SPECIAL FOCUS 2

Quantifying Uncertainties in Global Growth Forecasts
Global and regional growth projections have an important bearing on the assessment of individual country prospects and policy choices. However, these projections are subject to a range of uncertainties that could also influence policy decisions. Such uncertainties around baseline forecasts could be caused by low-probability but high-impact events, persistent forecast errors in models or expert judgment, or heightened volatility around economic turning points or during episodes of financial stress. The likelihood of actual outcomes deviating from projections is therefore significant, and might vary over time. Policy makers need to be informed about risks prevailing at the time of the forecast, and how these risks translate into confidence intervals around baseline projections.

A quantification of risks around global growth forecasts can be achieved in different ways. A first approach is to look at past prediction errors as a rough guide to likely future forecast deviations. This provides an objective but static and unconditional measure of uncertainty, which does not reflect current circumstances that might affect forecast errors. A second approach to partly address this shortcoming is to undertake scenario analysis. In this case, results will be largely dependent on the properties of the specific model used for simulations whereas most institutions derive their baseline forecasts from a variety of models and expert judgment. Measures of uncertainty should reflect this process, linking actual forecast errors with uncertainty regarding underlying assumptions.

This Special Focus essay derives confidence intervals around global growth forecasts by mapping the distribution of forecast errors to that of selected risk factors; including option prices on equities and oil prices as well as consensus forecasts for term spreads, i.e., the difference between long-term and short-term interest rates), across G20 economies (which account for 64 percent of global GDP). Signals from the market-implied or consensus forecast distribution of these forward-looking indicators are extracted and weighted to derive a fan-chart around global growth projections.

This Special Focus describes the fan chart approach, answering the following three questions:

1. What are the selected risk indicators used to assess forecast uncertainty?
2. How can different risk factors be combined in a single fan chart?
3. What is the current balance of risks to global growth?

**Selected risk indicators**

Various market— and survey-based indicators have been suggested as useful measures of forecast uncertainty. In particular, the pricing of options used by investors to hedge can provide information on market perception of underlying risks (Moschini and Lapan 1995; Carter 1999) and has predictive power in forecasting future uncertainty of the underlying assets (Christensen and Prabhala 1998; Andersen and Bondarenko
Equity prices. Equity prices futures—especially the Standard and Poor’s S&P500 Index—are positively correlated with prospects for the U.S. and global economy.

Term spreads. Term spreads (difference between long and short-term nominal interest rates) embed both inflation and real equilibrium interest rate expectations, both of which are tightly connected to growth prospects. A global term spread is proxied by GDP-weighted term spreads across G20 economies.

Average of Brent and WTI crude oil forward prices. Abrupt changes in oil prices make growth prospects more uncertain. A supply-driven decline in oil prices tends to improve global growth prospects.

For each risk factor, its forecast distribution captures both the degree of uncertainty and the balance of risks:

- The degree of uncertainty surrounding each risk factor is captured by the dispersion of its forecast distribution. The dispersions for the Brent and WTI prices and S&P 500 returns risk factors are measured by the implied volatility of at-the-money forward option prices. For the June GEP forecast vintages, the 6-month maturity implied volatility is used for current year forecast whereas 18-month maturity implied volatility is used for next year forecast. For global term spread, the dispersion is computed from the monthly Consensus Economics survey for each country of the G20 and aggregated using real GDP weights.

- The balance of risks for each factor is captured by the skewness of its forecast distribution. A negative skewness indicates a balance of risks that is tilted to the downside. The skewness of the risk-neutral probability distributions of option price on S&P 500 returns as well as on Brent and WTI prices are approximated from the slope of their respective implied volatility curves, following the methodology of Mixon (2011). For the term spread, the skewness is computed from the monthly Consensus Economics survey for each country of the G20 and aggregated using real GDP weights.

Several episodes of heightened uncertainty stand out from the analysis of these risk factors (Figure SF 2.1). The first one is the global financial crisis of 2008-09. Its unexpected severity was associated with financial market disruptions and a broad-based increase in volatility and risk aversion (Stock and Watson 2012; Allen, Bali, and Tang 2012). This was also reflected in the rising degree of uncertainty of all three risk factors. The second and third, albeit milder, episodes were around intensifications in the Euro Area sovereign debt crisis in 2011 and 2012, when financial market indicators also pointed towards a greater level of uncertainty surrounding global growth forecasts. Recent episodes of market stress, such as those associated with the Taper Tantrum in 2013,

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1. Only countries with available data on interest rates are used in the aggregations. These include: Australia, Canada, France, Germany, India, Indonesia, Italy, Japan, Netherlands, Republic of Korea, Sweden, Spain, Switzerland, the United Kingdom, and the United States.

