard price indices. In the appendix, we show that up to second-order approximation, we have

$$\Delta \log C_i \approx \frac{1 - \sigma}{\bar{\epsilon}_t} (\Delta \log E_t - \Delta \log \mathcal{P}_t), \quad (32)$$

where \mathcal{P}_t is the chained Törnqvist price index $\Delta \log \mathcal{P}_t \equiv \frac{1}{2} \sum_i (\omega_{it} + \omega_{it+1}) \Delta \log p_{it}$ and $\bar{\epsilon}_t$ is a correspondingly chained average of income elasticity parameters ϵ_i 's, $\bar{\epsilon}_t \equiv \frac{1}{2} \sum_i (\omega_{it} + \omega_{it+1}) \epsilon_i$. Using this result, and following the approach developed by Blundell and Robin (1999), we develop an iterative linear estimation scheme for identifying ϵ_i/ϵ_m . We have applied this scheme to the macro data where the fixed-effects specification is feasible, and have obtained results very similar to those presented here. The second scheme directly estimates equation (29) using the following expression for the average cost of consumption as a function of

Specification Without Price Data The specification in Equation (29) requires data on both household expenditure shares and relative prices. In the case of our Indian household consumption data, we do not have access to price data at the commodity-region level. In this case, we rely on an alternative approach inspired by the analysis of CEX data in [Aguiar and Bils \(2015\)](#). Specifically, we begin with equation (30) and introduce a full set of interactions among region r , time t , sector i , and household income quintile q as fixed-effects in the estimation to capture the effect of relative prices and the price indices \mathcal{P}_t^n . Formally, we estimate the following system of equations for $i \in \mathcal{I}_{-m}$

$$\log\left(\frac{\omega_{it}^n}{\omega_{mt}^n}\right) = (\epsilon_i - \epsilon_m) \log E_t^h + \pi_{it}^{rq} + \zeta_i^n + \nu_{it}^n, \quad (34)$$

where π_{it}^{rq} denotes fixed-effects. We also present the estimation results based on this specification in the U.S. household expenditure data, and show that it produces estimates very similar to the specification (30) that uses observed prices.

3.3 Household-Level Results

In this section, we present our estimation results using micro-level data on household consumption. Our goal is to show that nonhomothetic-CES preferences, despite their parametric parsimony, provide a good account of the relation between the composition of the consumption expenditure of households as a function of relative prices and total household expenditure.

Empirical Strategy and Identification We use the specifications introduced in Section 3.2, where in this case n denotes different households observed in the data, under the assumptions $\zeta_i^n \equiv \beta_i' X_i^n + \delta_{ir}$ and $\nu_{it}^n \equiv \delta_{it} + \tilde{\nu}_{it}^n$. The first assumption imposes the constraint that the cross-household heterogeneity in time-invariant taste parameters can be fully explained as a linear function of the vector X_i^n of household characteristics discussed above (age, household size, number of earners dummies) and region fixed effects. The second assumption allows for a dyad of time t fixed effects to absorb potential consumption shocks. This specification identifies income elasticities based on how the within-region variation in expenditure shares varies with total household expenditure, controlling for the household characteristics above.

To deal with potential measurement error and endogeneity issues, we use instruments for total expenditure, expenditure shares, and the model parameters as

$$P_t = E_t/C_t = \left[\sum_i (\Omega_i p_{it}^{1-\sigma})^{\chi_i} (\omega_{it} E_t^{1-\sigma})^{1-\chi_i} \right]^{\frac{1}{1-\sigma}}, \quad (33)$$

where we have defined $\chi_i \equiv (1-\sigma)/\epsilon_i$ to simplify the expression. In practice, the estimation relies on iterating on a series of linear regressions as in [Blundell and Robin \(1999\)](#). This approach also yields similar estimates. The details of these estimations and the results are available upon request.

the observed measures of household expenditure and relative prices. First, we follow [Aguiar and Bils \(2015\)](#) and instrument the total expenditure of households in a given quarter with the annual household income after taxes and the income quintile of the household. The instruments capture the permanent household income and are therefore correlated with household expenditure without being affected by transitory measurement error in total expenditures.²⁷ Second, we instrument household relative prices with a “Hausman” relative-price instrument. Each of the prices used in the relative-price instrument is constructed in two steps. First, for each sub-component of a sector, we compute the average price across regions excluding the own region. Then, the sectoral price for a region is constructed using the average region expenditure shares in each sub-component as weights.²⁸ These price instruments capture the common trend in U.S. prices while alleviating endogeneity concerns due to regional shocks (and measurement error of expenditure).²⁹

We perform our estimation of the system of equations instrumenting for relative prices and household consumption with the full set of instruments. We present our estimation results using two alternative weighting schemes. We use the household weights provided in the CEX data to map the household sample to be representative of the entire population. Additionally, we re-weight households by their total level of expenditure to bridge the gap with the estimates with aggregate-level data.³⁰ Comparing the alternative weighting schemes allows us to examine the stability of the estimated parameters across income groups.

3.3.1 Evidence from the U.S.

Baseline estimates Our baseline results are reported in [Table 1](#). They correspond to estimating household relative final-good expenditure shares as described in [equation \(30\)](#) with household quarterly expenditures deflated using Törnqvist price indices. Columns (1) and (2) correspond to the estimates from three-stage least squares when we control only for household characteristics X^h but we do not include any time or region fixed effects.³¹ Column

²⁷The measure of total household income corresponds to a separate question in the CEX and is not constructed adding household expenditures over the year. [Boppart \(2014a\)](#) also instruments quarterly expenditure levels by household income. In the NSS data, we use an analogous measure of annual household income as an instrument.

²⁸Formally, we instrument relative prices $\log(p_{it}^r/p_{mt}^r)$ with $\log(p_{it}^{-r}/p_{mt}^{-r})$, where $\log p_{it}^{-r}$ for $i \in \{a, m, s\}$ constructed as follows. Suppose that for sector j we have information on the price of subcomponents $k \in \{1, \dots, K\}$, then $\log p_{it}^{-r} = \sum_{k=1}^K \hat{\omega}_{kt}^r \log \hat{p}_{kt}^{-r}$ where $\hat{\omega}_{kt}^r$ denotes the average expenditure share of k in region r and \hat{p}_{kt}^{-r} denotes the log of the average price in the U.S. excluding region r . We have verified that constructing the instrument using the price in the own region or the average national price delivers similar results.

²⁹Using the average price in the U.S. excluding the own region addresses the concern of regional shocks, while capturing the common component of prices across regions. Using average expenditures in the region addresses the concern of mismeasurement of household expenditure shares in that region to the extent that the mismeasurement averages out in the aggregate.

³⁰We thank the editor for this suggestion.

³¹The first stage regressions include all household controls and fixed effects (if included in the second stage) in addition to the instruments. The first stage for consumption typically yields significant coefficients for total

Table 1: Baseline Regression, CEX Final Good Expenditure

	(1)	(2)	(3)	(4)	(5)	(6)
σ	0.53 (0.04)	0.54 (0.04)	0.49 (0.04)	0.51 (0.04)	0.42 (0.29)	0.38 (0.28)
$\epsilon_a - \epsilon_m$	-0.46 (0.02)	-0.47 (0.02)	-0.46 (0.02)	-0.46 (0.02)	-0.49 (0.02)	-0.49 (0.02)
$\epsilon_s - \epsilon_m$	0.43 (0.02)	0.43 (0.02)	0.43 (0.02)	0.42 (0.02)	0.43 (0.02)	0.43 (0.02)
Expenditure Re-Weighted	N	Y	N	Y	N	Y
Region FE	N	N	Y	Y	Y	Y
Year \times Quarter FE	N	N	N	N	Y	Y

Households controls (described in the main text) are included in all regressions. Standard errors clustered at the household level in (1) to (4), robust with small-sample adjustment for (5) and (6). The number of observations is 60,925 in all regressions.

(1) corresponds to the weighting scheme that replicates the U.S. population, while column (2) corresponds to the expenditure re-weighted estimates. In both cases we find very similar estimates. The estimates show that the income elasticity parameter is lower for agriculture relative to manufacturing ($\epsilon_a - \epsilon_m = -0.46$ in the first column) and higher for services relative to manufacturing ($\epsilon_s - \epsilon_m = 0.43$ in the first column). The price elasticity estimates are less than one ($\sigma = 0.53$ in the first column) suggesting that agriculture, manufacturing and services are gross complements in household preferences.

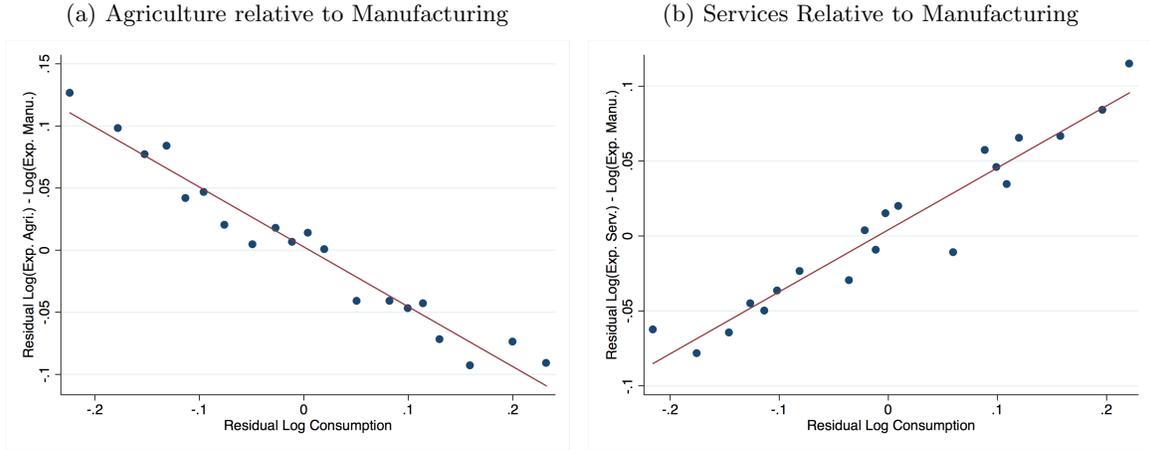
We subsequently add region and time fixed effects in columns (3) to (6). We find very similar coefficients to those in columns (1) and (2).³² A significant observation from Table 1 is that our estimates of relative income elasticities do not change significantly between the specifications with U.S. population weights (odd columns) and those with expenditure weights (even columns). This finding suggests that one of the critical assumptions in our model, the constancy of income elasticities parameters $\{\epsilon_i\}_{i \in \mathcal{I}}$ across income groups, provides a good description of the data.

We further explore how well the assumption of log-linear relative sectoral demand is supported by the data in Figure 2. The figure plots the (binned) residual variation used to identify the relative income elasticity parameters $\epsilon_i - \epsilon_m$ in specification (1), after all controls

annual income and income quintiles at 5% and insignificant coefficients for price instruments. Conversely, the first stage for price instruments yields significant first stage for the price instrument of interest and annual income, with more mixed results on the income quintiles depending on the specification. We have verified that using a more structural approach where we use only income instruments for consumption and price instruments for prices yields similar results. Table E.1 in the online appendix contains the first stage for column (3) as an example.

³²When including both region and time fixed effects, the variation left in prices to identify the price elasticity is substantially reduced and the household-level clustered standard errors become large (almost .3 in both weighting schemes). The point estimates of price elasticity in this case decline from around 0.5 to around 0.4.

Figure 2: Residual Variation Identifying Relative Income Elasticities in the CEX



These plots depict the (binned) residuals corresponding to column (1) in Table 1. Each point corresponds to the average value of 20 equal-sized bins of the data. The red line depicts the baseline regression fit reported in column (1).

have been partialled-out from our instrumented consumption measure and relative expenditure shares. As implied by our model, we find that residual variation in relative shares is well approximated by a log-linear function of residual consumption, both for agriculture relative to manufacturing (2a) and services relative to manufacturing (2b). We can think of this figure as the micro-level counterpart to the macro-level patterns depicted in Figure 1: both in the aggregate and household data, relative demand curves for sectoral goods show a stable log-linear relationship in income.

Table 2 further explores the stability of the slope of the relative demand in income across different subsamples of the data. We split households in two groups: above and below the annual median income in the sample. Columns (1) and (2) report the estimates of baseline specification (29) when we estimate it separately for each subsample. We find that the estimated elasticities are not significantly different from each other. We also study the stability of the estimates over time and estimate our baseline regression in the pre- and post-2005 sub-samples. Columns (3) and (4) report the estimates, where we again find estimates that are close in magnitude.

Columns (5) in Table 2 reports the estimates of the income elasticities in the case where we omit prices in our baseline specification (29) and include instead $\text{time} \times \text{region}$ fixed effects. Column (6) augments the previous regression by introducing $\text{time} \times \text{region} \times \text{household-income-quintile}$ fixed effects. This corresponds to the econometric specification without prices discussed in Section 3.2, where we approximate the household-specific price indices by household-income-quintile dummies interacted with $\text{time} \times \text{region}$ fixed effects. In both specifications, we also obtain similar estimates to our baseline results. This result is reassuring for our anal-

Table 2: Split-Sample and No-price Specifications, CEX Final Good Expenditure

	< P50	> P50	Pre '05	Post '05	$\delta_{t,r}$	$\delta_{t,r,q}$
	(1)	(2)	(3)	(4)	(5)	(6)
σ	0.57 (0.07)	0.56 (0.07)	0.64 (0.08)	0.77 (0.06)	—	—
$\epsilon_a - \epsilon_m$	-0.45 (0.07)	-0.37 (0.06)	-0.50 (0.03)	-0.46 (0.03)	-0.48 (0.02)	-0.61 (0.09)
$\epsilon_s - \epsilon_m$	0.41 (0.07)	0.49 (0.06)	0.40 (0.03)	0.48 (0.03)	0.44 (0.02)	0.49 (0.09)
Year \times Quarter \times Region	N	N	N	N	Y	Y
Yr. \times Qtr. \times Reg. \times Inc. Quintile	N	N	N	N	N	Y

Regressions estimated using CEX-replicate weights. Households controls included in all regressions (described in the main text). Standard errors clustered at the household level.

ysis of the Indian household-level data in Section 3.3.2 below, where we exclusively rely on specifications without price data.

Structural Estimates Next, we present the structural estimates from (31), which identify the relative income elasticity ϵ_i/ϵ_m for $i \in \{a, s\}$ and the price elasticity parameter σ . As we discussed in Section 3.2, under a set of assumptions laid out in the appendix, our baseline estimates $\epsilon_i - \epsilon_m$ are proportional to $\epsilon_i/\epsilon_m - 1$ up to an unknown scaling factor. Here, we also report the particular scaling factor (or, equivalently, the normalization of ϵ_m) that yields the closest mapping between our point estimates in the structural and baseline specifications.

Table 3 reports the same set of regressions as our baseline Table 1, but now based on the structural estimation implied by equation (31). The second and third rows correspond to the estimates of the difference in income elasticity parameters $\epsilon_i - \epsilon_m$ normalized by ϵ_m (note that this number is invariant to any re-scaling of $\{\epsilon_i\}_{i \in \mathcal{I}}$). As before, we find income elasticity parameters for agriculture and services that are higher and lower, respectively, relative to manufacturing. We also find that the estimates are stable across our two weighting schemes. The fourth and fifth rows correspond to the normalization of ϵ_m (shown in the sixth row) that yields the closest point estimates to our baseline estimates. We see that under this normalization the estimates of the income elasticities are very similar to the baseline regression. As in our baseline exercise, the estimates across the two different weighting schemes are similar in all specifications. Finally, our estimates of the price elasticity oscillate around 0.3 across the specifications. They appear to be somewhat smaller than in the baseline specification (which ranged between 0.38 and 0.54), but all of the estimates remain less than 1 and imply gross complementarity across goods.

Table 3: Structural Estimates, CEX Final Good Expenditure

	(1)	(2)	(3)	(4)	(5)	(6)
σ	0.26 (0.04)	0.28 (0.04)	0.28 (0.03)	0.20 (0.05)	0.31 (0.04)	0.33 (0.05)
$\epsilon_a/\epsilon_m - 1$	-0.80 (0.06)	-0.83 (0.07)	-0.81 (0.06)	-0.70 (0.07)	-0.95 (0.09)	-0.97 (0.10)
$\epsilon_s/\epsilon_m - 1$	0.65 (0.07)	0.68 (0.07)	0.75 (0.06)	0.67 (0.07)	0.82 (0.09)	0.85 (0.10)
Baseline Normalization						
$\epsilon_a - \epsilon_m$	-0.49 (0.03)	-0.49 (0.03)	-0.46 (0.02)	-0.45 (0.03)	-0.49 (0.03)	-0.49 (0.04)
$\epsilon_s - \epsilon_m$	0.40 (0.03)	0.40 (0.03)	0.43 (0.03)	0.43 (0.04)	0.43 (0.04)	0.43 (0.04)
ϵ_m	0.61 (0.02)	0.59 (0.02)	0.57 (0.02)	0.65 (0.03)	0.52 (0.02)	0.51 (0.02)
Expenditure Re-Weighted	N	Y	N	Y	N	Y
Region FE	N	N	Y	Y	Y	Y
Year \times Quarter FE	N	N	N	N	Y	Y

All regressions include household controls (described in the text). Standard errors shown in parenthesis. Standard errors are clustered at the household level for the first three lines. Standard errors in the last three lines are derived using the delta method to account for the fact that the optimal scaling of the parameters is computed using the estimated values. The number of observations is 60,925 in all regressions.

3.3.2 Evidence from India

Table 4 reports our estimation results from estimating equation (34).³³ Columns (1) and (2) report our baseline estimates using the full sample for the two weighting schemes. We find that the relative income elasticities between agriculture and manufacturing, $\epsilon_a - \epsilon_m$, and between services and manufacturing, $\epsilon_s - \epsilon_m$, are of the same sign and similar magnitude to those estimated in the CEX. Likewise, comparing the point estimates in column (1) and (2) we see that they again remain stable across the two weighting schemes. We further explore the stability of the parameter estimates by applying the same specification separately to the subset of households above and below the median income level. Columns (3) and (4) show that we find very similar estimates of income elasticities in the two sub-samples. Finally, we show in columns (5) and (6) that when we restrict our attention to urban households, we also

³³We use total household annual income as instrument for household quarterly expenditure. The first stage includes all controls used in the second stage. The coefficient on household annual income is positive and significant in all first-stage regressions. We note also that for columns (1) to (4) we augment specification (34) interacting the income-quintile \times time \times region with a dummy that indicates whether the household is classified as rural to account for potential constant difference between rural and urban households.

Table 4: Baseline Regression for India, NSS Expenditure

			< P50	> P50	Only Urban	
	(1)	(2)	(3)	(4)	(5)	(6)
$\epsilon_a - \epsilon_m$	-0.63 (0.05)	-0.55 (0.07)	-0.62 (0.05)	-0.69 (0.29)	-0.57 (0.05)	-0.52 (0.07)
$\epsilon_s - \epsilon_m$	0.49 (0.07)	0.42 (0.10)	0.69 (0.53)	0.51 (0.08)	0.56 (0.08)	0.45 (0.11)
Expenditure Re-Weighted	N	Y	N	N	N	Y
Time \times Region \times Inc. Quintile FE	Y	Y	Y	Y	Y	Y
Time \times Reg. \times Inc. Quint. \times Rural FE	Y	Y	Y	Y	N	N

Standard errors clustered at the year \times state \times district shown in parenthesis. All regressions include household controls (discussed in the main text). Observations for the full sample are 293,007. Urban observations are 118,681. Time fixed effects are the interaction of year \times month. Region fixed effects are the interactions of state \times district.

obtain very similar estimates regardless of the weighting scheme used.

We conclude from the estimation results on household expenditure in India and the U.S. that the elasticity parameters $\{\epsilon_i\}_{i \in \mathcal{I}}$ differ systematically across sectors: agriculture and services show robustly higher income elasticity parameters compared to manufacturing, when estimated across the two countries, time periods, and different income levels. Furthermore, all parameter estimates of the price elasticity of substitution between these broad sectors is below unity.

3.4 Cross-Country Aggregate-Level Results

After estimating the model with household data, we explore the ability of nonhomothetic CES preferences to account for the broad patterns of structural transformations observed across countries during the post-war period.

Empirical Strategy and Identification To estimate our model using aggregate data, we employ a strategy very similar to the one we used for micro data. Since our model assumes that each country is inhabited by homogeneous households, the specifications introduced in Section 3.2 apply, where n now stands for different countries observed in our data (see footnote 22 for further discussion). In our baseline exercise, we estimate our model from the patterns of structural change in employment. In particular, equation (19) implies that relative sectoral consumption expenditures are proportional to relative sectoral employment shares,

yielding³⁴

$$\log\left(\frac{L_{it}^n}{L_{mt}^n}\right) = (1 - \sigma) \log\left(\frac{p_{it}^n}{p_{mt}^n}\right) + (\epsilon_i - \epsilon_m) \log\left(\frac{E_t^n}{\mathcal{P}_t^n}\right) + \zeta_i^n + \nu_{it}^n, \quad i \in \mathcal{I}_-, \quad (35)$$

where $\log(E_t^n/\mathcal{P}_t^n)$ stands for the aggregate real consumption, as reported by the Penn World Table. An analogous argument leads us from equation (31) to the expression for the structural estimation of the model based on relative employment shares

$$\log\left(\frac{L_{it}^n}{L_{mt}^n}\right) = (1 - \sigma) \log\left(\frac{p_{it}^n}{p_{mt}^n}\right) + (1 - \sigma) \left(\frac{\epsilon_i}{\epsilon_m} - 1\right) \log\left(\frac{E_t^n}{p_{mt}^n}\right) + \left(\frac{\epsilon_i}{\epsilon_m} - 1\right) \log(\omega_{mt}^n) + \zeta_i^n + \nu_{it}^n, \quad (36)$$

where ω_{mt}^n denotes the share of manufacturing value added in country n at time t .

Using employment rather than value added shares, as in equations (35) and (36), is our favored specification for investigating the cross-country data because it does not use the price data (an explanatory variable) to construct the dependent variables. To account for the fact that some goods can be traded (thus affecting the sectoral composition of employment), we also control for log-sectoral exports and imports in equation (35).³⁵

In both cross-country estimation equations, (35) and (36), ζ_i^n denotes a country-sector fixed effect. Thus, our estimation relies on the within-country variation of employment shares, income, and relative prices to identify the price and income elasticities. The identification assumption to obtain consistent estimates is that, for each country, the shocks to relative prices and income are uncorrelated with the relative demand shocks ν_{it}^n . This assumption would be violated if, for example, sectoral taste shocks (which are part of ν_{it}^n) are correlated with aggregate demand or relative price shocks. To alleviate these endogeneity concerns, we estimate our model separately for OECD and Non-OECD countries and show that the estimates do not change significantly across sub-samples. While the estimates could in principle be biased in both cases, this would require sectoral taste shocks (or any other omitted variable) to be correlated with aggregate demand or relative price shocks in the same way across sub-samples, which we deem less likely.

³⁴Note that the sector-country fixed effects ζ_i^n also absorb country-specific heterogeneity in sectoral capital intensity α_i^n 's.

³⁵We use the trade detail data from the PWT to construct sectoral exports and imports. Agricultural trade flows correspond to trade in food and beverages. Manufacturing trade flows correspond to trade in industrial supplies, fuels and lubricants, capital goods, transport equipment and consumer goods. In the results presented here, we use a reduced-form strategy and directly control for log-sectoral exports and imports. Alternatively, we can rely on a model with exogenous trade flows to derive the estimation equations that control for trade flows and are consistent with the model. In this case, we need to assume that factor intensities are identical in the production function of the same sector across different countries approach. We can then use the accounting identity $p_{it}^n C_{it}^n = p_{it}^n Y_{it}^n - NX_{it}^n$, where NX_{it}^n denotes the nominal value of net exports in sector i , time t and country n . It follows that the expressions for sectoral employment in sector i should be adjusted by terms involving the observed values of $NX_{it}^n/p_{it}^n Y_{it}^n$. Using this alternative model-driven controls for trade flows, we have found results very similar to what is presented here.

Table 5: Baseline Estimates for the Cross-Country Sample

	World		OECD		Non-OECD	
	(1)	(2)	(3)	(4)	(5)	(6)
σ	0.89 (0.10)	0.87 (0.09)	0.75 (0.15)	0.76 (0.14)	0.94 (0.13)	0.91 (0.12)
$\epsilon_a - \epsilon_m$	-1.07 (0.07)	-1.06 (0.07)	-1.02 (0.17)	-0.91 (0.19)	-1.09 (0.08)	-1.08 (0.08)
$\epsilon_s - \epsilon_m$	0.28 (0.09)	0.29 (0.09)	0.39 (0.17)	0.48 (0.13)	0.19 (0.10)	0.21 (0.10)
Country \times Sector FE	Y	Y	Y	Y	Y	Y
Trade Controls	N	Y	N	Y	N	Y
R^2	0.78	0.78	0.78	0.80	0.76	0.76
Observations	1626	1626	492	492	1134	1134

Standard errors clustered by country shown in parenthesis.

Estimation Results Table 5 reports our baseline specification obtained from estimating (35) for the full sample of 39 countries and separately for OECD and Non-OECD sub-samples. Columns (1) and (2) report the estimates for our entire sample with and without trade controls, respectively. Once again, in our estimates the income elasticity parameter is lower for agriculture relative to manufacturing ($\epsilon_a - \epsilon_m = -1.07$ in the first column) and larger for services compared to manufacturing ($\epsilon_s - \epsilon_m = 0.28$ in the same column). The price elasticity is also less than unity ($\sigma = 0.89$). Introducing trade controls hardly changes our estimates, as shown in column (2).³⁶

Columns (3) and (4) report the estimated elasticities for OECD countries and columns (5) and (6) report the estimates for the Non-OECD sample. The estimates are similar for the two sub-samples. In fact, we cannot reject the null that the estimates for the elasticity parameters are the same for both sub-samples at conventional levels (with the only exception being the relative elasticity of services to manufacturing in the case including trade controls). As with the micro estimates, we find that our estimates of income elasticity parameters are stable across countries of different levels of income. Furthermore, the similarity of the estimates across sub-samples is reassuring, as we deem less likely that unobserved relative demand shocks maybe correlated with relative prices and income in the same way across two such different groupings of countries.

³⁶The previous version of the paper (Comin et al., 2015) used the Barro-Ursua measures of real consumption. In that case, we only had data for 25 countries. Almost all of the differences from our current sample come from the fact that we now have more Non-OECD countries. In the previous case we found estimates similar in magnitude: $\epsilon_a - \epsilon_m$ ranged between -0.81 and -1.16 , $\epsilon_s - \epsilon_m$, between 0.23 and 0.51 , and σ between 0.66 to 0.76 . With the Barro-Ursua dataset we can reject the null hypothesis of σ being equal to one in most specifications. Also, we can reject the null hypothesis that $\log(E_t)$ and $\log(E_t/P_t)$ have unit roots in our sample. Thus, the variables in our regression can not be cointegrated.

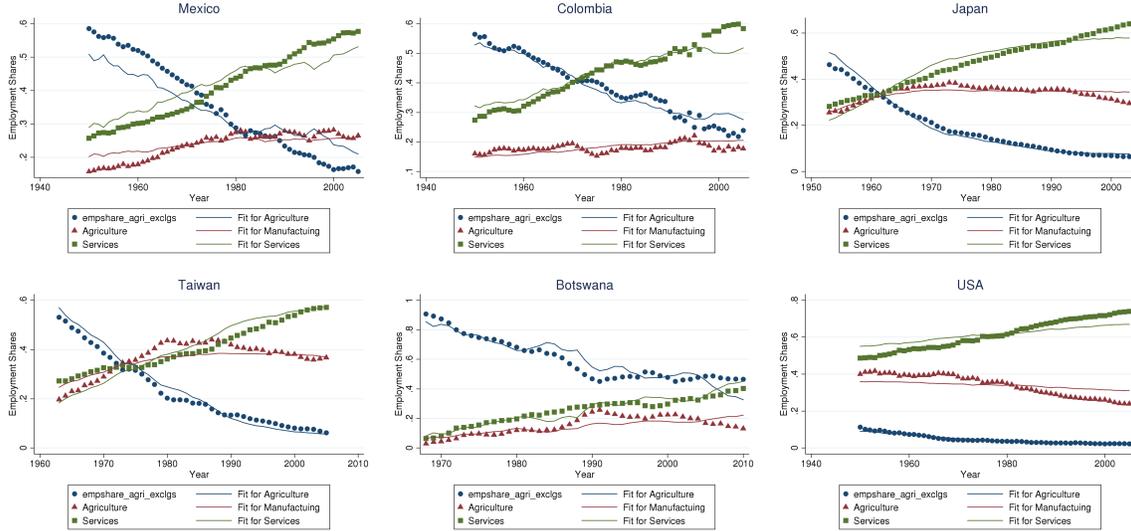
Table 6: Structural Estimates for the Cross-Country Sample

	World		OECD		Non-OECD	
	(1)	(2)	(3)	(4)	(5)	(6)
σ	0.57 (0.09)	0.50 (0.11)	0.20 (0.08)	0.27 (0.07)	0.63 (0.10)	0.48 (0.11)
$\epsilon_a/\epsilon_m - 1$	-0.98 (0.16)	-0.89 (0.17)	-0.93 (0.16)	-0.99 (0.05)	-0.91 (0.11)	-0.80 (0.16)
$\epsilon_s/\epsilon_m - 1$	0.17 (0.13)	0.21 (0.16)	0.48 (0.14)	0.67 (0.23)	0.18 (0.24)	0.37 (0.15)
Baseline Normalization						
$\epsilon_a - \epsilon_m$	-1.09 (0.08)	-1.07 (0.08)	-0.97 (0.06)	-0.85 (0.06)	-1.08 (0.09)	-0.97 (0.08)
$\epsilon_s - \epsilon_m$	0.18 (0.04)	0.25 (0.06)	0.50 (0.11)	0.57 (0.12)	0.22 (0.07)	0.45 (0.10)
ϵ_m	1.12 (0.09)	1.21 (0.10)	1.04 (0.08)	0.85 (0.07)	1.19 (0.08)	1.22 (0.10)
Country \times Sector FE	Y	Y	Y	Y	Y	Y
Trade Controls	N	Y	N	Y	N	Y
Observations	1626	1626	492	492	1134	1134

Notes: Standard errors shown in parenthesis. Standard errors are clustered at the country level in the first three rows (computed through bootstrapping of 50 samples with replacement). Standard errors in the last three rows are derived using the delta method to account for the fact that the optimal scaling of the parameters is computed using the estimated values. The estimation in columns (3) and (4) is performed imposing the constraint that $\epsilon_a/\epsilon_s \geq 0$ (by estimating an exponential transformation of the variable). The reported standard errors are also adjusted using the delta method.

Table 6 presents the structural estimates obtained from specification (36) for the same setting as that in the baseline table. Columns (1) and (2) report the world estimates. We find that the normalized relative income elasticities (reported in the second and third rows) are of the expected sign, $\epsilon_a/\epsilon_m < 1$ and $\epsilon_s/\epsilon_m > 1$. The fourth and fifth rows report the relative income measures under the normalization of ϵ_m that minimizes the difference with the baseline estimates. We find that the estimates of the income elasticity are very similar in magnitude. In fact, we cannot reject the null that they are the same. We find a smaller price elasticity (0.57 and 0.50) relative to the baseline exercise. Columns (3) to (6) report the estimation for the OECD sub-samples with and without trade controls, for which we have very similar findings for the income elasticities. The point estimates of σ vary more than in the baseline estimation, between 0.20 and 0.63, but always remain less than unity.

Figure 3: Example of Fit with common preference parameters $\{\sigma, \epsilon_a - \epsilon_m, \epsilon_s - \epsilon_m\}$.



4 Accounting for Structural Change

After estimating the model, we turn to our main question which is studying the drivers of structural change. To this end, we first assess the overall model fit and then assess the contribution of each of the two drivers, income and price effects, to the observed reallocations of sectoral employment shares across countries.

Figure 3 plots the actual and the predicted employment shares from column (2) of Table 5 for six countries, Mexico, Colombia, Japan, Taiwan, Botswana and the United States.³⁷ This figure confirms the good fit of the model despite its parsimony. Recall that we impose a common set of elasticities across all countries, $\{\sigma, \epsilon_a - \epsilon_m, \epsilon_s - \epsilon_m\}$, and only allow for constant taste differences across countries ($\{\log \Omega_i^n\}$, captured as country-sector fixed effects). The fitted model captures the employment share trends in all three sectors reasonably well, despite the fact that these countries are at different stages of development.³⁸ For Japan and Taiwan, we note that our model generates hump-shapes for the share of employment in manufacturing that are similar to those observed the data.³⁹

³⁷Figures F.3, F.4 and F.5 in the online Appendix show the plot for all countries in our sample.

³⁸For the USA, however, we see that the evolution of the employment shares in services and manufacturing are steeper than predicted by our model. This is the case also for other OECD countries. This reflects the fact that the estimated income elasticity of services is greater for this set of countries, as column (5) in Table 5 shows. If we plot the predicted fit using the estimates $\{\sigma, \epsilon_s - \epsilon_m, \epsilon_a - \epsilon_m\}$ for only OECD countries this problem goes entirely away, as shown in the working paper version Comin et al. (2015).

³⁹For example, for Taiwan, the predicted initial level of the employment share in manufacturing is 21%, it goes up to 39% and back to 35% at the end of the period. The observed levels are 20%, 43% and 37%. For Japan, the employment share in manufacturing is 26% in the initial period, it goes up to 38% in the mid 1970s and it is 30% by the end of our sample. The fitted time series starts at 26%, goes up to 35% and declines to 33% by the end of the period.

To gain a better intuition on the role played by relative prices, consumption, and net exports, we study in Figure 6 the case of Japan in detail. Panel (a) shows the overall fit of the data using the estimated parameters from the entire sample. Panels (b), (c) and (d) show the time series of relative prices, consumption and sectoral net exports. Then, we report the partial fit generated by each time series (and the country-sector fixed effects) at the estimated parameter values. Panel (e) shows the partial fit generated by the relative price time series. We see that, at the estimated parameter values, relative prices account for little of the variation. In contrast, the evolution of aggregate consumption accounts for most of the structural transformation (see panel f). In particular, income effects drive the observed hump-shape in manufacturing. Intuitively, as the Japanese economy became richer in the 1950s, it reallocated labor away from agriculture to both manufacturing and services. Subsequent income growth led to the expansion of services which absorbed employment from manufacturing. Finally, panel (g) shows that changes in sectoral net exports did not play a significant role in accounting for the structural transformation in Japan.