2. The skewness is approximated based on the following measure of implied volatility skew: (25 percent Delta Call implied volatility-25 percent Delta Put implied volatility)/50 percent Delta implied volatility where Delta is the degree to which an option is exposed to shifts in the price of the underlying asset. The framework assumes that the option-implied distribution generates a volatility curve that is linear in delta. This model is considered as more empirically plausible than one assuming linearity in the percentage strike model (Mixon 2011). Among measures based on the slope of the implied volatility curve, this measure is the least correlated with the level of implied volatility. Robustness checks are undertaken with alternative skew measures such as the widely used (90 percent moneyness implied volatility – 110 percent moneyness implied volatility)/100 percent moneyness implied volatility measure, with largely similar results.

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3. The data sample is from January 2006 to April 2016.

4. Ideally, the fan chart would extend into 2018. However, reliable market-based indicators derived from liquid option markets are not available at this horizon.
sharply declining oil prices since mid-2014 and the ongoing emerging market slowdown have also raised forecast uncertainty. Around these episodes, downside risks to growth have been more prevalent.

Risk indicators and global growth

To characterize the evolution of uncertainty around global growth forecasts, a similar approach to that proposed by Blix and Sellin (1998) and Kannan and Elekdag (2009) is used. More specifically, changes in the degree of uncertainty (dispersion) and balance of risks (skewness) of underlying risk factors are used to assess the potential size and direction of forecast errors at any point in time.

In order to calculate the degree of uncertainty and balance of risks of global growth forecasts from the selected risk factors, a number of assumptions are needed regarding the functional form of their respective distributions, as well as the weight given to individual risk factors. In line with other authors (Blix and Sellin 1998 and Kannan and Elekdag 2009), a two-piece normal distribution is used to characterize both global growth forecasts and individual risk factors. The uncertainty and balance of risks (measured by the dispersion and skewness) of global growth forecasts is recovered from the corresponding statistics of the distribution of the three risk factors, assuming a linear relationship between them.

To aggregate risk factors into a measure of global risk, weights of each risk factor need to be estimated. A first option is to use model simulations to estimate the risk weights (Österholm 2009; Michal et al 2014; Alvaro and Maximiano 2003). This consists of simulating the forecast distribution under alternative scenarios and then minimizing the distance between the baseline forecast distribution and a weighted average of the distributions under each scenario. This approach provides a useful illustration to discuss forecast uncertainty, but depends heavily on individual model properties and scenario assumptions.

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**FIGURE SF2.1 Uncertainty and balance of risks for risk factors**

**Uncertainty.** The implied current-year volatility of equity price options and next-year volatility of term-spread forecasts have edged up since the second half of 2015 but remain near their historical medians. In contrast, the implied volatility of oil price futures widened to post-crisis highs, pointing to increased uncertainty. **Balance of risks.** The distribution of future equity prices is increasingly tilted to the downside while that of oil price futures is tilted to the upside. Movements in the skewness of term spread forecasts are mixed. Together, these developments suggest higher uncertainty and rising downside risk to global growth for 2016 and 2017.

A. S&P500: Degree of uncertainty

- Implied volatility
  - Current year
  - Next year

B. S&P500: Balance of risks

- Skewness
  - Current year
  - Next year

C. Term spread: Degree of uncertainty

- Standard deviation
  - Current year
  - Next year

D. Term spread: Balance of risks

- Skewness
  - Current year
  - Next year

E. Oil price: Degree of uncertainty

- Implied volatility
  - Current year
  - Next year

F. Oil price: Balance of risks

- Skewness
  - Current year
  - Next year

Sources: World Bank, Bloomberg, Consensus Economics.

Note: Gray areas represent the global financial crisis of 2008-09, the intensifications of the Euro area debt crisis in 2010 and 2012.

A. The implied volatility of option prices on the S&P 500 is recovered using the Black-Scholes formula from 6- and 18-month-ahead put and call option contracts. B. The skewness of option prices on the S&P 500 is approximated using (25 Delta Put volatility-25 Delta Call volatility)/50 Delta volatility where Delta stands for the degree to which an option is exposed to shifts in the price of the underlying asset. This skewness measure is scaled down by a factor of 3 to match the equivalent skewness parameter of the risk neutral density function (Mixon 2011). C.D. The degree of uncertainty (proxied by dispersion) and balance of risks (proxied by skewness) of current and next-year term spread forecasts are compiled from monthly surveys of professional forecasters.
Uncertainty surrounding global growth forecasts has increased since the January 2016 Global Economic Prospects and is slightly above the historical median. Upside risks have decreased while downside risks for the current year have reached post-crisis highs. The probability of a 1 percentage point decline below current global growth projections in 2016 is estimated at 12.5 percent.