Contribution of Relative Price and Income Effects After studying the case of Japan, next we explore the drivers of structural transformation in a more systematic manner. We want to quantify how much of the variation in employment shares that countries experience can be accounted for by changes in relative prices and aggregate consumption. We proceed by constructing the predicted values of our estimating equation when we exclude prices or real consumption. We then compare how much of the overall within-country predicted variation in employment shares can be accounted for by the predicted values excluding prices or real consumption. For this exercise we use the world estimates in column (2) of Table 6 of our structural model (31).⁴⁰ We use as a measure of fit for any given subset of regressors the within- R^2 , which is the R^2 we obtain from regressing the log-employment shares, net of the estimated country-sector fixed effects ζ_i^c 's, on the predicted relative log-employment shares when we only include those regressors.

Table 7 reports the results of this exercise. The first row shows the within- R^2 when only relative prices are used to construct the predicted values. We find that for the log-relative employment share of agriculture to manufacturing the within- R^2 is 0.06, which represents 6.1% of the 0.98 total within- R^2 that we obtain when we include all regressors. The same exercise when only consumption is included in the regression yields 0.94, implying that 96% of the within-country variation can be accounted for by consumption. Note that since prices and consumption measures are not orthogonal, the overall within- R^2 is less than the sum of the partial within- R^2 's. Thus, the within- R^2 is an upper bound on how much each regressor

⁴⁰Thus, the measure of real consumption is the one implied by our model, Equation (9). Table F.2 reports the same exercise when we use our baseline estimates instead of the structural estimates. We find very similar results.

Table 7: Drivers of Structural Change, Within- R^2 Decomposition

	$\ln \left(\frac{\text{Agriculture}}{\text{Manufacturing}} \right)$			$\ln \left(\frac{\text{Services}}{\text{Manufacturing}} \right)$		
	World	OECD	Non-OECD	World	OECD	Non-OECD
Prices Only	0.06	0.11	0.03	0.05	0.02	0.06
Consumption Only	0.94	0.98	0.92	0.80	0.91	0.75
Prices and Consumption	0.94	0.98	0.92	0.82	0.91	0.78
All Regressors	0.98	0.99	0.98	0.87	0.92	0.85

can account for. In fact, the third row shows that if we include prices and consumption to create our predicted values, the within- R^2 is still 0.94. The remaining gap up to the overall within- R^2 of 0.98 is accounted for the sectoral trade controls.

If we break down the predicted values between OECD and Non-OECD countries (using the same estimated coefficients), a similar picture emerges. The within- R^2 with only prices is higher for OECD countries, 0.11, than for Non-OECD countries, 0.03. Still, the bulk of the variation is accounted for by real consumption. Finally, we find that the within- R^2 when we include prices and consumption accounts for almost all the within variation for OECD countries, while for Non-OECD countries there is a 0.06 gap that is accounted by sectoral trade. This suggests that international trade plays a more prominent role for structural transformation in developing countries.

The within- R^2 of log-relative services to manufacturing for the World sample shows a similar overall pattern. We find that when we construct predicted values with only prices and consumption the within- R^2 's are 0.05 and 0.80, respectively, accounting for 5.8% and 92% of the overall within- R^2 . In the OECD sample, prices play a somewhat smaller role than in Non-OECD countries, but consumption still has an overwhelmingly large explanatory power. We also find that trade plays a larger gap in Non-OECD countries, since the gap between the predicted values when we include all regressors vis-a-vis when we include prices and consumption is larger for this group of countries, 0.07, than for OECD countries, 0.01.

Overall, Table 7 paints a picture consistent with the view that the nonhomotheticity of demand plays a dominant role in accounting for structural change. If we attribute all the covariation in prices and consumption to prices, we find that the within- R^2 accounted for by real consumption is 89.8% ($= (0.94 - 0.06) / 0.98$) in the log-relative agriculture to manufacturing regression, and 88.5% ($= (0.82 - 0.05) / 0.87$) in the log-relative service to manufacturing regression. Even if we take the most favorable grouping of countries for prices in each regression (and keep attributing all the covariance to prices), we find that real consumption accounts for 87.8% of OECD's within- R^2 variation in agriculture relative to manufacturing and 84.7% of Non-OECD's within- R^2 variation in services relative to agriculture. Thus, we conclude that

Table 8: Correlation of Nominal and Real Value Added

	Data	Model
Agriculture/Manufacturing	0.96	0.94
Services/Manufacturing	0.87	0.71

Note: Model-predicted values constructed using the structural estimates from column (2) in Table 6.

nonhomotheticities account for over 80% of the structural change in our sample.^{41,42}

Structural Change in Real and Nominal Value Added A salient feature of the patterns of structural transformation observed in the data on sectoral value added is that they appear regardless of whether we document them in nominal or real terms (Herrendorf et al., 2014). To investigate our model’s ability to account for this fact, we combine our structural cross-country estimates from Table 6 and the sectoral demands, (15), to generate the predicted evolution of nominal and real sectoral demands.

Table 8 reports the correlation between nominal and real shares both in our estimated model and in the data. We find that the model is able to generate correlation similar to the data. In particular, the correlation between the nominal and real relative demand of agricultural goods to manufactures is 0.94 in our model, while in the data it is 0.96. For services, the model generates a correlation of 0.71 while in the observed correlation in the data is 0.87.⁴³

The success in generating a correlation between nominal and real measures of the same magnitude as in the data is important. Note that it is an out-of-sample test of the predictions

⁴¹In the working paper version of the paper (Comin et al., 2015), we also show that the likelihood-ratio tests of including price and consumption data to a model with only country-sector fixed effects are significant. The increase in the likelihood is much higher when we add consumption data than when we add relative price data.

⁴²This conclusion differs from Boppart (2014a) who studies the evolution of services relative to the rest of the economy in the U.S. during the postwar period. He finds that the contribution of price and income effects are roughly of equal sizes. First, the differences in the results are partly due to the differences in the level of sectoral aggregation. If we confine our analysis to the U.S. and lump together agriculture and manufacturing into one sector, we find that price effects account for 26% of the variation. Second, our specification of demand is different from Boppart (2014a) because in our specification the price elasticity is constant. In contrast, Boppart’s demand system implies that the price elasticity of services relative to the rest of consumption is declining as the economy grows. As noted by Buera and Kaboski (2009), since the relative expenditure and value added of services grows at a faster rate than services relative price, a declining price elasticity automatically increases the explanatory power of relative prices. We have checked that a declining variable elasticity is quantitatively important for the decomposition exercise. We have generated a fake panel of countries with two sectors (agriculture plus manufacturing, and services) with preferences given by nonhomothetic CES calibrated to capture the key features of our true cross-country panel. We then do two decomposition exercises with these data: one estimating a nonhomothetic CES demand, and another estimating a PIGL demand. We find that the within variation accounted for by prices is four times larger with PIGL than with nonhomothetic CES.

⁴³Table F.3 in the online appendix reports the same exercise when we use the reduced-form estimates. We find very similar correlations of the predicted values, 0.92 for agriculture to manufacturing and 0.65 for services to manufacturing.

of our model, since our estimation has not targeted the evolution of sectoral shares of real or nominal value added. As discussed in Section 2, if we had used a homothetic CES framework, the correlation generated by the model would have been negative because the price elasticity of substitution is smaller than one. This implies that real and nominal variables can not have positive co-movement with homothetic CES.⁴⁴ Of course, any specification of preferences that asymptotically converges to a homothetic CES (e.g., Stone-Geary) would face a similar problem in explaining the nominal-real co-movement. In our framework, there is a second force that makes the positive co-movement possible: income effects. The nonhomothetic effect of real consumption affects in an identical way real and nominal variables. At the estimated parameter values, the implied income effects are sufficiently strong to overcome the relative price effect and make both time series co-move positively. Therefore, we argue that the ability to simultaneously account for the evolution of real and nominal sectoral shares is in fact a key feature of our specification of nonhomotheticity.

5 Additional Empirical Results and Robustness Checks

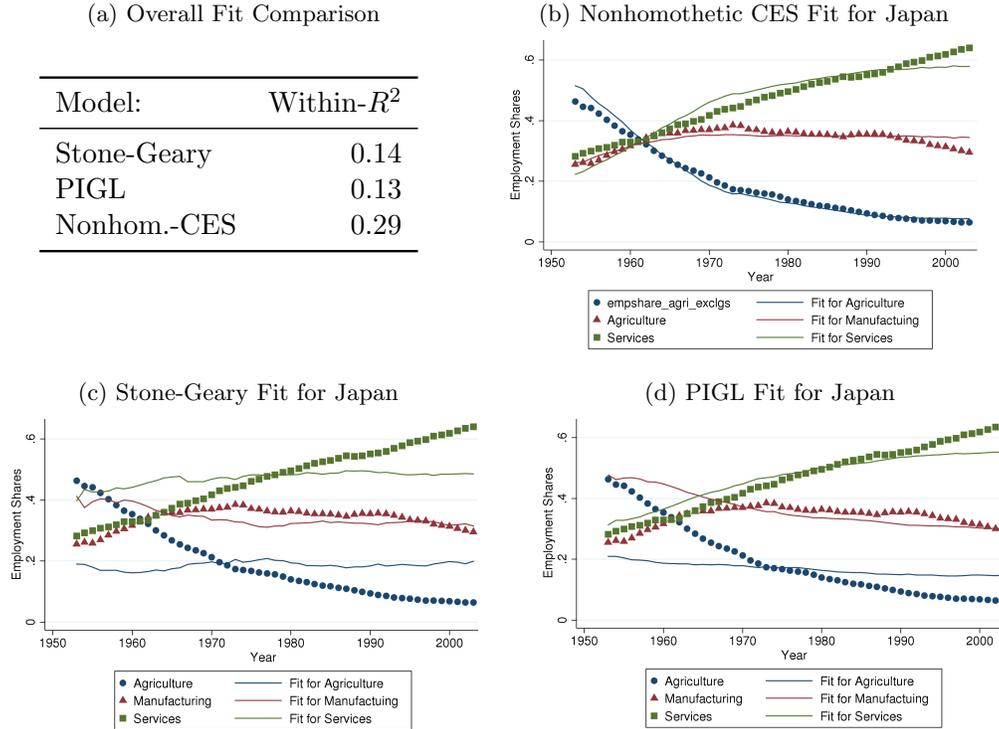
In this section, we first present a fit comparison of nonhomothetic CES to Stone-Geary and PIGL demand systems. We then briefly discuss several extensions and robustness checks of our empirical results for both micro and macro data.

Fit Comparison with Stone-Geary and PIGL preferences We compare the cross-country fit of our model to alternative specifications where we replace the nonhomothetic CES aggregator with Stone-Geary (Herrendorf et al., 2014) and PIGL preferences (augmented to three sectors as described in Boppart (2014b)). Appendix D introduces these two demand systems and the estimation procedure. Here, we highlight two similarities between these demand specifications and ours that allow us to perform this comparison. First, the number of parameters to be estimated in these two demand systems is the same as that in nonhomothetic CES. Second, as with nonhomothetic CES, there exist sets of parameters for these two demand systems such that the expenditure shares are constants for each country (they correspond to Cobb-Douglas with expenditure shares equal to country-sector averages). Thus, we benchmark the fit of these three demand systems relative to using the country-sector average as a prediction for each sector. This amounts to computing the R^2 for agriculture, manufacturing and services shares after subtracting country-sector means in each sector.⁴⁵

⁴⁴To see that, note that the relative trend in nominal values $\omega_{i,t}/\omega_{j,t}$ would be proportional to $(p_{i,t}/p_{j,t})^{1-\sigma}$. For real values, $c_{i,t}/c_{j,t}$, would be proportional to $(p_{i,t}/p_{j,t})^{-\sigma}$. As $0 < \sigma < 1$, both trends would move in opposite directions.

⁴⁵The R^2 compares the sum of squared errors of the model fit to the sum of squared errors obtained by using the country-sector average as a prediction. Formally, $R^2 = 1 - \frac{1}{I} \sum_{i=1}^I \left(\sum_{t=1}^N (y_{it}^c - \hat{y}_{it}^c)^2 / \sum_{t=1}^N (y_{it}^c - \bar{y}_i^c)^2 \right)$ where N denotes the total number of observations per sector, I , the number of sectors, y_{it}^c , observed employment

Figure 4: Comparison of Demand Systems and Fit for Japan



We find that the within- R^2 for Stone-Geary is 0.14, meaning that 14% of the residual variation in agricultural, manufacturing and service shares after we partial out country-sector averages is accounted for by the Stone-Geary demand system. The corresponding number for nonhomothetic CES is more than two-times larger, 0.29. The intuition for the worse fit of Stone-Geary is that income effects are very low for rich countries, since for high levels of income the subsistence levels responsible for introducing the nonhomotheticity become negligible (see, also, [Dennis and Iscan, 2009](#)).⁴⁶ For the PIGL demand system we find an R^2 of 0.13, which is very similar to that of Stone-Geary. PIGL preferences track the trends in services more accurately than Stone-Geary due to the fact that they feature a non-vanishing nonhomotheticity of the service sector. However, they under-perform relative to nonhomothetic CES

shares in sector i and country c , \hat{y}_{it}^c , predicted employment shares, \bar{y}_i^c the sample average of y_{it} for country c in sector i , and $i \in \mathcal{I} = \{a, m, s\}$. We also note that the estimates used to compute the within- R^2 for nonhomothetic CES correspond to the structural estimates in column (1) of Table 6. Finally, note that in this exercise we are computing the R^2 on employment shares (and not relative log-shares). The reason is that the estimation of the three demand systems is based on different left-hand-side variables (e.g., Stone-Geary is not log-linear and it is estimated on shares directly). We chose to benchmark the fit of the three demand systems based on the level of employment shares as it is arguably the most basic object of interest.

⁴⁶For the U.S., the value of the nonhomothetic terms $p_{it}\bar{c}_i$ relative to total expenditure is never higher (in absolute terms) than 0.1%, which suggests that nonhomotheticity are insignificant. The highest values of the nonhomotheticities in the sample are 37% for agriculture and 18% for services.

mostly because they assume a homothetic composite between agriculture and manufacturing, while nonhomothetic CES allows for sector-specific nonhomotheticities. Figure 4 illustrates the fit for the case of Japan for the three demand systems.⁴⁷

Beyond 3 Sectors Estimation Jorgenson and Timmer (2011) have pointed out that in order to understand how structural transformation progresses in rich countries, it is important to zoom in the service sector, as it represents the majority of rich economies’ consumption shares (see also Buera and Kaboski, 2012b). Our framework lends itself to this purpose, as it can accommodate an arbitrary number of sectors. We estimate our demand system to more than three sectors for both micro and macro data. In the CEX data, the largest broad expenditure category is housing services. We extend our analysis by separating housing from the rest of services and re-estimate our baseline model with four sectors.⁴⁸ Table 11 in the appendix reports our findings. The price elasticity σ and the relative elasticity of agriculture to manufacturing, $\epsilon_a - \epsilon_m$, and services to manufacturing (excluding housing), remain very similar to our baseline estimates in Table 1. We find that the relative income elasticity of housing to manufacturing $\epsilon_{\text{housing}} - \epsilon_m$ is around 0.69, and thus somewhat larger than for the rest of services.⁴⁹

For the macro data, we extend our estimation to the 10 sectors in Groningen’s data: (1) agriculture, forestry and fishing, (2) mining and quarrying, (3) manufacturing, (4) public utilities, (5) construction, (6) wholesale and retail trade, hotels and restaurants, (7) transport, storage and communication, (8) finance, insurance, real state, (9) community, social and personal services, (10) government services.⁵⁰ Table 13 in the appendix reports the results estimating the demand system in an analogous manner to our baseline estimation (35). We find that the smallest income elasticities correspond to agriculture and mining, while the highest correspond to service sectors, such as finance, insurance, real state, and retail. Columns (2) and (3) show that the ranking of sectors in terms of their income elasticity is very similar when we estimate OECD and Non-OECD countries separately. Finally, the estimated price elasticity of substitution is very close to our baseline estimate (0.84 versus 0.89).⁵¹

⁴⁷We report the fit for all countries for both Stone-Geary and PIGL in Online Appendix H.

⁴⁸We define housing as expenditure in dwellings plus utilities. We use the same set of instruments plus a price instrument for housing constructed in an analogous way to the other price instruments.

⁴⁹Table E.2 in the online appendix shows the same estimates using the structural equation. The estimated income elasticities are also very similar to the ones obtained in the baseline estimation.

⁵⁰The data set also contains information on dwellings that are not constructed within the period, but this information is very sparse and we abstract from them. Note that in this case, the manufacturing sector is more narrowly defined than in the baseline estimation as it excludes mining and construction.

⁵¹Table F.7 in the Online Appendix reports the estimation results from a nested CES structure. That is, we estimate the demand for each of the sectors that belong to services or manufacturing separately. This is done at the expense of not having sectoral consumption data. We use aggregate sectoral value added instead. The results are similar to the ones reported in the main text.