A. Risks to global growth

B. Uncertainty of global growth forecasts

C. Contribution of risk factors to forecast uncertainty

D. Balance of risks to global growth forecasts

E. Contribution of risk factors to the balance of risks to global growth

F. Probability of global growth being 1 percentage point below baseline forecasts

Sources: World Bank, Bloomberg, Consensus Economics.


A. “90 percent in JAN16” is the 90 percent confidence interval of a fan chart based on data available for the January 2016 Global Economic Prospects report.

B. Dispersion is measured by the standard deviation. Gray areas represent the global financial crisis of 2008-09 and the intensification of the Euro area debt crisis in 2010 and 2012.

C. “Other factors” denotes the contribution of own shocks of global growth forecast error in VAR variance decomposition.

D. The balance of risk is measured by the skewness. Gray areas represent the global financial crisis of 2008-09 and the intensification of the Euro area debt crisis in 2010 and 2012.

In Kannan and Elekdag (2009) the risk factors selected include inflation, term spread, S&P500 and oil prices. Inflation is excluded here for two reasons: first, changes in oil prices will eventually feed into inflation and, second, monetary policy tightening risk in response to increases in inflation is captured by the term spread.

Kannan and Elekdag (2009) and Blix and Sellin (1998) use a simpler approach, estimating elasticities of global growth with respect to risk factors by running an OLS regression on each risk factor individually. The same OLS coefficients are used as weights for the skewness and variance at all forecast horizons, ignoring any lag structure and potential interactions between risk factors.

However, the lag structure and interactions may matter. For example, the performance of the yield curve predictor of future growth depends on the forecast horizon (Ang et al. 2006; Wheelock and Wohar 2009). More refined approaches to estimating the risk weights include Bayesian methods (Cogley et al. 2005) and Vector Autoregressive analysis (Novo and Pinheiro 2003, Smets and Wouters 2004).

The baseline approach adopted here departs from the OLS-based approach proposed by Kannan and Elekdag (2009) in two ways: in the selection of the risk factors and in the estimation of the weight parameters used in the aggregation of risk factors. In the approach used here, the weights assigned to each risk factor in the aggregation into global risks are different and vary over the forecast horizon. The weights for the computation of the global growth forecast uncertainty are estimated as the share of the variance of the global growth forecast error explained by each risk factor at various forecast horizons (see technical discussion in Annex SF2.1).

Instead of using the same weights for uncertainty and balance of risks (as in the OLS-based approach), the weights to aggregate the balance of risks of individual risk factors into a global balance of risks are the impulse responses of global growth to each risk factor at different forecast horizon. This approach allows individual risk factors to tilt the global balance of risks differently at different forecast horizons. The variance decomposition and impulse responses are derived from the recursive identification also used in the analysis of...
spillovers (World Bank 2016a). Using this weighted average global uncertainty and balance of risks, a fan chart can be drawn around the baseline global growth forecast (Figure SF 2.2).

**Balance of risks to global growth**

The resulting fan chart shows confidence intervals at 50, 80 and 90 percent probability around the growth projections in the June 2016 *Global Economic Prospects* (Figure SF 2.2). The fan chart uses information available up to May 2016. It illustrates that the uncertainty surrounding global growth forecasts has risen marginally above the long-term average and increasingly tilted to the downside.

The period around the global financial crisis in late 2008– early 2009 illustrates the risks captured in the fan chart. After sharp corrections in most asset prices (financial, housing, or commodities), the balance of risks may have been on the upside. However, this bias was negligible compared to the record-high uncertainty that opened up.

Uncertainty about growth forecasts for 2016 and 2017 is estimated to be near the historical median but has increased since early January 2016, especially for 2017. This reflects heightened volatility in oil prices, term spreads and equity prices since the start of 2016. That said, forecast uncertainty remains significantly below levels observed during the Euro Area crisis in late 2011, let alone the global financial crisis of 2008-09. The balance of risks is tilting increasingly to the downside for 2016. Rising downside risks reflect, especially, growth concerns captured in falling equity price futures.

Compared to the January 2016 projection, upside risks to the baseline forecast have decreased, with equity markets suggesting a lower probability of strengthening growth. For 2017, risks may be turning more balanced. Downside risks to oil prices could boost global growth in 2017 and expectations for rising term spreads may signal receding recession risks. Similarly, a sharp deterioration in investor sentiment could disrupt financial markets and present a downside risk to global growth.

Some of the downside risks have materialized since the January forecasts, resulting in forecast downgrades for 2016 and 2017. That said, the probability of global growth being 1 percentage point lower than currently projected in 2017 remains around the average for the decade (19 percent as of May 2016), still well below the probability in 2008 on the eve of the global financial crisis (26 percent). The probability of growth falling to or below 1 percent (the growth rate likely associated with global recession) is somewhat above the 10-year average for 2006-16.

**Conclusion**

A complete assessment of global economic prospects requires baselines forecasts as well as an assessment of risks. The latter conveys to policymakers a sense of the uncertainty prevailing at the time of forecasting, which might vary with incoming data, past forecast performance, and changing expectations. In this special focus three questions have been addressed:

- What are the selected risk indicators used to assess forecast uncertainty? Three risk factors were chosen for their tight connection with global growth prospects: equity prices, term spreads, oil prices. Changes in the distribution of forecasts for these underlying risk factors are mapped into a distribution of risks for global growth.
- How can different risk factors be combined in a single fan chart? Signals extracted from the distribution of individual risk factors are

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6 The ordering of the variables used in the Cholesky decomposition is as follows: global term spread, stock-market returns (S&P500), oil prices, and global growth. The variance decomposition results show that, historically, other factors not included in the analysis explain more the variance of global growth forecast errors than the three selected risk factors in the short-run (see Annex Table SF2.1).

7 The risk weights can be adjusted to reflect judgment when there are significant divergences between market perceptions and World Bank Group assessments of risks (Blix and Sellin 1997).

8 The probability of global growth being 1 percent lower than currently projected for next year averaged 18 percent during 2006 and 2015, peaking at 26 percent in 2008.

9 The probability of growth falling below 1 percent in 2006-16 average (excluding the global recession 2009) of 3.5 percent for the current year and 10 percent for the next year.
aggregated using weights estimated from a vector autoregression model of global growth on the risk indicators. This approach allows individual risk factors to impact forecast uncertainty and the balance of risks differently at different forecast horizons.

• What is the current balance of risks to global growth? Uncertainty about growth forecasts for 2016 and 2017 is estimated to be near the historical median but has increased since early January 2016, especially for 2017. The balance of risks to global growth forecasts for 2016-17 has tilted further to the downside since January 2016.

Some downside risks have materialized since the January 2016 forecasts. As a result, a forecast downgrade has accompanied rising uncertainty around global growth forecasts and a balance of risks that is increasingly tilted to the downside. Given the already-weak global growth prospects in 2016, the probability of global growth falling to or below 1 percent in 2016 is above its historical average.
ANNEX SF2.1 Estimating the distribution of the global growth forecast

This annex provides the technical details of assessing the uncertainty surrounding the GEP global growth forecast. For computational tractability and in line with previous authors, a two-piece normal distribution is used to characterize both global growth forecasts and individual risk factors.\textsuperscript{10} The assumption of a two-piece normal distribution for global growth allows asymmetry to be captured by a combination of the mode and standard deviation of two individual normal distributions. The skewness and standard deviations of the risk factors are directly computed from the distributions of the market-based and survey-based data.

Assessing uncertainty in the global growth forecast

The degree of uncertainty surrounding the forecast points relative to historical levels of uncertainty is measured by the mean square errors of historical forecast. As in Kannan and Elekdag (2009) and Blix and Sellin (1998) global growth risk is assumed to be based on an assessment of the selected risk factors.

Broadly in line with Kannan and Elekdag (2009), the selected global risk factors include risks to oil prices, global stock markets (as proxied by the S&P500 index) and a GDP-weighted average of term spreads in G20 countries (see more extensive discussion in the main text of this Special Focus).\textsuperscript{11}

- Equity prices. Equity prices futures—especially the Standard and Poor’s S&P500 index—are positively correlated with prospects for the U.S. and global economy. Their implied volatility signals changes in global investors’ risk aversion (Adrian and Shin 2014, Bekaert, Hoerova and Lo Duca 2013). The implied volatility of the S&P 500 returns at 6- and 18-months-ahead option contracts at 100 percent moneyness is obtained from Bloomberg and used as a proxy for equity market uncertainty.