Alternative Instrument for the CEX For the 2001-2002 subsample of our baseline CEX data, we can use an alternative instrumental variable approach for household quarterly expenditure developed by [Johnson et al. \(2006\)](#). The instrument is based on the fact that the timing of the 2001 Federal income tax rebates for each household was a function of the last digit of the recipient’s social security number. Thus, it was effectively random.⁵² We re-do our baseline formulation using the tax rebate as an instrument (and we maintain the same price instruments). The estimates we find are imprecise with large standard errors (see table 12 in the appendix). However, the point estimates are in line with our baseline estimates in Table 1.

CEX Value-Added Demand Formulation So far, we have estimated household demand defined over households’ final-good expenditure. Previous work has shown that the patterns of structural transformation are qualitatively similar whether we measure sectoral economic activity in terms of value-added or final-good expenditure shares ([Herrendorf et al., 2013](#)). We estimate our model defining household utility over the value added provided by each sector (rather than final good expenditure) and show that we obtain similar results. Table 10 in the appendix presents the estimates of our baseline specification (29) where we use as dependent variable household expenditure shares measured in value added (instead of final good expenditure). We find that the estimates of $\epsilon_a - \epsilon_m$ are around -0.56, while the estimates of $\epsilon_s - \epsilon_m$ are between 0.50 and 0.53. The estimates do not vary once we re-weight by expenditure, suggesting that estimates are stable across the income distribution. The point estimates we obtain for the price elasticity are in the 0.4 to 0.5 range. Column (5) perform the same estimation using as the dependent variable the time-series for the aggregate consumption expenditure shares in the U.S. data, when specified in value-added terms. The macro estimates appear to be close in magnitude to the CEX estimates.

Macro Estimation with Value-Added Shares We investigate whether we find similar estimates to the baseline cross-country results when we use value-added output shares as dependent variable (instead of employment shares). Some statistical agencies count all the employment in the investment sector in the manufacturing sector, while the service component of investment has been increasing over time ([Herrendorf et al., 2013](#)). By measuring sectoral activity in terms of their employment shares, we are implicitly adopting this assumption. Table F.4 in the online appendix reports the estimation results using shares of the sectoral in output value added as dependent variables in our baseline estimation, equation (35). The estimates appear with the expected signs and overall similar magnitudes as in the baseline regression. The income elasticity of agriculture relative to manufacturing ranges from -0.94 to -1.07 (versus -0.91 to -1.08 with employment shares), the income elasticity of services relative

⁵²We thank Bart Hobijn for suggesting this instrument to us.

Table 9: Model Parameters for the Calibration Exercise

Parameter	γ_a	$\gamma_m = \gamma_0$	γ_s	σ	ϵ_a	ϵ_m	ϵ_s	α	θ	β	δ
Value	0.029	0.013	0.011	0.50	0.05	1.00	1.20	0.33	2.20	0.96	0.10

to manufacturing oscillates between 0.11 and 0.28 (versus 0.19 to 0.48 with employment shares). The price elasticity appears somewhat smaller, ranging from 0.42 to 0.59 (versus 0.75 to 0.94).⁵³

6 Calibration Exercise

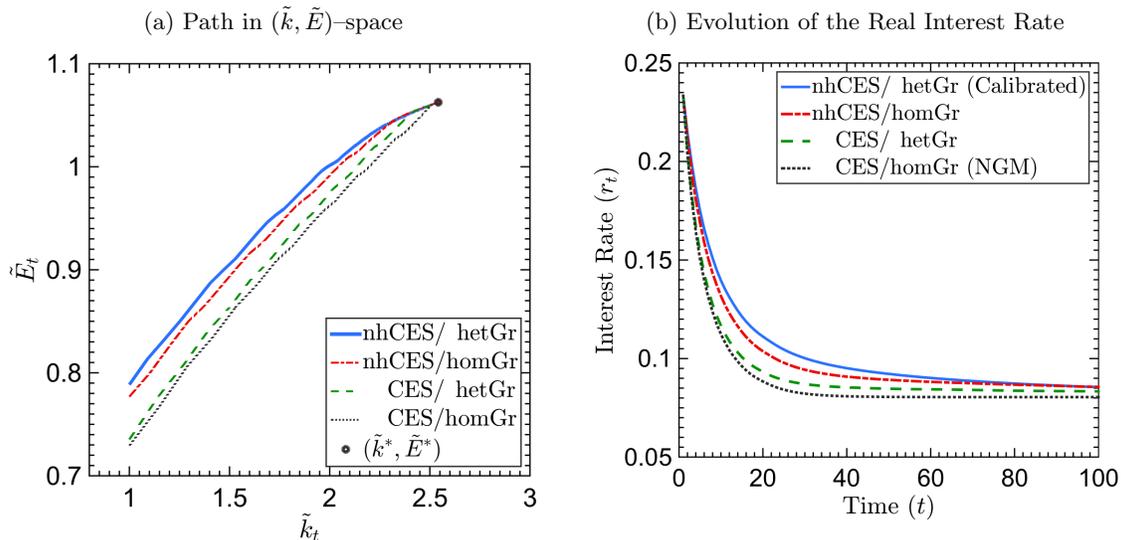
So far, we have only focused the predictions of the model regarding the intratemporal allocation of consumption expenditures. In this section, we rely on a simple calibration exercise to study the dynamic predictions of the model. As we discussed in Section 2.2.3, the qualitative properties of the transitional dynamics of the model heavily depend on the relationship between sectoral income elasticity and rates of productivity growth. In this section, we study the dynamics of the economy calibrated for the set of parameters estimated for the nonhomothetic CES preferences in Section 3. We then compare our model with simpler versions where we strip off different drivers of structural change. Relative to the Neoclassical Growth Model benchmark, we find that including any drivers of structural change in the model generates a slow-down of the convergence toward the long-run value.

Model Calibration For the preferences, we rely on the values estimated for the sectoral income elasticity parameters ϵ_i 's and the elasticity of substitution σ using the macro data in Section 3 and set $\Omega_i \equiv 1$ for all $i \in \{a, m, s\}$. We assume that capital intensity is the same across sectors and choose the standard value $\alpha = 0.33$ and a rate of depreciation of $\delta = 0.10$ for capital. For the sectoral rates of productivity growth, we assume that the rates of productivity growth in the investment sector and manufacturing are the same $\gamma_m = \gamma_0$, and calibrate them to the rate of growth of labor productivity observed in the in the postwar period in the US.⁵⁴ We then use the rates of decline in relative sectoral prices within the same period to calibrate the rates of growth of sectoral productivity for agriculture and services. Finally, we choose the value of the parameter θ such that the asymptotic value of the elasticity of intertemporal substitution matches 0.5, a reasonable number within the range of various

⁵³As an additional robustness check, we have also tested whether a constant price elasticity of substitution across sectors the three sector is a good approximation, by not constraining σ to be the same across sectoral regressions. Table F.5 in the online appendix shows that the estimated price elasticities in the unconstrained regression are very similar, and we cannot reject the null that they are identical at standard p-values.

⁵⁴Note that based on the model the rate of growth of labor productivity growth exceeds the rate of growth of multifactor productivity (TFP) γ_m by a factor $\alpha/(1 - \alpha)$.

Figure 5: Transitional Dynamics: Comparison with Neoclassical Growth Model (NGM)



The evolution of the economy starting from initial per-capita stock of capital of $\tilde{k}_0 = 1 < \tilde{k}^* = 2.10$. The parameters for the Calibrated Model are given in Table 9. The nhCES/hetGr corresponds to the calibrated model with nonhomothetic CES and heterogeneous rates of sectoral productivity growth. The nhCES/homGr model corresponds to the case with nonhomothetic CES preferences and homogeneous rates of sectoral productivity growth, $\gamma_i = 0.011$ for $i \in \{a, m, s\}$. The CES/hetGr model corresponds to the case with homothetic CES preferences, $\epsilon_i = 1.20$ for $i \in \{a, m, s\}$. The CES/homGr corresponds to the case of the Neoclassical Growth Model (NGM) both the rates of productivity growth and the income elasticity parameters are homogeneous across sectors. \tilde{k}^* and \tilde{E}^* denote the asymptotic normalized per-capita stock of capital and total consumption expenditure, respectively.

estimates provided in the literature (e.g., Guvenen, 2006; Havránek, 2015). Table 9 presents the set of model parameters used for the calibration.⁵⁵

Dynamics of Capital Accumulation First, we study how the presence of nonhomothetic CES demand changes the dynamics of the process of capital accumulation and the real interest rate. For this exercise, we compare the transitional dynamics of the calibrated model, in which both the parameters of income elasticity and the rates of productivity growth vary across sectors, with the following three different increasingly simpler models: 1) a model where the rates of productivity growth are homogeneous across sectors and the evolution of sectoral allocations is *exclusively* driven by nonhomotheticity in demand, 2) a model with homothetic preferences where income elasticity parameters are identical and, following Ngai and Pissarides (2007), the evolution of sectoral allocation are *exclusively* driven by the heterogeneity in sectoral rates of productivity growth, and 3) a standard neoclassical growth model (NGM) with homothetic CES preferences and homogeneous rates of productivity growth across sec-

⁵⁵Section C discusses the details of the method used for solving Equations (25) and (26) to derive the transitional dynamics of the model under these model parameters.

tors. We choose the parameters such that all models asymptotically converge to the same steady state as that of the calibrated model.⁵⁶

Beginning at an initially low level of per-capita stock of capital of $\tilde{k}_0 = 1 < \tilde{k}^* = 2.5$, Figure 5a shows the path of the economy from this initial condition toward its steady state in the space of the normalized per capita stock of capital and per-capita consumption expenditure. The figure compares these paths for all four models. All three models featuring structural change have higher values of total consumption expenditure relative to the NGM, which does not feature structural change, at all levels of per-capita stock of capital along the transitional path. As a result, we conclude based on this calibration that the presence of structural change implies a slower process of capital accumulation compared to the NGM, whether it is driven through the price or the income channel.

The slowdown in capital accumulation relative to the NGM benchmark is driven by the same two forces that shape the evolution of sectoral shares, namely, the inter-sectoral heterogeneity in the elasticities of income and the rates of productivity growth. In Section 2.2.3, we explained the mechanism behind the former force: the elasticity of intertemporal substitution gradually rises as their consumption shifts toward more income elastic goods that are also more intertemporally substitutable. The latter force is present in the benchmark theory of Ngai and Pissarides (2007): over time, household consumption shifts toward the sectors with the slower rates of productivity growth, lowering the rate of fall in the price of consumption. If household consumption is intertemporally inelastic (in the sense that $\theta > 1$), conditional on a given level of interest rate, the slowdown in the rate of decline of prices results in faster growth of consumption expenditure.⁵⁷ As the figure shows, these two forces, as well as their potential interactions, contribute to the slowdown in the accumulation of capital in the calibrated model, although nonhomotheticity plays a larger role.

Dynamics of Interest Rate Figure 5b compares the implications of all four models above for the evolution of the real interest rate. The slower process of capital accumulation implies that the real interest rate also converges toward its steady state more slowly in all three models featuring structural change, relative to the NGM benchmark. Once again, the model that solely features nonhomotheticity grows more slowly compared to the one solely featuring heterogeneous sectoral rates of productivity growth. Nevertheless, the overall difference between the evolution of the real interest rate between the calibrated model and the corresponding

⁵⁶Given the calibrated model parameters, the share of the service sector in consumption and employment converges to 1. Therefore, asymptotically all four models behave identical to a single-sector Neoclassical Growth Model where the instantaneous utility is defined as $C_t = C_{st}^{\epsilon_s/(1-\sigma)}$ and the productivity in the final good sector grows at rate γ_s .

⁵⁷To better see this point, consider a constant income elasticity parameter across sectors, $\epsilon_i = \epsilon$, and log-linearize the Euler Equation (13) to find $\theta \Delta \log E_t \approx (1 - \theta) (\gamma_0 - \bar{\gamma}_t) + r_t - (1 - \beta)$, where $\bar{\gamma}_t \equiv \sum_i \omega_{it} \gamma_i$ is the consumption-weighted average of the sectoral rates of productivity growth (see also Equation 22 in Ngai and Pissarides, 2007). When $\sigma < 1$, over time $\bar{\gamma}_t$ falls and therefore $\Delta \log E_t$ grows if $\theta > 1$.

NGM is relatively small: the time it takes for the real interest rate to go from 200% to 150% of its steady state level (half-life) is 9.1 years in the former relative to 4.4 years in the latter.⁵⁸

7 Conclusion

This paper presents a tractable model of structural transformation that accommodates both long-run demand and supply drivers of structural change. Our main contributions are to introduce the nonhomothetic CES utility function to growth theory, show its empirical relevance and use its structure to decompose the overall observed structural change into the contribution of income and price effects. These preferences generate nonhomothetic Engel curves at any level of development, which are in line with the evidence that we have from both rich and developing countries. Moreover, for this class of preferences, price and income elasticities are independent and they can be used for an arbitrary number of sectors. We argue that these are desirable theoretical and empirical properties.

We estimate these preferences using household-level data for the U.S. and India, and aggregate data for a panel of 39 countries during the post-war period. We argue that non-homothetic CES preferences provide a good fit of the data despite their parsimony. A key property is that they generate log-linear relative Engel curves. Armed with the estimated price and income elasticity parameters, we then use the demand structure to decompose the broad patterns of reallocation observed in our cross-country data into the contribution of nonhomotheticities and changes in relative prices. We find that over 80% of the variation is accounted for nonhomotheticities in demand.

To conclude, we believe that the proposed preferences provide a tractable departure from homothetic preferences. They can be used in other applied general equilibrium settings that currently use homothetic CES and monopolistic competition as their workhorse model, such as international trade. Also, as we discuss in Appendix A, it is possible to generalize non-homothetic CES to generate non-constant elasticity parameters, which may be useful in some applications. Even in this case, nonhomothetic CES remains a local approximation (with constant elasticity parameters) and can be used to guide how the varying elasticities should be parametrized, e.g., by estimating nonhomothetic CES across sub-samples.

⁵⁸The corresponding numbers in the model with nonhomotheticity and in the model with differential rates of productivity growth are 7.2 and 5.0, respectively.

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Appendix [Click Here to Download the Online Appendix](#)

A General Nonhomothetic CES Preferences

In this section of the appendix, we provide an overview of the properties of the general family of nonhomothetic Constant Elasticity of Substitution (CES) preferences. We first introduce the general family and then specialize them to the case of isoelastic nonhomothetic CES functions of Section 2.1.

Prior Work Sato (1975) derived a general family of CES functions as the solution to a partial differential equation that imposes the constancy of elasticity of substitution. This family includes standard homothetic CES functions as well as two classes of separable and non-separable nonhomothetic functions. Hanoch (1975) showed that additivity of the direct or indirect utility (or production) function results in price and income effects that are non-trivially dependent on each other. He then introduced *implicit additivity* and derived a family of functions where the income elasticity of demand is not fully dependent on the elasticity of substitution. Our nonhomothetic CES functions correspond to the non-separable class of functions in the sense of Sato (1975), which also satisfy the condition of implicit additivity in the sense of Hanoch (1975). Finally, Blackorby and Russell (1981) have proved an additional property that is unique to this class of functions. In general, different generalizations of the elasticity of substitution to cases involving more than two variables, e.g., the Allen-Uzawa definition or the Morishima definition, are distinct from each other. However, for the class of nonhomothetic CES functions they become identical and elasticity of substitution can be uniquely defined similar to the case of two-variable functions.

General Definition Consider now preferences over a bundle \mathbf{C} of I goods defined through an implicit utility function:

$$\sum_{i=1}^I \Omega_i^{\frac{1}{\sigma}} \left(\frac{C_i}{g_i(U)} \right)^{\frac{\sigma-1}{\sigma}} = 1, \quad (\text{A.1})$$

where functions g_i 's are differentiable in U and $\sigma \neq 1$ and $\sigma > 0$. To emphasize that this is a more general utility function, here we use U instead of \mathbf{C} , which we reserve for the nonhomothetic CES presented in Section 2. Theorem 2 in Blackorby and Russell (1981) implies that property (6) holds if and only if the preferences can be written as equation (A.1). In this sense, the definition above corresponds to the most general class of nonhomothetic CES preferences. Standard CES preferences are a specific example of Equation (A.1) with $g_i(U) = U$ for all i 's.

These preferences were first introduced, seemingly independently, by Sato (1975) and Hanoch (1975) who each characterize different properties of these functions. Here, we state and briefly prove some of the relevant results to provide a self-contained exposition of our theory in this paper.