- Term spreads. Term spreads (difference between long and short-term nominal interest rates) embed both inflation and real equilibrium interest rate expectations, which are tightly connected to growth prospects (Cieslak and Povala 2014). A rapid decline in term spreads is seen as a predictor of increased recession risks.\textsuperscript{12} The dispersion of term spread forecasts therefore captures uncertainty surrounding growth prospects while a left hand shift in their distribution signals a predominance of downside risks. Current and next-year term spread forecasts are compiled from monthly surveys published by Consensus Economics from January 2006 to May 2016. A global term spread is proxied by GDP-weighted (at 2010 prices and exchange rates) term spreads across G20 economies.

- Average of Brent and WTI crude oil forward prices. Abrupt changes in oil prices make growth prospects more uncertain. A supply-driven decline in oil prices raises global growth prospects (Baffes et al 2015; Kilian 2014). As in the case of the S&P 500, the implied volatility at 100 percent moneyness of Brent and WTI crude oil prices at 6- and 18-months-ahead option contracts are obtained from Bloomberg and used as a proxy for crude oil market uncertainty. Crude oil prices implied volatility is obtained by taking a simple average of that of Brent and WTI.

\textsuperscript{10}For more properties of the two-piece normal distribution, see Kannan and Elekdag (2009).

\textsuperscript{11}Kannan and Elekdag (2009) also include U.S. inflation as a risk factor to proxy the risks to U.S. monetary policy. However, for most countries, especially emerging markets and developing countries (EMDE), the most immediate risks to monetary policy are already captured by equity prices and term spreads.

Changes in risk stemming from these risk factors signal a change in the distribution of global growth forecast. The current measure of the $h$-period ahead global growth forecast uncertainty $\sigma^2(h_k)$ is assumed to be linearly related to its historical level as:

$$\sigma^2(h_k) = \phi(h_k) \sigma^2_{hist}(h_k) \quad (1),$$

where $\sigma^2_{hist}(h_k)$ is the mean square errors of the historical global growth forecast and $\phi(h_k)$ is a scaling parameter for a given forecast horizon $h$ calculated as:

$$\phi(h) = \beta_0(h) + \sum_j \beta_j(h) \frac{\sigma_j^2(h)}{\sigma^2_{hist}(h)} \quad (2),$$

where $\sigma_j^2(h)$ is the current measure of $h$-period ahead forecast uncertainty extracted from risk factor $j$ and $\sigma^2_{hist}(h)$ the corresponding historical measures and $\beta_j(h)$ is the weight (defined to be positive) of factor $j$ in explaining the forecast errors variance of global growth. $\beta_0(h)$ is the weight of other factors not included in the analysis. By construction, the weight parameters for a given forecast horizon $h$ are constrained to add up to 1:

$$\beta_0(h) + \sum_j \beta_j(h) = 1 \quad (3)$$

The parameter $\phi$ amplifies or dampens historical uncertainty by the uncertainty surrounding individual risk factors. A $\phi$ responsive to variations in uncertainty of the risk factors adds objectivity to the assessment of global growth uncertainty. Notice that subjective judgements can be allowed in the computation of the parameter $\phi$ to reflect the forecaster’s view of the current state of uncertainty (Blix and Sellin 1997). For example, if for any reason the forecaster view is different from the market predictions, the parameter $\phi$ can be modified to allow the forecaster to discount the signals extracted from the market.

A starting point (baseline) of the analysis would be the case where $\phi = 1$. In this case global growth forecast uncertainty is equal to its historical level.

When $\phi > 1$, current information on risk factors (market- and survey-based) signals that global growth forecast uncertainty is larger than historical uncertainty and vice versa when $\phi < 1$. The $\phi$ intuition behind the expression of is that an increase in uncertainty of the risk factors relative to their historical levels will increase $\phi$ and thus signal an increase in uncertainty of global growth. For example, if there is no change in uncertainty of all risk factors relative to their historical levels—that is $\sigma_j^2 = \sigma^2_{hist}$ —this would imply that $\phi = 1$ and that the global growth forecast uncertainty remains unchanged relative to historical levels.

The weight parameters $\beta_j$ are estimated as the shares of the variance of global growth forecast errors explained by forecast error of risk factor $j$. This is calculated in the variance decomposition at a given horizon $h$ of global growth forecast error in a vector autoregression (VAR) with orthogonalized error terms:

$$\sigma^2(h_k) = \sigma^2_{hist}(h_k) + \sum_j \sigma_j^2(h) \quad (4),$$

where $\sigma^2_{hist}(h_k)$ is the variance of global growth own $h$-period ahead forecast error, that is, the forecast errors due to other factors of global growth not included in the analysis.

Both sides of equation (4) are divided by the historical uncertainty of global growth $\sigma^2_{hist}$ for a given forecast horizon:

$$\frac{\sigma^2(h_k)}{\sigma^2_{hist}} = \frac{\sigma^2_{hist}(h_k)}{\sigma^2_{hist}} + \sum_j \left( \frac{\sigma^2_{hist}(h_k)}{\sigma^2_{hist}(h_k)} \right) \frac{\sigma^2_j(h)}{\sigma^2_{hist}(h_k)} \quad (5).$$

Equations (5) and (1) – (2) are equivalent with the terms $\beta_j = \frac{\sigma^2_{hist}(h_k)}{\sigma^2_{hist}(h_k)}$ and $\beta_0 = \frac{\sigma^2_{hist}(h_k)}{\sigma^2_{hist}(h_k)}$ as the shares of variance of the global growth forecast errors at a given horizon explained by risk factor $j$ and by global growth’s own forecast error, respectively. Estimates of these parameters can be obtained by a variance decomposition analysis in a VAR including global growth and the selected risk factors. Notice that in the analysis, the contribution of other factors to the uncertainty of global growth forecast at a given horizon $h$ —that is, $\frac{\sigma^2_j(h)}{\sigma^2_{hist}(h_k)}$—is kept constant at its historical average.
Assessing the balance of risk to global growth

It is assumed that the skewness of the global growth distribution can be approximated as a linear combination of the skewness of the selected risk factors as in Blix and Sellin (1998). Denote by \( \gamma(h) \) and \( \gamma_j(h) \) the skewness of \( h \)-period ahead forecast of global growth and risk factor \( j \) respectively:

\[
\gamma(h) = \sum_j a_j(h) \gamma_j(h) \tag{6}
\]

where \( a_j(h) \) is the weight associated with the risk factor \( j \) at the forecast horizon \( h \).\(^\text{13}\) Equation (6) is an approximation of the true forecast error skewness. The parameter \( a_j(h) \) can be thought of as the elasticity of \( h \)-period ahead global growth with respect to risk factor \( j \). Notice that the contribution of each risk factor to the skewness of global growth depends on both its skewness and weight. The weight parameters are thus as important as the skewness of the risk factors in the estimation of the balance of risk of global growth.

Estimating risk weights

As baseline, a vector autoregression (VAR) approach is used. The VAR includes the global term spread, the first difference in the log S&P500 Index, the log of crude oil prices (de-trended), and global growth. Two lags are selected based on one or more information criteria. Impulse responses and variance decompositions are evaluated at the forecast horizons of 2, 4, 6, and 8 quarters (Annex Table SF2.1). Quarterly data from 1991Q1-2015Q3 are used in the estimation.

- **Estimating weights for skewness.** The weight assigned to each risk factor in the estimation of global growth skewness is estimated using the impulse responses of global growth at the forecast horizons of interest (2, 4, 6, and 18 quarters) from the same VAR. The Cholesky ordering is the same as for the estimation of weights for dispersion.

As a result, each of these recovered statistics characterizing the shape of the global growth forecast distribution is time-varying, reflecting shifts in the distribution of the underlying risk factors.

- **Estimating weights for dispersion.** The weight assigned to each risk factor is estimated as the share of the global growth forecast error variance explained by each of the risk factors. The variance decomposition is derived from a recursive identification based on the assumption that oil prices are mostly driven by supply factors as in the analysis of spillovers (World Bank 2016a). The ordering of the variables used in the Cholesky decomposition is as follows: global term spread, S&P500, oil prices, and global growth. The derived dispersion of the forecast error distribution is scaled to match the mean square root of past forecast errors since 2010.

\(^{13}\)To ensure that risks are not perpetually skewed in one direction, over the full horizon of historically available data, the skewness of each risk factor is adjusted for its mean over the full time series.
The methodology described above yields the moments of the two-piece normal distribution of global growth: dispersion, mode (the June 2016 Global Economic Prospects forecasts), and skewness. The corresponding cumulative density function of global growth can be derived. Based on this, the probability of growth falling below any threshold can be calculated (Annex Figure SF2.2).

### ANNEX FIGURE SF2.2 Probability of growth outcomes

The probability of global growth falling below the baseline forecast of 2.4 percent in 2016 and 2.9 percent in 2017 is 54 and 52 percent, respectively and, based on OLS estimates, 40 and 51 percent, respectively.