Lemma 2. *If $\sigma > 0$ and functions $g_i(\cdot)$ are positive and monotonically increasing for all i , the function $U(\mathbf{C})$ defined in Equation (A.1) is monotonically increasing and quasi-concave for all $\mathbf{C} \gg 0$.*

Proof. Establishing monotonicity is straightforward. To establish quasi-concavity, assume to the contrary that there exists two bundles of \mathbf{C}' and \mathbf{C}'' and their corresponding utility values U' and U'' , such that $U \equiv U(\alpha\mathbf{C}' + (1 - \alpha)\mathbf{C}'')$ is strictly smaller than both U' and U'' . We then have for the

case $\sigma \geq 1$

$$\begin{aligned}
1 &= \sum_i \Omega_i^{1/\sigma} \left(\alpha \frac{C'_i}{g_i(U)} + (1-\alpha) \frac{C''_i}{g_i(U)} \right)^{\frac{\sigma-1}{\sigma}}, \\
&> \sum_i \Omega_i^{1/\sigma} \left(\alpha \frac{C'_i}{g_i(U')} + (1-\alpha) \frac{C''_i}{g_i(U'')} \right)^{\frac{\sigma-1}{\sigma}}, \\
&\geq \alpha \sum_i \Omega_i^{1/\sigma} \left(\frac{C'_i}{g_i(U')} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \sum_i \Omega_i^{1/\sigma} \left(\frac{C''_i}{g_i(U'')} \right)^{\frac{\sigma-1}{\sigma}},
\end{aligned}$$

where in the second inequality we have used monotonicity of the g_i 's and in the third we have used Jensen's inequality and the assumption that $\infty > \sigma > 1$. Since the last line equals 1 from the definition of the nonhomothetic CES functions valued at U' and U'' , we arrive at a contradiction. For the case that $0 < \sigma < 1$, we can proceed analogously. In this case, the inequality signs are reversed in both lines and we also reach a contradiction. \square

Demand Function Henceforth, we assume the conditions in Lemma 2 are satisfied. The next lemma characterizes the demand for general nonhomothetic CES preferences and provides the solution to the expenditure minimization problem.

Lemma 3. *Consider any bundle of goods that maximizes the utility function defined in Equation (A.1) subject to the budget constraint $\sum_i p_i C_i \leq E$. For each good i , the real consumption satisfies:*

$$C_i = \Omega_i \left(\frac{p_i}{E} \right)^{-\sigma} g_i(U)^{1-\sigma}, \quad (\text{A.2})$$

where U satisfies

$$E = \left[\sum_{i=1}^I \Omega_i (g_i(U) p_i)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \quad (\text{A.3})$$

and the share in consumption expenditure is given by

$$\omega_i \equiv \frac{p_i C_i}{E} = \Omega_i^{\frac{1}{\sigma}} \left(\frac{C_i}{g_i(U)} \right)^{\frac{\sigma-1}{\sigma}} = \Omega_i \left[g_i(U) \left(\frac{p_i}{E} \right) \right]^{1-\sigma}. \quad (\text{A.4})$$

Proof. Let λ and ρ denote the Lagrange multipliers on the budget constraint and constraint (A.1), respectively:

$$\mathcal{L} = U + \rho \left(1 - \sum_i \Omega_i^{\frac{1}{\sigma}} \left(\frac{C_i}{g_i(U)} \right)^{\frac{\sigma-1}{\sigma}} \right) + \lambda \left(E - \sum_i p_i C_i \right).$$

The FOCs with respect to C_i yields:

$$\rho \frac{1-\sigma}{\sigma} \frac{\omega_i}{C_i} = \lambda p_i, \quad (\text{A.5})$$

where we have defined $\omega_i \equiv \Omega_i^{\frac{1}{\sigma}} \left(\frac{C_i}{g_i(U)} \right)^{\frac{\sigma-1}{\sigma}}$. Equation (A.5) shows that expenditure $p_i C_i$ on good i is proportional to ω_i . Since the latter sums to one from constraint (A.1), it follows that ω_i is the expenditure share of good i , and we have: $E = \sum_{i=1}^I p_i C_i = \frac{1-\sigma}{\sigma} \frac{\rho}{\lambda}$. We can now substitute the

definition of ω_i from Equation (A) in expression (A.5) and use the budget constraint above to find Equation (A.2), as well as Equations (A.2) and (A.4). \square

Elasticities of Demand Lemma 3 implies that the equation defining the expenditure function (and implicitly the indirect utility function) for general Nonhomothetic CES preferences is given by Equation (A.2). The expenditure function is continuous in prices p_i 's and U , and homogeneous of degree 1, increasing, and concave in prices. The elasticity of the expenditure function with respect to utility is

$$\eta_E^U \equiv \frac{U \partial E}{E \partial U} = \sum_i \omega_i \eta_{g_i}^U = \overline{\eta_{g_i}^U}, \quad (\text{A.6})$$

which ensures that the expenditure function is increasing in utility if all g_i 's are monotonically increasing. It is straightforward to also show that the elasticity of the utility function (A.1) with respect to consumption of good i is also given by

$$\eta_{U^i}^{C_i} \equiv \frac{C_i \partial U}{U \partial C_i} = \frac{\omega_i}{\eta_{g_i}^U}, \quad (\text{A.7})$$

where ω_i is the ratio defined in Equation (A).

Examining sectoral demand from Equation (A.2) along indifference curves, we can derive the main properties of nonhomothetic CES preferences. As expected, *on a given indifference curve*, the elasticity of substitution is constant

$$\eta_{C_i/C_j}^{p_i/p_j} \equiv \left. \frac{\partial \log(C_i/C_j)}{\partial \log(p_i/p_j)} \right|_{U=\text{const.}} = \sigma. \quad (\text{A.8})$$

More interestingly, the elasticity of relative demand with respect to utility, in constant prices, is in different from unity:⁵⁹

$$\eta_{C_i/C_j}^U \equiv \left. \frac{\partial \log(C_i/C_j)}{\partial \log U} \right|_{p=\text{const.}} = (1 - \sigma) \frac{\partial \log(g_i/g_j)}{\partial \log U}. \quad (\text{A.9})$$

Since utility has a monotonic relationship with real income (and hence expenditure), it then follows that the expenditure elasticity of demand for different goods are different. More specifically, we can use (A.6) to find the expenditure elasticity of demand:

$$\eta_{C_i}^E \equiv \frac{\partial \log C_i}{\partial \log E} = \sigma + (1 - \sigma) \frac{\eta_{g_i}^U}{\eta_{g_i}^U}. \quad (\text{A.10})$$

⁵⁹Preferences defined by Equation (A.1) belong to the general class of preferences with *Direct Implicit Additivity*. Hanoch (1975) shows that the latter family of preferences have the nice property that is illustrated by Equations (A.8) and (A.9): the separability of the income and substitution elasticities of the Hicksian demand. This is in contrast to the stronger requirement of *Explicit Additivity* commonly assumed in nonhomothetic preferences, whereby the utility is explicitly defined as a function $U = F(\sum_i f_i(C_i))$. In Section G.1 of the Online Appendix, we will show examples of how substitution and income elasticities of Hicksian demand are *not* separable for preferences with explicitly additivity in direct utility, e.g., generalized Stone-Geary preferences (Kongsamut et al., 2001), or indirect utility, e.g., PIGL preferences (Boppart, 2014a).

Convexity of the Expenditure Function in Utility First, we express the second derivative of the expenditure function in terms of elasticities,

$$\frac{\partial^2 E}{\partial U^2} = \frac{E}{U^2} \eta_E^U \left(\eta_E^U + \eta_{\eta_E^U}^U - 1 \right), \quad (\text{A.11})$$

where $\eta_{\eta_E^U}^U$ is the second order elasticity of expenditure with respect to utility. We can compute this second order elasticity as follows:⁶⁰

$$\begin{aligned} \eta_{\eta_E^U}^U &= U \frac{\partial}{\partial U} \log \sum_i \eta_{g_i}(U) (g_i(U) p_i)^{1-\sigma} - (1-\sigma) \frac{\partial \log E}{\partial \log U}, \\ &= \frac{\sum_i \eta_{\eta_{g_i}} \cdot \eta_{g_i} (g_i(U) p_i)^{1-\sigma} + (1-\sigma) \sum_i \eta_{g_i}^2 (g_i(U) p_i)^{1-\sigma}}{\sum_i \eta_{g_i} (g_i(U) p_i)^{1-\sigma}} - (1-\sigma) \overline{\eta_{g_i}}, \\ &= \overline{\eta_{g_i}} \left[\frac{\overline{\eta_{\eta_{g_i}} \cdot \eta_{g_i}}}{(\overline{\eta_{g_i}})^2} + (1-\sigma) \text{Var} \left(\frac{\eta_{g_i}}{\overline{\eta_{g_i}}} \right) \right], \end{aligned} \quad (\text{A.13})$$

where $\overline{X_i}$ and $\text{Var}(X_i)$ denote the expected value and variance of variable X_i across sectors with weights given by expenditure shares ω_i for prices \mathbf{p} and utility U .

Income-Isoelastic Nonhomothetic CES Preferences We discussed that a class of preferences satisfies equation (6) if and only if we can write it as (A.1). It is easy to see that if we further impose condition (5), we find $\log g_i(U) = \epsilon_i / (1-\sigma) \log U + g(U)$ for all i . Furthermore, imposing the condition that the case of $\epsilon_i = 1-\sigma$ should correspond to homothetic CES implies that $g(U) = 0$. This gives us the definition of our basic model in Section 2, where the isoelastic functions g_i are defined as: $g_i(U) = U^{\frac{\epsilon_i}{1-\sigma}}$, where $\eta_{g_i}^U = \epsilon_i / (1-\sigma)$, and we retrieve standard CES preferences when $\epsilon_i = 1-\sigma$ for all i 's. Equations (2) and (3) follow by substituting for g_i , with $C = U$, in the results of Lemma 3 above. From (A.6), the real income elasticity of the expenditure function is: $\eta_E^C \equiv \frac{C}{E} \frac{\partial E}{\partial C} = \frac{\bar{\epsilon}}{1-\sigma}$, where $\bar{\epsilon} = \sum_i \omega_i \epsilon_i$. Therefore, a sufficient condition for the function $E(C; \mathbf{p})$ to be a one-to-one mapping for all positive prices is that all sectors have an income elasticity larger than the elasticity of substitution $\epsilon_i > 0$ if $\sigma < 1$ (and $\epsilon_i < 0$ if $\sigma > 1$). This directly follows from Lemma 2. Combining Equations (A.11) and (A.13), we find

$$\frac{\partial^2 E}{\partial C^2} = \frac{E}{C^2} \frac{\bar{\epsilon}}{1-\sigma} \left(\frac{\bar{\epsilon}}{1-\sigma} - 1 \right) + \frac{\text{Var}(\epsilon)}{\bar{\epsilon}}. \quad (\text{A.14})$$

Therefore, a sufficient condition for the expenditure function to be convex in C for all prices is that $\epsilon_{\min} \geq 1-\sigma$.

⁶⁰To make sense of (A.13), consider the choice of $g_i(U) \equiv g(U)^{\epsilon_i}$ for some monotonically increasing function $g(\cdot)$ (which corresponds to the aggregator introduced in Section A of the online appendix). We have that $\eta_{g_i} = \eta_g \epsilon_i$ and $\eta_{\eta_{g_i}} = \eta_{\eta_g}$, implying:

$$\eta_{\eta_E^U}^U = \eta_g \overline{\epsilon_i} \left[\eta_{\eta_g} + (1-\sigma) \text{Var} \left(\frac{\epsilon_i}{\overline{\epsilon_i}} \right) \right]. \quad (\text{A.12})$$

B Proofs of Propositions and Lemmas

Proof of Lemma 1. First, we show that the household problem has a unique solution that is characterized by an Euler equation along with a standard transversality condition. Let $E_t = w_t + (1 + r_t) \mathcal{A}_t - \mathcal{A}_{t+1}$ be the consumption expenditure when the representative household has current stock of assets \mathcal{A}_t and chooses an allocation \mathcal{A}_{t+1} of assets for the next period. We can decompose the problem into two independent parts. The intratemporal problem involves allocating the expenditure E_t across I goods so as to maximize the aggregator C_t defined by Equation (1). The solution is given by Equations (2) and (3).

Let $\tilde{C}_t(E) \equiv \max C_t$ subject to the constraint $E = \sum_{i=1}^I p_{it} C_{it}$. The intertemporal problem then involves finding the sequence of assets $\{\mathcal{A}_{t+1}\}_{t=0}^{\infty}$ such that

$$\max \sum_{t=0}^I \beta^t \frac{\tilde{C}_t (w_t + (1 + r_t) \mathcal{A}_t - \mathcal{A}_{t+1})^{1-\theta} - 1}{1 - \theta}. \quad (\text{B.1})$$

From Section A, we know that when $\epsilon_i \geq 1 - \sigma$ for all i , the expenditure function is monotonically increasing and strictly convex for all prices. Therefore, its inverse, the indirect aggregate consumption function $\tilde{C}(E; \mathbf{p}_t)$ exists and is monotonically increasing and strictly concave for all prices. Standard results from discrete dynamic programming (e.g., see Acemoglu, 2008, Chapter 6) then imply that the Euler equation

$$C_t^{-\theta} \frac{\partial \tilde{C}_t}{\partial E_t} = \beta (1 + r_t) C_{t+1}^{-\theta} \frac{\partial \tilde{C}_{t+1}}{\partial E_{t+1}},$$

and the transversality condition

$$\lim_{t \rightarrow \infty} \beta^t (1 + r_t) \mathcal{A}_t C_t^{-\theta} \frac{\partial \tilde{C}_t}{\partial E_t} = 0, \quad (\text{B.2})$$

provide necessary and sufficient condition for a sequence $\{\mathcal{A}_{t+1}\}_{t=0}^{\infty}$ to characterize the solution.

Using the results of Section A, we can simplify the Euler equation above to

$$C_t^{-\theta} \frac{C_t}{E_t} \frac{1 - \sigma}{\bar{\epsilon}_t} = \beta (1 + r_t) C_{t+1}^{-\theta} \frac{C_{t+1}}{E_{t+1}} \frac{1 - \sigma}{\bar{\epsilon}_{t+1}},$$

and the transversality condition to

$$\lim_{t \rightarrow \infty} \beta^t (1 + r_t) \frac{\mathcal{A}_t}{E_t} C_t^{1-\theta} \frac{1 - \sigma}{\bar{\epsilon}_t} = 0.$$

□

Proof of Proposition 1. Our proof for the proposition involves two steps. First, we use the second Welfare Theorem and consider the equivalent centralized allocation by a social planner. Due to the concavity of the aggregator C_t as a function of the bundle of goods (C_1, \dots, C_I) , which is ensured by the condition $\epsilon_i \geq 1$ for all i , we can use standard arguments to establish the uniqueness of the equilibrium allocations (see Stockey et al., 1989, p. 291). Next, we construct a unique constant growth path (steady state) that satisfies the equilibrium conditions. It then follows that the equilibrium

converges to the constructed Constant Growth Path (CGP).

Consider an equilibrium path along which consumption expenditure E_t , aggregate stock of capital K_t , and the capital allocated to the investment sector K_{0t} all asymptotically grow at rate $(1 + \gamma_0)^{\frac{1}{1-\alpha_0}}$, and the labor employed in the investment sector asymptotically converges to $L_0^* \in (0, 1)$. Henceforth, we use the tilde variables to denote normalization $A_{0t}^{-\frac{1}{1-\alpha_0}}$, for instance, $\tilde{K}_t \equiv A_{0t}^{-\frac{1}{1-\alpha_0}} K_t$. Accordingly, we can write the law of evolution of aggregate stock of capital as

$$\tilde{K}_{t+1} = \frac{1 - \delta}{(1 + \gamma_0)^{1/(1-\alpha_0)}} \tilde{K}_t + \frac{1}{(1 + \gamma_0)^{1/(1-\alpha_0)}} \tilde{K}_{0t}^{\alpha_0} L_{0t}^{1-\alpha_0}, \quad (\text{B.3})$$

and the interest rate and wages as

$$r_t = R_t - \delta = \alpha_0 \left(\frac{\tilde{K}_{0t}}{L_{0t}} \right)^{\alpha_0 - 1} - \delta, \quad (\text{B.4})$$

$$\tilde{w}_t = (1 - \alpha_0) \tilde{K}_{0t}^{\alpha_0} L_{0t}^{-\alpha_0}. \quad (\text{B.5})$$

From the assumptions above, it follows that \tilde{K}_{0t}/L_{0t} asymptotically converges to a constant, which from Equation (B.4) implies that the rate of interest also converges to a constant r^* .

We first derive an expression for the asymptotic growth of nominal consumption expenditure shares (and sectoral employment shares) of different sectors, using in equation (2),

$$\begin{aligned} 1 + \xi_i &\equiv \lim_{t \rightarrow \infty} \frac{\omega_{it+1}}{\omega_{it}} = \lim_{t \rightarrow \infty} \left(\frac{E_t}{E_{t+1}} \right)^{1-\sigma} \left(\frac{p_{it+1}}{p_{it}} \right)^{1-\sigma} \left(\frac{C_{t+1}}{C_t} \right)^{\epsilon_i}, \\ &= \left(\frac{1}{1 + \gamma_0} \right)^{\frac{1-\sigma}{1-\alpha_0}} \left(\frac{(1 + \gamma_0)^{\frac{1-\alpha_i}{1-\alpha_0}}}{1 + \gamma_i} \right)^{(1-\sigma)} (1 + \gamma^*)^{\epsilon_i}, \\ &= \frac{(1 + \gamma^*)^{\epsilon_i}}{\left[(1 + \gamma_0)^{\frac{\alpha_i}{1-\alpha_0}} (1 + \gamma_i) \right]^{1-\sigma}}, \end{aligned} \quad (\text{B.6})$$

where in the second line we have used the definition of the constant growth path as well as the fact that from Equations (B.4) and (B.5), the relative labor-capital price grows as rate $(1 + \gamma_0)^{\frac{1}{1-\alpha_0}}$ and therefore from Equation (17) we have

$$\lim_{t \rightarrow \infty} \frac{p_{it+1}}{p_{it}} = \frac{1 + \gamma_0}{1 + \gamma_i} (1 + \gamma_0)^{\frac{\alpha_0 - \alpha_i}{1-\alpha_0}}. \quad (\text{B.7})$$

Equation (B.6) shows that the expenditure shares asymptotically grow (or diminish) monotonically. Since the shares belong to the compact $I - 1$ dimensional simplex, they asymptotically converge to a time-constant set of shares.