#### A. Probability of global growth below threshold: VAR-based estimates

#### B. Probability of global growth below threshold: OLS-based estimates

![Graph showing probability of growth outcomes](image)

Source: World Bank Staff estimates.

### ANNEX TABLE SF2.1 Global growth dispersion weights: VAR estimates

<table>
<thead>
<tr>
<th>VAR-based weights</th>
<th>OLS-based weights</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6-months ahead</td>
</tr>
<tr>
<td>S&amp;P500 ($\beta_1$)</td>
<td>0.25</td>
</tr>
<tr>
<td>Term spread ($\beta_2$)</td>
<td>0.01</td>
</tr>
<tr>
<td>Oil price ($\beta_3$)</td>
<td>0.01</td>
</tr>
<tr>
<td>Other factors ($\beta_4$)</td>
<td>0.73</td>
</tr>
</tbody>
</table>

Source: World Bank staff estimates.

Note: VAR-based results are variance decomposition weights derived from a VAR of oil prices, S&P500, term spread and global growth—they are calculated as the share of variance of global growth explained by selected risk factors and other risk factors not included in the analysis. OLS-based weights are the absolute values of the coefficients obtained from an OLS-regression of global growth on oil prices, S&P500 and global term spreads. Regressions use annual data for 1982-2014.

### ANNEX TABLE SF2.2 Global growth skewness weights: OLS estimates

<table>
<thead>
<tr>
<th>VAR-based weights</th>
<th>OLS-based weights</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6-months ahead</td>
</tr>
<tr>
<td>S&amp;P500 ($\alpha_1$)</td>
<td>0.36</td>
</tr>
<tr>
<td>Term spread ($\alpha_2$)</td>
<td>0.1</td>
</tr>
<tr>
<td>Oil price ($\alpha_3$)</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

Source: World Bank staff estimates.

Note: Impulse responses-based weights are derived from a VAR including in this order: oil prices, S&P500 Index, term spread and global growth. The responses of global growth to own innovations are not presented here. OLS-based weights are derived from OLS-regression of global growth on oil prices, S&P500 and global term spreads. Regressions use annual data for 1982-2014.
### ANNEX TABLE SF2.3.A Literature review: Fan chart construction methodology

<table>
<thead>
<tr>
<th>Author</th>
<th>Country/ Variable of interest</th>
<th>Methodology</th>
<th>Variables included</th>
</tr>
</thead>
<tbody>
<tr>
<td>Michal et al. (2014)</td>
<td>Czech Republic. Inflation, real GDP growth, interest rate, exchange rate</td>
<td>BVAR forecast model and minimum distance method in the construction of a fan chart for inflation, real GDP growth, and interest and exchange rate forecasts. Assess the Zero Lower Bound of interest rate effect on the fan chart and propose a test procedure that evaluates the severity of macroeconomic risk factors included in the central bank financial stress and macroeconomic outlook test model. Data are from 1998Q1 - 2012Q2.</td>
<td>CPI inflation, real GDP growth, 3-month interest rate, nominal CZK/EUR exchange rate.</td>
</tr>
<tr>
<td>Reserve Bank of India, Apr. 2016 (Banerjee and Das 2011)</td>
<td>India. Inflation, real GDP growth</td>
<td>Fan charts for inflation and GDP growth are constructed based on historical forecast error variances.</td>
<td>Wholesale price index inflation rate, real GDP growth, international investment position, real effective exchange rate, M1 money aggregate.</td>
</tr>
<tr>
<td>Bank of Israel, Monetary Policy Report, H12015</td>
<td>Israel. Inflation, interest rates</td>
<td>Uncertainty results from shocks to endogenous variables whose distribution is based on their past developments.</td>
<td>External and domestic developments: global demand, commodity prices, financial market index, real GDP growth, inflation.</td>
</tr>
<tr>
<td>Bank of Japan, Outlook for Economic Activity and Prices, Jan. 2016</td>
<td>Japan. Inflation, real GDP growth</td>
<td>Distributions of forecasts are based on the Policy Board members’ assessment of uncertainty and their judgement of the balance of risk associated with their forecasts. The distributions of Board members forecast are presented to illustrate the extent of uncertainty and balance of risk associated with the projections.</td>
<td>Inflation, real GDP growth.</td>
</tr>
<tr>
<td>Author</td>
<td>Country/Variable of interest</td>
<td>Methodology</td>
<td>Variables included</td>
</tr>
<tr>
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<tr>
<td>Board of Governors of the Federal Reserve System, Feb. 2016</td>
<td>United States, PCE (personal consumption expenditure), inflation, real GDP growth, unemployment</td>
<td>Density of individual forecast series are based on Board members’ assessment and judgement of the balance of risk. Histograms of individual series are presented to illustrate the distributions of the forecasts.</td>
<td>Global demand, commodity prices, financial markets index, unemployment, GDP growth, inflation.</td>
</tr>
<tr>
<td>Cogley et al. (2003)</td>
<td>United Kingdom, Inflation</td>
<td>BVAR forecast model and minimum entropy method in the construction of a fan chart for inflation forecast. The paper assesses the effect of parameter uncertainty on inflation forecasts and compares the BVAR fan chart with the one produced by the Bank of England. The sample used is from 1957Q1 to 2002Q4.</td>
<td>Output gap, RPIX inflation, 3-month Treasury Bill rate.</td>
</tr>
<tr>
<td>U.S. Congressional Budget Office (2007)</td>
<td>United States, Budget balance</td>
<td>Fan chart for budget balance elements including revenue, expense, and debt. Fan chart is based on historical forecast errors. The revenue historical forecast error is decomposed into cyclical and noncyclical components using OLS regressions of revenue forecast error on a measure of business cycle (output gap). The distribution of the revenue forecast is predicted by making assumptions on the cyclical and non-cyclical components and using the OLS estimation coefficients as weights.</td>
<td>Primary surplus/deficit, debt/GDP, GDP growth.</td>
</tr>
</tbody>
</table>
**ANNEX TABLE SF2.3.B Literature review: Estimation of weight parameters for risk factors**