Since shares have to add up to 1, we need to have that $\xi_i \leq 0$ for all i . Moreover, this inequality has to be satisfied with equality at least for one non-vanishing sector. Now, consider the expression defined in (22) for the growth rate of real consumption. For sectors $i \in \mathcal{I}^*$ that achieve the minimum, the growth of nominal expenditure share becomes zero, and their shares converge to constant values ω_i^* . For sectors $i \notin \mathcal{I}^*$, we find the following expression for the growth rate of nominal shares ξ_i in Equation (B.6) becomes negative. Assuming $\sigma < 1$ and $\epsilon_i > \sigma$, the expression on the right hand side

becomes strictly less than 1, since we know sector i does not achieve the minimum in (22). Therefore, $\xi_i < 0$ and the nominal shares asymptotically vanish for $i \notin \mathcal{I}^*$.

Asymptotically, the expenditure-weighted average income elasticity and expenditure-weighted capital intensity in the consumption sector both converge to constants $\bar{c}^* \equiv \lim_{t \rightarrow \infty} \sum_{i=1}^I \epsilon_i \omega_{it} = \sum_{i \in \mathcal{I}^*} \epsilon_i \omega_i^*$ and $\bar{\alpha}^* \equiv \lim_{t \rightarrow \infty} \sum_{i=1}^I \alpha_i \omega_{it} = \sum_{i \in \mathcal{I}^*} \alpha_i \omega_i^*$. Henceforth, we extend our notation to use tilde to indicate variables normalized by their corresponding asymptotic rate of growth (or decline) along our proposed constant growth path. For instance, we let $\tilde{p}_{it} \equiv p_{it}(1+\gamma_0)^{-\frac{1-\alpha_i}{1-\alpha_0}t}(1+\gamma_i)^{-t}$ and $\tilde{C}_t \equiv C_t(1+\gamma^*)^{-t}$. Furthermore, we define starred notation to indicate the asymptotic value of each variable along the constant growth path, for example, we let $p_i^* \equiv \lim_{t \rightarrow \infty} \tilde{p}_{it}$ and $\tilde{C}^* \equiv \lim_{t \rightarrow \infty} \tilde{C}_t$.

We now show that a constant growth path exists and is characterized by γ^* as defined by equation (22). We also show the existence of the asymptotic values $\{\tilde{K}^*, \tilde{C}^*, \tilde{K}_0^*, L_0^*\}$. From the Euler equation (13), we have that asymptotically

$$(1 + \gamma^*)^{1-\theta} = \frac{(1 + \gamma_0)^{\frac{1}{1-\alpha_0}}}{\beta(1 + r^*)}, \quad (\text{B.8})$$

which pins down r^* , the asymptotic real interest rate in terms of γ^* given by Equation (22). Then from Equation (B.4), we find the asymptotic capital-labor ratio in the investment sector in terms of the asymptotic real interest rate

$$\kappa \equiv \frac{\tilde{K}_0^*}{L_0^*} = \left(\frac{\alpha_0}{r^* + \delta} \right)^{\frac{1}{1-\alpha_0}}. \quad (\text{B.9})$$

This gives us the asymptotic relative labor-capital price from Equations (B.4) and (B.5) as

$$\frac{\tilde{w}^*}{R^*} = \frac{1 - \alpha_0}{\alpha_0} \frac{\tilde{K}^*}{L_0^*} = \frac{1 - \alpha_0}{\alpha_0} \left(\frac{\alpha_0}{r^* + \delta} \right)^{\frac{1}{1-\alpha_0}}. \quad (\text{B.10})$$

From Equation (17), we find

$$\tilde{p}_i^* = \frac{\alpha_0^{\alpha_0} (1 - \alpha_0)^{1-\alpha_0}}{\alpha_i^{\alpha_i} (1 - \alpha_i)^{1-\alpha_i}} \left(\frac{\tilde{w}^*}{R^*} \right)^{\alpha_0 - \alpha_i} \frac{A_{0,0}}{A_{i,0}}, \quad (\text{B.11})$$

where \tilde{w}^*/R^* is given by Equations (B.10) and (B.8) and $A_{i,0}$ denotes the initial state of technology in sector i and $A_{0,0} \equiv 1$. Given asymptotic prices

$$\tilde{E}^* = \left[\sum_{i \in \mathcal{I}^*} \left(\tilde{C}^* \right)^{\epsilon_i - \sigma} \left(\tilde{p}_i^* \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad (\text{B.12})$$

and

$$\omega_i^* = \left(\frac{\tilde{p}_i^*}{\tilde{E}^*} \right)^{1-\sigma} \left(\tilde{C}^* \right)^{\epsilon_i - \sigma}. \quad (\text{B.13})$$

Next, we combine the equation for accumulation of capital (B.3), the household budget constraint (16) the market clearing condition of consumption goods to establish that there exists a unique $\{\tilde{K}^*, \tilde{C}^*, \tilde{K}_0^*, L_0^*\}$ satisfying the asymptotic equilibrium conditions and $\kappa = \tilde{K}_0^*/L_0^*$ where κ is given by Equation (B.9). From market clearing, the sum of payments to labor in the consumption sector is

$\sum_{i=1}^I (1 - \alpha_i) \omega_{it} E_{it}$, which implies $(1 - \bar{\alpha}_t) \tilde{E}_t = \tilde{w}_t (1 - L_{0t})$. Asymptotically, we find that

$$(1 - \bar{\alpha}^*) \tilde{E}^* = (1 - \alpha_0) \kappa^{\alpha_0} (1 - L_0^*). \quad (\text{B.14})$$

Similarly, from Equation (B.3) it follows that $\left[(1 + \gamma_0)^{\frac{1}{1-\alpha_0}} - (1 - \delta) \right] \tilde{K}^* = \kappa^{\alpha_0} L_0^*$. Defining the expression within the square brackets at a positive constant ϑ , we use write the asymptotic employment in the investment sector in terms of the aggregate stock of capital as

$$L_0^* = \vartheta \kappa^{-\alpha_0} \tilde{K}^*. \quad (\text{B.15})$$

Finally, using the market clearing condition in the assets market $\mathcal{A}_t = K_t$ and Equation (16), we find that $\tilde{E}_t = \tilde{w}_t + R_t \tilde{K}_t - \left(\frac{\tilde{K}_{0t}}{L_{0t}} \right)^{\alpha_0} L_{0t}$ for all t . Taking the limit, it follows that

$$\tilde{E}^* = (1 - \alpha_0) \kappa^{\alpha_0} + \alpha_0 \kappa^{\alpha_0 - 1} \tilde{K}^* - \kappa^{\alpha_0} L_0^*. \quad (\text{B.16})$$

Substituting from Equation (B.15) into Equations (B.14) and (B.16) yields,

$$\bar{\alpha}^* \tilde{E}^* = \alpha_0 (\kappa^{\alpha_0 - 1} - \vartheta) \tilde{K}^*. \quad (\text{B.17})$$

We can show that the left hand side of this equation is a monotonically increasing function of \tilde{C}^* with a given κ .⁶¹ From condition (23), we have that $\kappa^{\alpha_0 - 1} - \vartheta > 0$ and therefore the right hand side is a linear increasing function of \tilde{K}^* . Therefore, Equation (B.17) defines \tilde{C}^* , and correspondingly \tilde{E}^* , as an increasing function of \tilde{K}^* . Finally, substituting this function and Equation (B.15) in Equation (B.16), we find

$$\tilde{E} + (\vartheta - \alpha \kappa^{\alpha_0 - 1}) \tilde{K} = (1 - \alpha_0) \kappa^{\alpha_0}. \quad (\text{B.18})$$

From condition (23), we know that the left hand side is a monotonically increasing function of \tilde{K}^* for constant κ . This function is 0 when \tilde{K}^* and limits to infinity as the latter goes to infinity. Therefore, Equation (B.18) uniquely pins down \tilde{K}^* as a function of κ , which in turn is given by Equation (B.9). Condition (23) also ensures that the transversality condition (14) is satisfied. Finally, we verify that $L_0^* \in (0, 1)$. Combining equations (B.15), (B.14) and (B.16) we obtain that

$$L_0^* = \frac{\bar{\alpha}}{\left[\frac{1 - \bar{\alpha}}{1 - \alpha_0} (\alpha_0 \kappa^{\alpha_0 - 1} \vartheta - 1) + 1 \right]} \quad (\text{B.19})$$

Assuming that the term in square brackets is positive, we have that $L_0^* \in (0, 1)$ if and only if $\vartheta < \kappa^{\alpha_0 - 1}$, which in terms of fundamental parameters requires that $\beta(1 + \gamma^*)^{1 - \theta} < \frac{(1 + \gamma_0)^{-\frac{\alpha_0}{1 - \alpha_0}}}{\alpha_0 + (1 - \alpha_0)(1 + \gamma_0)^{-\frac{1}{1 - \alpha_0}}(1 - \delta)}$ which is the condition stated in (23). Also, it is readily verified that as long as $\vartheta < \kappa^{\alpha_0 - 1}$, L_0^* cannot be negative.

Therefore, we constructed a unique constant growth path that asymptotically satisfies the equilibrium conditions whenever the parameters of the economy satisfy Equation (23). Together with the

⁶¹We have that $\frac{\partial(\bar{\alpha}^* \tilde{E}^*)}{\partial \tilde{C}^*} = \frac{\bar{\alpha}^* \tilde{E}^*}{\tilde{C}^*} \frac{\bar{\epsilon}}{1 - \sigma} [1 + (1 - \sigma) \rho_{\epsilon_i, \alpha_i}]$ where $\rho_{\epsilon_i, \alpha_i}$ is the correlation coefficient between ϵ_i and α_i under a distribution implied by expenditure shares (see online Appendix for details of the derivation). Therefore, the derivative is always positive and the function is a monotonic of \tilde{C}^* .

uniqueness of the competitive equilibrium, this completes the proof. \square

Derivations for the Results in Section 2.2.3 We first characterize the dynamics of the state variable, the normalized per-capita stock of capital $\tilde{k}_t \equiv \tilde{K}_t/L$. Substituting in $K_{t+1} = A_{0t}K_{0t}^\alpha L_{0t}^{1-\alpha} + K_t(1 - \delta)$ and noting the equality of per-capita stock of capital across sectors, we find

$$(1 + \gamma_o)^{\frac{1}{1-\alpha}} \tilde{k}_{t+1} = \tilde{k}_t^\alpha l_{0,t} + \tilde{k}_t(1 - \delta),$$

where $l_{0,t} \equiv L_{0,t}/L$ is the share of labor employed in the investment sector. We can show that this share is given by $l_{0,t} = 1 - \tilde{E}_t/\tilde{k}_t^\alpha$ (see the online appendix), therefore establishing Equation (25).

For the evolution of per-capita consumption, we need to write C_{t+1}/C_t in terms of variables known at time t . Rwriting the the Euler Equation (13) as $(C_{t+1}/C_t)^{1-\theta} \beta(1 + r_t) = (E_{t+1}/E_t) \bar{\epsilon}_{t+1}/\bar{\epsilon}_t$, first note that the interest rate is given from Equation (B.4) as $r_t = \alpha \tilde{k}_t^{\alpha-1} - \delta$. Substituting for the normalized variables, we find

$$\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{1-\theta} \frac{(1 + \gamma^*)^{1-\theta}}{(1 + \gamma_o)^{\frac{1}{1-\alpha}}} \beta(1 + r_t) = \left(\frac{\tilde{E}_{t+1}}{\tilde{E}_t} \right) \frac{\bar{\epsilon}_{t+1}}{\bar{\epsilon}_t}.$$

Using the expression for the asymptotic rate of interest r^* from (B.8) then gives us Equation (26).

Next, we can write the growth in per-capita consumption expenditure as

$$\begin{aligned} \left(\frac{E_{t+1}}{E_t} \right)^{1-\sigma} &= \sum_{i=1}^I \Omega_i \left(\frac{p_{it}}{E_t} \right)^{1-\sigma} C_t^{\epsilon_i} \left(\frac{p_{it+1}}{p_{it}} \right)^{1-\sigma} \left(\frac{C_{t+1}}{C_t} \right)^{\epsilon_i}, \\ &= \sum_{i=1}^I \omega_{it} \left(\frac{C_{t+1}}{C_t} \right)^{\epsilon_i} \left(\frac{1 + \gamma_o}{1 + \gamma_i} \right)^{(1-\sigma)t}, \end{aligned}$$

where we have used Equation (3), Equation (17), and the expression for expenditure shares $\omega_{it} = \Omega_i(p_{it}/E_t)^{1-\sigma} C_t^{\epsilon_i}$ under the assumption of $\alpha_i \equiv \alpha$. Substituting for the normalized variables \tilde{E}_t and \tilde{C}_t in the expression above gives Equation (27).

Finally, we use the same idea to rewrite the term $\bar{\epsilon}_{t+1}$ as follows

$$\begin{aligned} \frac{\bar{\epsilon}_{t+1}}{\bar{\epsilon}_t} &= \sum_{i=1}^I \left(\frac{p_{it+1}}{E_{t+1}} \right)^{1-\sigma} C_{t+1}^{\epsilon_i} \epsilon_i, \\ &= \left(\frac{E_t}{E_{t+1}} \right)^{1-\sigma} \sum_{i=1}^I \left(\frac{p_{it}}{E_t} \right)^{1-\sigma} C_t^{\epsilon_i} \left(\frac{p_{it+1}}{p_{it}} \right)^{1-\sigma} \left(\frac{C_{t+1}}{C_t} \right)^{\epsilon_i} \left(\frac{\epsilon_i}{\bar{\epsilon}_t} \right), \\ &= \frac{\sum_{i=1}^I \omega_{it} \left(\frac{C_{t+1}}{C_t} \right)^{\epsilon_i} \left(\frac{1 + \gamma_o}{1 + \gamma_i} \right)^{(1-\sigma)t} \left(\frac{\epsilon_i}{\bar{\epsilon}_t} \right)}{\sum_{i=1}^I \omega_{it} \left(\frac{C_{t+1}}{C_t} \right)^{\epsilon_i} \left(\frac{1 + \gamma_o}{1 + \gamma_i} \right)^{(1-\sigma)t}}. \end{aligned}$$

Multiplying both the numerator and the denominator by $(1 + \gamma_o)^{-\frac{t}{1-\alpha_o}}$ and substituting again for the normalized variables \tilde{E}_t and \tilde{C}_t gives us Equation (28).

Proof of Equation (32). From the definition of the expenditure function in Equation (3), we have

$$\begin{aligned}
\left(\frac{E_{t+1}}{E_t}\right)^{1-\sigma} &= \frac{\sum_i \Omega_i C_{t+1}^{\epsilon_i} P_{it+1}^{1-\sigma}}{\sum_i \Omega_i C_t^{\epsilon_i} P_{it}^{1-\sigma}} \\
&= \frac{\sum_i \Omega_i C_t^{\epsilon_i} P_{it}^{1-\sigma} \times \left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i} \left(\frac{P_{it+1}}{P_{it}}\right)^{1-\sigma}}{\sum_i \Omega_i C_{t+1}^{\epsilon_i} P_{it+1}^{1-\sigma} \times \left(\frac{C_{t+1}}{C_t}\right)^{-\epsilon_i} \left(\frac{P_{it+1}}{P_{it}}\right)^{-(1-\sigma)}} \\
&= \left(\frac{E_{t+1}}{E_t}\right)^{-(1-\sigma)} \frac{\sum_i \omega_{it} \times \left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i} \left(\frac{P_{it+1}}{P_{it}}\right)^{1-\sigma}}{\sum_i \omega_{it+1} \times \left(\frac{C_{t+1}}{C_t}\right)^{-\epsilon_i} \left(\frac{P_{it+1}}{P_{it}}\right)^{-(1-\sigma)}}.
\end{aligned}$$

Assuming that $\Delta \log E_t = \log(E_{t+1}/E_t) \ll 1$ and $\Delta \log P_{it} = \log(P_{it+1}/P_{it}) \ll 1$ for all i , we can rewrite the expression above up to the second order in $\Delta \log E_t$, $\Delta \log C_t$, and $\Delta \log P_{it}$ as

$$\begin{aligned}
\log \frac{E_{t+1}}{E_t} &\approx \frac{1}{2(1-\sigma)} \sum_i (\omega_{it} + \omega_{it+1}) \left((1-\sigma) \log \frac{P_{it+1}}{P_{it}} + \epsilon_i \log \frac{C_{t+1}}{C_t} \right), \\
&= \underbrace{\left[\frac{1}{2} \sum_i (\omega_{it} + \omega_{it+1}) \log \frac{P_{it+1}}{P_{it}} \right]}_{\equiv \Delta \log \mathcal{P}_{it}} + \frac{1}{1-\sigma} \underbrace{\left[\frac{1}{2} \sum_i (\omega_{it} + \omega_{it+1}) \epsilon_i \right]}_{\equiv \bar{\epsilon}_t} \times \log \frac{C_{t+1}}{C_t},
\end{aligned}$$

from which Equation (32) follows. \square

C Discussion of the Estimation Strategy

To simplify the exposition of the derivations, we define the following notation, only to be used within this section of the Appendix: let $Y_{it}^n \equiv \log(\omega_{it}^n/\omega_{mt}^n)$, $P_{it}^n \equiv \log(p_{it}^n/p_{mt}^n)$, $X_t^n \equiv \log C_t^n$, and $Z_{it}^n \equiv \log(E_t^n/P_{it}^n)$ for all $i \in \mathcal{I}_- = \mathcal{I} \setminus \{m\}$. We can then rewrite Equation (15) as

$$Y_{it}^n = (1-\sigma) P_{it}^n + (\epsilon_i - \epsilon_m) X_t^n + \zeta_i^n + \nu_{it}^n, \quad i \in \mathcal{I}_-. \quad (\text{C.1})$$

Henceforth, we assume i is always within set \mathcal{I}_- and drop the reference to the set.

Throughout, we maintain the following assumptions.⁶²

Assumption 1. *Relative prices and income are orthogonal to the errors, that is, $\mathbb{E}[P_{jt}^n \nu_{it}^n] = \mathbb{E}[X_t^n \nu_{it}^n] = 0$ for all i, j . Moreover, relative prices are not perfectly correlated with either the real income index X_t^n or the proxy Z_t^n , that is, $|\mathbb{E}[X_t^n P_{it}^n]| < (\mathbb{E}[X_t^n] \mathbb{E}[P_{it}^n])^{1/2}$ and $|\mathbb{E}[Z_t^n P_{it}^n]| < (\mathbb{E}[Z_t^n] \mathbb{E}[P_{it}^n])^{1/2}$.*

The first approach used in Section 3.2 involves replacing the unobserved index of real consumption X_t^n by a *proxy* variable, which is the consumption expenditure deflated by a standard price index Z_t^n . For any population-level distribution of relative prices and income, without loss of generality, we can

⁶²In the case of household-level data, instead of assuming the orthogonality of the covariates and the error, we use instruments for both relative prices and income, which would slightly complicate the derivations that follows. However, the main insights will remain intact whether we assume the orthogonality of the covariates or the existence of instruments for them.

write

$$X_t^n = \sum_i \eta_i P_{it}^n + \gamma Z_t^n + l^n + u_t^n, \quad (\text{C.2})$$

such that $\mathbb{E}[u_t^n] = \mathbb{E}[P_{it}^n u_t^n] = \mathbb{E}[Z_t^n u_t^n] = 0$ for all $i \in \mathcal{I}_{-m}$. It follows that we can write

$$\begin{aligned} Y_{it}^n &= (1 - \sigma + \eta_i (\epsilon_i - \epsilon_m)) P_{it}^n + \sum_{j \neq i} \eta_j (\epsilon_i - \epsilon_m) P_{jt}^n \\ &+ (\epsilon_i - \epsilon_m) \gamma Z_t^n + \zeta_i^n + (\epsilon_i - \epsilon_m) (l^n + u_t^n) + \nu_{it}^n. \end{aligned} \quad (\text{C.3})$$

The lemma below establishes that we can identify the model's elasticity parameters up to a constant factor using a system OLS estimate or a feasible GLS estimate of log relative shares on log relative prices and log real consumption expenditure, of the form⁶³

$$Y_{it}^n = \sum_j \alpha_{ij} P_{jt}^n + \beta_i Z_t^n + \tilde{\zeta}_i^n + \tilde{\nu}_{it}^n. \quad (\text{C.4})$$

Lemma 4. *Assume that the model in Equation (C.1) is well-specified, Assumption 1 holds, and that $\gamma \neq 0$, i.e., the real income index of nonhomothetic CES and the real income calculated based on standard price indices, our proxy variable, are correlated after controlling for relative prices. Let $\hat{\beta}_i$ denote the coefficients on the real consumption expenditure based on estimating the system of Equations C.4. Then, the coefficients on the proxy variable Z_t^n satisfy $\text{plim } \hat{\beta}_i / \hat{\beta}_j = (\epsilon_i - \epsilon_m) / (\epsilon_j - \epsilon_m)$.*

Next, we consider the system of linear equations implied by Equation (15). This system further imposes the following constraints: 1) $\alpha_{ij} = 0$ for $i \neq j$, and 2) $\alpha_{ii} = 1 - \sigma$. To simplify the setup, let us consider the case where we have one fixed effect per each good, that is, $\zeta_i^n = \zeta_i$. Define $\mathbf{Y}_t^n \equiv (Y_{1t}^n, \dots, Y_{It}^n)'$, $\boldsymbol{\nu}_t^n \equiv (\nu_{1t}^n, \dots, \nu_{It}^n)'$, \mathbf{X}_{it}^n , \mathbf{Z}_{it}^n , $\mathbf{1}_i$ as $(I - 1)$ -dimensional vectors that with X_t^n , Z_t^n , and 1, respectively, in their i -th location and 0 elsewhere. Furthermore, define the matrix $\mathbf{Z}_t^n \equiv (\mathbf{P}_t^n, \mathbf{Z}_1^n \dots, \mathbf{Z}_I^n, \mathbf{1}_1, \dots, \mathbf{1}_I)$ and $\boldsymbol{\beta} \equiv (\alpha, \beta_1, \dots, \beta_I, \tilde{\zeta}_1, \dots, \tilde{\zeta}_I)'$. The system of equations is now defined by $\mathbf{Y}_t^n = \mathbf{Z}_t^n \boldsymbol{\beta} + \boldsymbol{\nu}_t^n$.

Define the linear projection of a random vector \mathbf{D}_t^n into \mathbf{Z}_t^n as

$$L^* [\mathbf{D}_t^n | \mathbf{Z}_t^n] \equiv \mathbf{Z}_t^n \mathbb{E} [(\mathbf{Z}_t^n)' \mathbf{Z}_t^n]^{-1} \mathbb{E} [(\mathbf{Z}_t^n)' \mathbf{D}_t^n].$$

We can write the model (C.1) as $\mathbf{Y}_t^n = (1 - \sigma) \mathbf{P}_t^n + \sum_i (\epsilon_i - \epsilon_m) \mathbf{C}_{it}^n + \sum_i \zeta_i \mathbf{1}_i + \boldsymbol{\nu}_t^n$, where $\boldsymbol{\nu}_t^n$ is orthogonal to \mathbf{Z}_t^n by Assumption (1). Therefore, the model implies

$$\begin{aligned} L^* [\mathbf{Y}_t^n | \mathbf{Z}_t^n] &= (1 - \sigma) \mathbf{P}_t^n + \sum_i (\epsilon_i - \epsilon_m) L^* [\mathbf{C}_{it}^n | \mathbf{Z}_t^n] + \sum_i \zeta_i \mathbf{1}_i, \\ &= \left[1 - \sigma + \sum_i (\epsilon_i - \epsilon_m) \tau_i \right] \mathbf{P}_t^n + \sum_i \left[\sum_j (\epsilon_j - \epsilon_m) \gamma_{ji} \right] \mathbf{Z}_{it}^n + \sum_i \left[\zeta_i + \sum_i \vartheta_{i1} \right] \mathbf{1}_i, \end{aligned}$$

where we have defined $(\tau_i, \gamma_{i1}, \dots, \gamma_{iI}, \vartheta_{i1}, \dots, \vartheta_{iI})' = \mathbb{E} [(\mathbf{Z}_t^n)' \mathbf{Z}_t^n]^{-1} \mathbb{E} [(\mathbf{Z}_t^n)' \mathbf{C}_{it}^n]$.

⁶³This result is fairly general and can be applied to all cases where one unobserved covariate appears on the right hand side of more than one equation a system of equations, and a proxy variable exists that is correlated with the unobserved covariate and is orthogonal to the error.

Below, we show that

$$\tau_i = \frac{\mathbb{E}[\xi_{it}^n \psi_t^n]}{\sum_j \mathbb{E}[(\xi_{jt}^n)^2]}, \quad \gamma_{ii} = \gamma + \sum_i \eta_i \chi_i - \chi_i \tau_i, \quad \gamma_{ij} = -\chi_j \tau_i, \quad (\text{C.5})$$

where $P_{it}^n = \chi_i Z_t^n + \xi_{it}^n$ and $X_t^n = (\gamma + \sum_i \eta_i \chi_i) Z_t^n + \psi_t^n$ and such that $\mathbb{E}[Z_t^n \psi_t^n] = \mathbb{E}[Z_t^n \xi_{it}^n] = 0$ for all i .

The lemma below establishes that we can identify the model's elasticity parameters up to a constant factor using a system OLS estimate or a feasible GLS estimate log relative shares on log relative prices and log real consumption expenditure, of the form

$$Y_{it}^n = \alpha P_{it}^n + \beta_i Z_t^n + \tilde{\zeta}_i + \tilde{\nu}_{it}^n. \quad (\text{C.6})$$

Lemma 5. *Assume that the model in Equation (C.1) is well-specified, Assumption (1) holds, $\mathbb{E}[X_t^n Z_t^n] \neq 0$, and $\mathbb{E}[\xi_{it}^n \psi_t^n] = 0$ for all i . Let $\hat{\beta}_i$ denote the coefficients on the real consumption expenditure. Then, the coefficients on the proxy variable Z_t^n satisfy $\text{plim } \hat{\beta}_i / \hat{\beta}_j = (\epsilon_i - \epsilon_m) / (\epsilon_j - \epsilon_m)$ and the coefficient on relative prices identifies the elasticity of substitution $\text{plim } \hat{\alpha} = 1 - \sigma$.*

The assumptions in Lemma 5 require, in particular, that $\sum_j \eta_j \mathbb{E}[\xi_{it}^n \xi_{jt}^n] = 0$ (from Equation C.2). This assumption is hard to directly check in practice. However, there are ways for us to indirectly test the assumptions in Lemma 5. In particular, they imply that the ratios of $\hat{\beta}_i / \hat{\beta}_j$ estimated either from the unconstrained system of equations (C.4) or from (C.6) should have the same plim (and coincide with the true parameter values). In the main text, we show that the reduced-form (corresponding to C.6) and the structural estimation strategies yield very similar ratios for the estimated income elasticity parameters across the board, making the case for the validity of the assumptions above.

Derivation of Equation (C.5) Let us consider the following moment conditions that are implied by the system OLS estimator: $\mathbb{E}[(Z_t^n)' (C_{it}^n - Z_t^n (\tau_i, \gamma_{i1}, \dots, \gamma_{iI}, \vartheta_{i1}, \dots, \vartheta_{iI})')] = 0$. Expanding this moment condition, we find

$$\mathbb{E} \left[\begin{pmatrix} P_{1t}^n & Z_t^n & 0 & 0 & 1 & 0 & \dots & 0 \\ P_{2t}^n & 0 & Z_t^n & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ P_{It}^n & 0 & 0 & Z_t^n & 0 & 0 & \dots & 1 \end{pmatrix}' \times \begin{pmatrix} -\tau_i P_{1t}^n - \gamma_{i1} Z_t^n - \vartheta_{i1} \\ \vdots \\ X_t^n - \tau_i P_{2t}^n - \gamma_{ii} Z_t^n - \vartheta_{ii} \\ \vdots \\ -\tau_i P_{It}^n - \gamma_{iI} Z_t^n - \vartheta_{iI} \end{pmatrix} \right] = 0.$$

For $j \neq i$, we find the moment condition:

$$\begin{aligned} -\tau_i \mathbb{E}[P_{jt}^n Z_t^n] - \gamma_{ij} \mathbb{E}[(Z_t^n)^2] - \vartheta_{ij} \mathbb{E}[Z_t^n] &= 0, \\ -\tau_i \mathbb{E}[P_{jt}^n] - \gamma_{ij} \mathbb{E}[Z_t^n] - \vartheta_{ij} &= 0. \end{aligned}$$

This implies that ϑ_{ij} / τ_i and γ_{ij} / τ_i are the coefficients corresponding to a linear projection of P_{jt}^n to 1 and Z_t^n , respectively. Defining $P_{it}^n = \chi_i Z_t^n + \delta_i + \xi_{it}^n$ and $X_t^n = (\gamma + \sum_i \eta_i \chi_i) Z_t^n + \nu + \psi_t^n$, this implies $\vartheta_{ij} = -\tau_i \delta_j$ and $\gamma_{ij} = -\tau_i \chi_j$.

The remaining condition give us

$$\begin{aligned}\mathbb{E}[P_{it}^n X_t^n] - \tau_i \mathbb{E}[(P_{it}^n)^2] - \gamma_{ii} \mathbb{E}[P_{it}^n Z_{it}^n] - \vartheta_{ii} \mathbb{E}[P_{it}^n] - \sum_{j \neq i} \mathbb{E}[P_{jt}^n (\tau_i P_{jt}^n + \gamma_{ij} Z_t^n + \vartheta_{ij})] &= 0, \\ \mathbb{E}[Z_t^n X_t^n] - \tau_i \mathbb{E}[Z_t^n P_{it}^n] - \gamma_{ii} \mathbb{E}[(Z_t^n)^2] - \vartheta_{ii} \mathbb{E}[Z_t^n] &= 0, \\ \mathbb{E}[X_t^n] - \tau_i \mathbb{E}[P_{it}^n] - \gamma_{ii} \mathbb{E}[Z_{it}^n] - \vartheta_{ii} &= 0.\end{aligned}$$

Substituting from the expressions above for P_{jt}^n , ϑ_{ij} and γ_{ij} , we find that $\sum_{j \neq i} \mathbb{E}[P_{jt}^n (\tau_i P_{jt}^n + \gamma_{ij} Z_t^n + \vartheta_{ij})] = \tau_i \sum_{j \neq i} \mathbb{E}[(\xi_{jt}^n)^2]$. Substituting this expression in the system above, we find a simple way to interpret the system of moment conditions: the coefficients $(\gamma_{ii}, \vartheta_{ii}, \tau_i)$ correspond to a linear projection of X_t^n into 1, Z_t^n , and a third covariate constructed by summing P_{it}^n with an i.i.d noise (e.g., measurement error) with variance $\sum_{j \neq i} \mathbb{E}[(\xi_{jt}^n)^2]$.

D Comparison with Stone-Geary and PIGL preferences

We compare the cross-country fit of our model to alternative specifications where we replace the nonhomothetic CES aggregator with Stone-Geary and PIGL preferences. A brief discussion of these preferences and estimation is given here. We relegate a detailed discussion to Online Appendix G.

We start considering a generalized Stone-Geary formulation (Herrendorf et al., 2014). These preferences define the intra-period consumption aggregator as

$$C_t^c = \left[\Omega_a^c (C_{at}^c + \bar{c}_a)^{\frac{\sigma-1}{\sigma}} + \Omega_m^c (C_{mt}^c)^{\frac{\sigma-1}{\sigma}} + \Omega_s^c (C_{st}^c + \bar{c}_s)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (\text{D.1})$$

where C_t^c denotes aggregate consumption of country c at time t , $\Omega_i^c > 0$ are constant preference parameters that are country specific, C_{it}^c denotes consumption in sector $i = \{a, m, s\}$, \bar{c}_a and \bar{c}_s are constants that govern the nonhomotheticity of these preferences, and σ is a parameter that tends to the price elasticity of substitution as $C_{it}^c \gg \max\{\bar{c}_a, \bar{c}_s\}$.⁶⁴ We use the first-order conditions of the intra-period problem to estimate the model. As with nonhomothetic CES preferences, we estimate three parameters that are common across countries $\{\sigma, \bar{c}_a, \bar{c}_s\}$ that govern the price and income elasticities and country-specific taste parameters $\{\Omega_i^c\}_{i \in \mathcal{I}, c \in C}$.

Our estimation results (reported in Table G.1 of the online appendix) imply that the three sectors are gross complements and that nonhomotheticities are significantly different from zero and of the expected sign, $\bar{c}_a < 0$ and $\bar{c}_s > 0$. To assess the goodness of fit, we compute the within- R^2 for the predicted time path of employment shares in agriculture, manufacturing and services relative to a model with only country-sector fixed effects.⁶⁵ We find that the within- R^2 of Stone-Geary is 0.14, meaning

⁶⁴Since these preferences are not implicitly additive, the price and income elasticities are not independent. In Appendix G.1 we show that the elasticity of substitution between i and j is $\sigma_{ij} = \sigma \eta_i \eta_j$, where η_i 's denote income elasticities.