<table>
<thead>
<tr>
<th>Author</th>
<th>Country/Variable</th>
<th>Methodology</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Linear Approximation: Forecast points as input in the construction of the forecast distributions from key risk factors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IMF, WEO Oct 2015</td>
<td>World /Forecast of global real GDP growth</td>
<td>Same as in Elekdag and Kannan(2009)</td>
</tr>
<tr>
<td>Blix and Sellin (1998)</td>
<td>Sweden/Inflation, GDP growth</td>
<td>Elasticity of inflation or the variable of interest with respect to each risk factor. Does not suggest any estimation method</td>
</tr>
<tr>
<td>2. VARs and DSGE Methods: Forecast distribution is inferred directly from the forecasting model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cogley et al (2005)</td>
<td>United Kingdom/Inflation</td>
<td>The weight parameters are obtained from BVAR with stochastic volatility</td>
</tr>
<tr>
<td>Alvaro and Maximiano (2003)</td>
<td>Euro Area/ Inflation, GDP growth</td>
<td>The weight parameters are estimated in VAR forecasting model</td>
</tr>
<tr>
<td>Smets and Wouters (2004)</td>
<td>Euro Area/ Many macro variables</td>
<td>The weight parameters are obtained from the sticky prices DSGE model using a Bayesian technique</td>
</tr>
<tr>
<td>Gürkaynak et al (2013)</td>
<td>U.S./Inflation, real GDP growth, Interest rate.</td>
<td>The weight parameters are obtained from an estimated DSGE and a VAR and forecast performances of the two approaches are compared</td>
</tr>
<tr>
<td>Wolters (2013)</td>
<td>U.S./Inflation, real GDP growth, Interest rate.</td>
<td>The weight parameters are obtained from a DSGE model and BVAR model. The forecast performance of the two approaches are compared</td>
</tr>
</tbody>
</table>

**ANNEX TABLE SF2.3.C Literature review: Measurement of dispersion and skewness**

<table>
<thead>
<tr>
<th>Author</th>
<th>Country/Variable</th>
<th>Methodology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixon (2011)</td>
<td>U.S./S&amp;P500</td>
<td>(25delta Put - 25delta Call)/50 delta</td>
</tr>
<tr>
<td>Baksi et al (2003)</td>
<td>U.S./S&amp;P100</td>
<td>The slope from regression log of Implied Volatility (IV) on log of moneyness</td>
</tr>
<tr>
<td>Chicago Board of Option Exchange (CBO) (2010)</td>
<td>U.S./SPX 500</td>
<td>Skewness = 100-10*(Price of Skewness )</td>
</tr>
<tr>
<td>Elekdag and Kannan (2009)</td>
<td>U.S./SPX 500</td>
<td>Skewness of the risk neutral density</td>
</tr>
</tbody>
</table>
References


Banco Central de Chile. 2016. Informe de Política Monetaria. Santiago: Banco Central de Chile.


Chicago Board Options Exchange. 2010. CBOE Skew Index FAQ. Chicago: CBOE.