⁶⁵The within R^2 compares the sum of squared errors of the model fit to the sum of squared errors obtained by using the country-sector average as a prediction (in other words, assuming flat lines at the average country level in Figure 4). Formally, $R^2 = 1 - \frac{1}{I} \sum_{i=1}^I \left(\sum_{t=1}^N (y_{it}^c - \hat{y}_{it}^c)^2 / \sum_{t=1}^N (y_{it}^c - \bar{y}_i^c)^2 \right)$ where N denotes the total number of observations per sector, I , the number of sectors, y_{it}^c , observed employment shares in sector i and country c , \hat{y}_{it}^c , predicted employment shares, \bar{y}_i^c the sample average of y_{it}^c for country c in sector i , and $i \in \mathcal{I} = \{a, m, s\}$. Note also that Stone-Geary and nonhomothetic CES collapse to the same demand system

that 14% of the residual variation in sectoral shares after we partial out country-sector averages is accounted for by the Stone-Geary demand system. The corresponding number for nonhomothetic-CES is 0.29. The intuition for the worse fit of Stone-Geary is that income effects are very low for rich countries, since for high levels of income the subsistence levels responsible for introducing the nonhomotheticity $\{\bar{c}_a, \bar{c}_s\}$ are negligible.⁶⁶ Thus, only variation in prices (and trade shares) are left to account for the variation in employment shares for rich countries, and the model can miss a substantial part of structural change. This is illustrated in Figure 4 for Japan.⁶⁷

Next, we study the cross-country fit of PIGL preferences as specified in Boppart (2014b). This preference structure features a homothetic CES aggregator between agriculture and manufacturing with price elasticity σ and a nonhomothetic aggregator between services and the agriculture-manufacturing composite. The within-period indirect utility V of a household with total expenditure E^c in country c is

$$V = \frac{1}{\varepsilon} \left(\frac{E^c}{p_{st}^c} \right)^\varepsilon - \frac{\Omega_s^c}{\gamma} \cdot \frac{\left(\Omega_a^c \cdot (p_{at}^c)^{1-\sigma} + \Omega_m^c \cdot (p_{mt}^c)^{1-\sigma} \right)^{\frac{\gamma}{1-\sigma}}}{(p_{st}^c)^\gamma} - \frac{1}{\varepsilon} + \frac{\Omega_s^c}{\gamma}, \quad (\text{D.2})$$

with $0 \leq \varepsilon \leq \gamma < 1$ and $\Omega_i^c > 0$ for $i \in \{a, m, s\}$. The nonhomotheticity and price elasticity between services and the agriculture-manufacturing CES composite are governed by two parameters, ε and γ . The nonhomotheticity is not vanishing as income grows, and the price elasticity grows with income but is bounded above by 1.⁶⁸

We use the demand implied by these preferences to estimate the demand parameters. As with nonhomothetic CES and Stone-Geary, we estimate three elasticities that are common across countries $\{\varepsilon, \gamma, \sigma\}$ and we allow for country-specific constant taste parameters, $\{\Omega_i^c\}_{i \in \mathcal{I}, c \in \mathcal{C}}$. We find that, at our estimated parameter values, manufacturing and agriculture are gross complements and nonhomotheticities are significantly different from zero. In fact, the nonhomotheticity parameter that we estimate is similar in magnitude to the U.S. estimate reported in Boppart (2014a) (see table G.3). The overall fit of the PIGL demand system as measured by the within R^2 is similar in magnitude

in the case that we set $\sigma = 1$ and $\bar{c}_i = 0$ or $\epsilon_i = 1 - \sigma$. In this case, differences across countries and sectors would be only accounted for through Ω_i^c , which would be exactly the country-sector average levels that we use as a reference for the within R^2 (this is also true for PIGL). Finally, we also note that the estimates used to compute the within- R^2 for nonhomothetic CES correspond to the structural estimates in column (1) of Table 6.

⁶⁶To have a better grasp of the magnitude of the income effects, we compute the values of $\frac{p_{a,t}^c \bar{c}_a}{\sum_{i \in \{a, m, s\}} p_{it}^c C_{it}^c}$ and $\frac{p_{s,t}^c \bar{c}_s}{\sum_{i \in \{a, m, s\}} p_{it}^c C_{it}^c}$ which are the nonhomothetic part of the demand function. For the U.S., they are never higher (in absolute terms) than .1%, which suggests that nonhomotheticities are insignificant when compared to aggregate consumption. The highest values of the nonhomotheticities in the sample are 37% and 18% for services.

⁶⁷We report the fit for all countries for both Stone-Geary and PIGL in Online Appendix H.

⁶⁸The parameter ε governs the nonhomotheticity of preferences between services and the composite of agricultural and manufacturing goods. If $\varepsilon > 0$, the expenditure elasticity is larger than one for services and less than one for agricultural and manufacturing goods (and identical for both). The price elasticity of substitution between services and the agriculture-manufacturing composite never exceeds one, it is increasing with the level of income and it asymptotes to $1 - \gamma$. The baseline model in Boppart contains only two sectors. Here we follow the extension proposed in Appendix B.3.3 (Boppart, 2014b) to account for three sectors such that there can be a hump-shape in manufacturing. We have generalized the demand to allow for constant taste parameters heterogeneous across countries and not symmetric between agricultural and manufacturing goods. We have also experimented with another proposed extension such that the expenditure share in the manufacturing sector constant (Appendix B.3.2) obtaining a worse fit.

to Stone-Geary, 0.13. As illustrated in Figure 4 for the case of Japan, PIGL preferences track the trends in services more accurately than Stone-Geary due to the fact that they feature a non-vanishing nonhomotheticity of the service sector. However, they under-perform relative to nonhomothetic CES mostly because they assume a homothetic composite between agriculture and manufacturing, while nonhomothetic CES allows for sector-specific nonhomotheticities.

E Value Added Expenditure Evidence from the U.S.

So far, we have assumed that utility is defined over households' final-good expenditure. In this section, we extend our analysis in two dimensions. First, we estimate our model defining household utility over the value added provided by each sector (rather than final good expenditure). The relevance of this extension stems from the fact that structural change is qualitatively similar when measuring sectoral economic activity with value added shares or final good expenditure shares (Herrendorf et al., 2013). Therefore, we want to verify that our model can account for structural change when using this alternative representation. Second, we present the aggregate counterparts for the baseline regressions in final good and value added. Even though our model does not satisfy the necessary conditions for Gorman aggregation, we find it useful to compare the household-level estimates to the macro estimates.

Micro Evidence Columns (1) to (4) in Table 10 present the estimates of our baseline specification (29) where we use as dependent variable household expenditure shares measured in value added (instead of final good expenditure). Thus, they are the value-added counterparts of our baseline estimates in Table 1. Note that we use the same set of instruments in this regression. We find that the estimates of $\epsilon_a - \epsilon_m$ are around -0.56 , while the estimates of $\epsilon_s - \epsilon_m$ are between 0.50 and 0.53 . The estimates do not vary once we re-weight by expenditure, suggesting again that estimates are stable across the income distribution.⁶⁹ The point estimates we obtain for the price elasticity are in the 0.4 to 0.5 range. Thus, overall, the point estimates appear to be quite similar to the formulation with final good expenditure in Table 1.

Macro Evidence Columns (5) and (6) perform the same estimation (equation 29) using U.S. aggregate data with expenditure shares specified over value added and final good expenditures, respectively.⁷⁰ The estimates for value-added shares reported in column (5) are -0.63 for $\epsilon_a - \epsilon_m$ and 0.62 for $\epsilon_s - \epsilon_m$. They have the expected sign and similar in magnitude to the micro estimates. In fact, the micro estimates albeit somewhat smaller lie within the 95% confidence interval of the macro estimates. The same is true for the estimate of 0.57 for the price elasticity.

Column (6) reports the estimates for expenditure shares. In this case, we find that the elasticity of agriculture to manufacturing, $\epsilon_a - \epsilon_m$, is -0.87 and an elasticity of services to manufacturing, $\epsilon_s - \epsilon_m$, 0.39 . Comparing these estimates to the ones we obtain in Table 1, we find that the estimate $\epsilon_a - \epsilon_m$

⁶⁹In Table 10 we only report column (2) with expenditure-weighted estimates. The re-weighted counterparts of columns (3) and (4) yield very similar estimates and are omitted. In Table E.3 in the online appendix we also report the estimates using the exact price index.

⁷⁰This exercise is analogous to Herrendorf et al. (2013) with the difference that we estimate the sectoral allocation implied by nonhomothetic CES demand rather than Stone-Geary. In fact, we use the data provided by Herrendorf et al. (2013). The data are constructed from the Bureau of Economic Analysis aggregate data and input output tables. See Herrendorf et al. for the details.

Table 10: CEX Value Added and U.S. Macro Estimates

	Household VA from CEX				Macro	
	(1)	(2)	(3)	(4)	VA	Exp.
$\epsilon_a - \epsilon_m$	-0.56 (0.03)	-0.57 (0.03)	-0.56 (0.03)	-0.56 (0.06)	-0.63 (0.12)	-0.87 (0.04)
$\epsilon_s - \epsilon_m$	0.53 (0.03)	0.51 (0.03)	0.51 (0.03)	0.50 (0.02)	0.62 (0.06)	0.39 (0.09)
σ	0.42 (0.02)	0.45 (0.02)	0.43 (0.02)	0.49 (0.17)	0.57 (0.10)	0.78 (0.09)
Expenditure Re-Weighted	N	Y	N	N	—	—
Region FE	N	N	Y	Y	—	—
Year \times Quarter FE	N	N	N	Y	—	—

Standard errors clustered at the household level in (1) to (4). Number of observations is 44268 in columns (1) to (4) and 63 in (5) and (6).

is almost twice as large, while it is almost identical for $\epsilon_s - \epsilon_m$. The price elasticity we find is 0.78, which is also larger than what we obtain for household level data.⁷¹

Herrendorf et al. (2013) use the same aggregate data to estimate the demand system that arises from Stone-Geary preferences. They also find that the elasticity of substitution is greater when estimating the model with expenditure than with value added data.⁷² Their estimate of σ with expenditure data is 0.85, which is similar to ours (0.78). However, their estimate of σ with value added data is close to (and not significantly different from) zero.⁷³ This result is in contrast with our estimated price elasticity of 0.57. The intuition for this discrepancy is as follows. Expenditure shares in services raise at a faster rate than the relative price of services. The Stone-Geary demand system imposes that the income effects become less important as aggregate consumption grows. This implies that the estimation has to load the late increase in service expenditure to increases in the relative price of services. Thus, as the relative prices of services grows at a slower rate than value added, the estimation selects the minimal price elasticity to maximize the explanatory power of relative prices. In contrast, the non-homothetic CES does not impose declining income effects. As a result, both income and price effects help account for the secular increase in expenditure shares in services and the estimation does not need to select a price elasticity of zero.

⁷¹As discussed above, there is no a priori reason for these estimates to coincide since our preference do not belong to the Gorman form.

⁷²To be precise, the parameter σ they estimate does not exactly coincide with the elasticity of substitution unless the non-homothetic terms are negligible or σ tends to zero.

⁷³Buera and Kaboski (2009) report a similar finding for the period 1870-2000 when also estimating Stone-Geary preferences.

F Additional Tables and Figures

Figure 6: Role Played by Different Components of the Regression: the Case of Japan

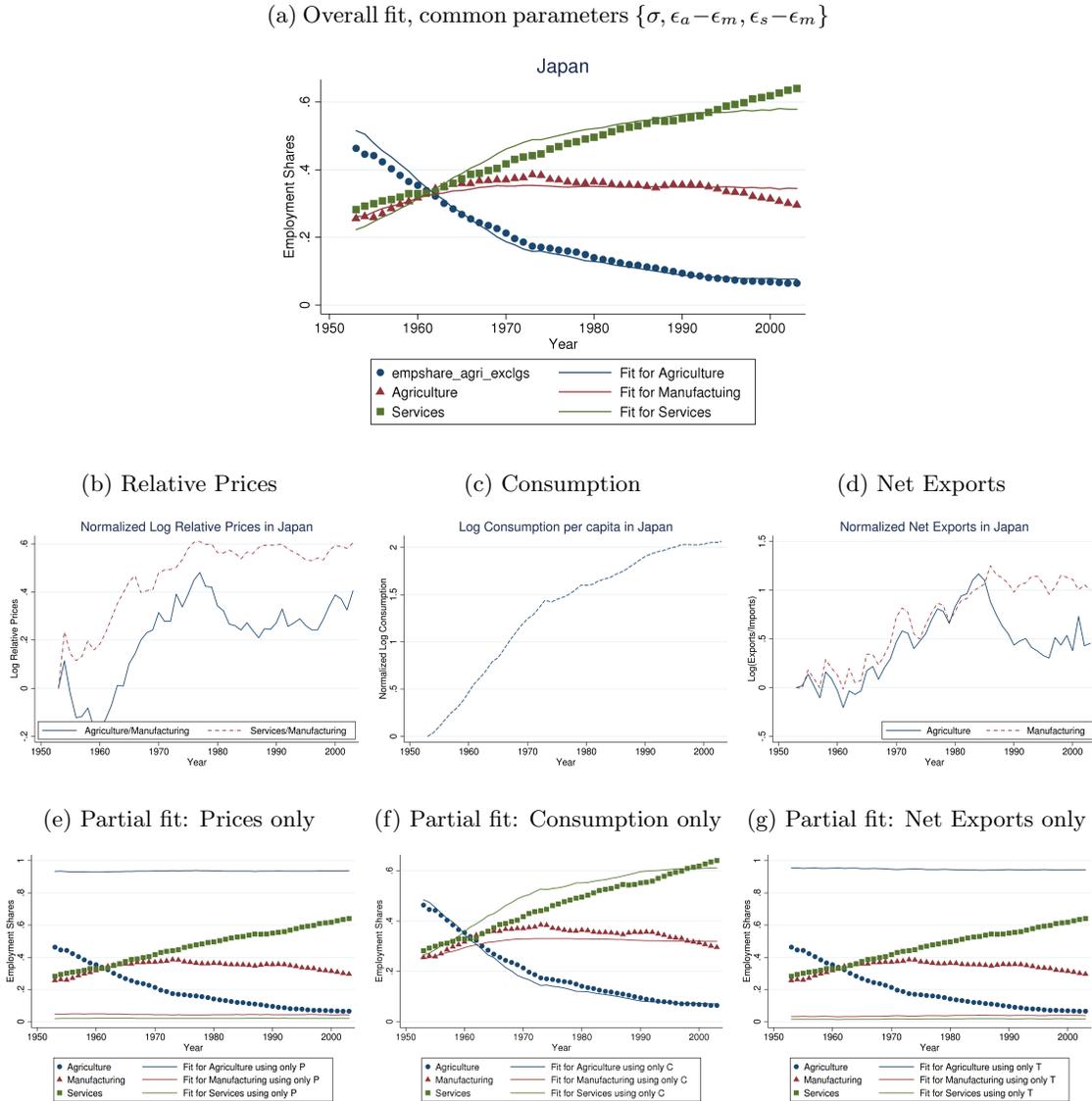


Table 11: 4-Sector Decomposition, CEX Expenditure

	(1)	(2)	(3)	(4)	(5)	(6)
σ	0.53 (0.04)	0.56 (0.04)	0.49 (0.04)	0.51 (0.04)	0.58 (0.25)	0.52 (0.28)
$\epsilon_a - \epsilon_m$	-0.40 (0.02)	-0.40 (0.02)	-0.39 (0.02)	-0.40 (0.02)	-0.41 (0.02)	-0.42 (0.02)
$\epsilon_s - \epsilon_m$	0.45 (0.02)	0.45 (0.02)	0.43 (0.02)	0.43 (0.02)	0.46 (0.02)	0.46 (0.03)
$\epsilon_{\text{housing}} - \epsilon_m$	0.68 (0.03)	0.69 (0.03)	0.68 (0.03)	0.69 (0.03)	0.69 (0.03)	0.70 (0.03)
Expenditure Re-Weighted	N	Y	N	Y	N	Y
Region FE	N	N	Y	Y	Y	Y
Year \times Quarter FE	N	N	N	N	Y	Y

Note: All regressions have 58655 observations. Standard errors clustered by household shown in parenthesis. Housing includes expenditures in shelter and utilities.

Table 12: Instrumental Variables Strategy
Consumption Expenditure Survey, 2001-2002

	(1)	(2)
$\epsilon_a - \epsilon_m$	-0.58 (0.35)	-0.73 (0.35)
$\epsilon_s - \epsilon_m$	0.50 (0.49)	0.26 (0.40)
σ	0.50 (0.61)	0.44 (0.55)
Expenditure Re-Weighted	N	Y
Household Charact.	Y	Y
Region FE	N	N
Year and Quarter FE	N	N

Note: Standard Errors clustered at the household level.

Table 13: 10-Sector Regression

	World	OECD	Non-OECD
Price Elasticity σ	0.84 (0.04)	0.86 (0.04)	0.86 (0.04)
Income Elasticity (relative to Manufacturing)			
Agriculture	-0.93 (0.09)	-0.79 (0.16)	-1.01 (0.11)
Mining	-0.92 (0.21)	-1.11 (0.16)	-0.79 (0.31)
Public Utilities	0.05 (0.10)	0.12 (0.07)	0.00 (0.15)
Transp., Storage, Communications	0.08 (0.09)	0.33 (0.10)	-0.05 (0.13)
Government Services	0.11 (0.12)	0.53 (0.21)	-0.11 (0.14)
Construction	0.26 (0.10)	0.27 (0.09)	0.28 (0.14)
Community, Social and Personal Serv.	0.26 (0.15)	0.85 (0.25)	-0.06 (0.17)
Wholesale and Retail	0.38 (0.11)	0.57 (0.16)	0.27 (0.14)
Finance, Insurance, Real State	0.73 (0.16)	1.20 (0.22)	0.48 (0.20)
R^2	0.91	0.91	0.87
Observations	1626	492	1134

Note: All sectoral elasticities computed relative to Manufacturing. Standard errors clustered at the country level. All regressions include a sector-country fixed effect.