Long-Term Growth Model (LTGM)  
MTI Forum Learning Module

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www.worldbank.org/LTGM (internet)  
http://LTGM (intranet FURL)

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Model Overview

• Countries want to grow at high rates
  • What growth rates are feasible? What would it take to achieve these goals?

• A simple model to analyze long-term growth
  • Based on celebrated Solow-Swan Model: savings and investment key
  • Also TFP, human capital, demographics, labor participation, FDI & external debt
  • Implications of growth for poverty

• Toolkit for use by country economists/policymakers in many countries
  • Spreadsheet-based for simplicity.
  • No macros; transparent, flexible & easy-to-learn

• Many extensions: public investment, WB HCI, TFP, Natural Resources....
Objectives of the Main LTGM

• Help policy makers in finding answers to 3 important policy questions:
  • Submodel 1: How much growth from a given investment profile?
  • Submodel 2: How much investment is needed to achieve given growth profile?
  • Submodel 3: How much growth from a given savings profile?
    • Requires assumptions on debt or current account balance

• Allow policy makers ample flexibility
  • Scenario analysis using many other variables: Productivity, Human Capital, Demographics, External sector
  • Growth → Poverty

• For long-run scenario analysis -- not short-run analysis or forecasting
Some examples of work using the LTGM

Used in 40+ countries for growth analysis and country reports (CEMs and SCDs):

- **Sub-Saharan Africa:** Cameroon (CEM), Cape Verde (SCD), Eswatini, Gabon, Guinea (SCD), Seychelles (SCD), Ghana (SCD), Malawi, South Africa, Ivory Coast, Mauritania, Zambia (SCD), Zimbabwe

- **South Asia:** Bangladesh, Nepal (CEM), Sri Lanka (CEM)

- **Latin America & Caribbean:** Brazil, Costa Rica, El Salvador, Guatemala, Honduras, Nicaragua, Panama, Peru

- **East Asia & Pacific:** Cambodia, Korea, Laos, Malaysia, Philippines, Vietnam, Thailand

- **Europe and Central Asia:** Armenia (SCD), Bosnia, Georgia (SCD), Kyrgyz Republic (SCD), Tajikistan (CEM)

- **MENA:** Egypt (CEM), Syria

- **Eg Cameroon CEM 2016** – goal to boost growth to 8% become an UMI country by 2035.
  - Planned ↑Investment insufficient higher TFP growth → ↑competition to boost TFP

- **Honduras, Panama, Peru, Zambia, Bangladesh, Malaysia, Cambodia** – LTGM Training for govt officials
Outline of the Rest of the Talk

Part A: Main LTGM
1. Explanation of how the growth model works
   • Equations, parameters, assumptions and drivers of growth

2. Hands-on demonstration and tutorial
   • Examples: investment path -> growth, growth+inequality-> poverty, growth target -> required investment, savings+ CAB -> growth

Part B: Public Capital Extension (and other extensions)
• List of extensions
• Overview of LTGM-Public Capital extension
• Hands-on demonstration using LTGM-PC

Comments/Questions/Suggestions
A1. The Growth Model
Three Building Blocks of the Model

1. Production Function

\[ Y_t(GDP) = A_t K_t^{1-\beta} (h_t L_t)^\beta \]

2. Capital Accumulation

\[ K_{t+1} = (1 - \delta)K_t + I_t \]

3. Demographics and Labor Market:

\[ y^p_t(GDP \text{ per capita}) = \frac{Y_t}{N_t} = \frac{Y_t}{L_t} \frac{L_t}{W_t} \frac{W_t}{N_t} = A_t k_t^{1-\beta} h_t^\beta \rho_t \omega_t \]

\( W_t \): working-age pop; \( N_t \): total population; \( \rho_t \): participation rate; \( \omega_t \): working-age-pop. to pop. ratio; \( A_t \): TFP; \( K_t \): capital; \( h_t \): human capital per worker; \( L_t \): workers)
Growth Drivers

\[ g_{y,t+1} \approx g_{A,t+1} + \beta (g_{h,t+1} + g_{\omega,t+1} + g_{N,t+1} + g_{\rho,t+1}) + \left[ \frac{1-\beta}{K_t/Y_t} \right] \frac{I_t}{Y_t} - (1-\beta)\delta \]

- **Common policy message**: investment-led growth [by itself] is not sustainable in long run
  - ↑ K/Y reduces the effectiveness of investment over time (↓ MPK)
  - Leads to an increase in the \( mICOR_t = \frac{1}{1-\beta} \frac{K_t}{Y_t} \) (ppt increase I/Y needed for extra 1% growth)
  - Needs to be accompanied by other sources (e.g., human capital, TFP, participation)
External Sector (how to fund investment?)

1. Current Account Balance (CAB): 
\[ I_t/Y_t = S_t/Y_t - CAB_t/Y_t \]

2. External Debt (we assume \( \Delta NFA_t \approx 0 \))

\[ CAB_t = \Delta NFA_t - \Delta NFL_t = -(FDI_t + (D_t - D_{t-1})) \]

Change Net Foreign Liabilities; Foreign Direct Inv.; Change Total External Debt

\[ \Rightarrow \frac{I_t}{Y_t} = \frac{S_t}{Y_t} + \frac{FDI_t}{Y_t} + \frac{D_t}{Y_t} - \frac{D_{t-1}/Y_{t-1}}{(1 + g_{y,t}^{pc})(1 + g_{N,t})} \]

- Common policy message: need to increase savings or attract FDI to fund investment plans
Saving and Investment
Average, 1980-2008

\[ y = 0.5611x + 0.1127 \]
\[ t = 14.49 \]
\[ R^2 = 0.6775 \]

Source: Hevia & Loayza (2012)
Solving the Model - Parameters

• Can solve the model in simple spreadsheet *without macros*

• Minimal Data requirements - only need data on **three** parameters
  - Labor share ($\beta$)
  - Depreciation rate ($\delta$)
  - Initial Capital-to-Output Ratio ($\frac{K_0}{Y_0}$)

• $\uparrow \beta$, $\uparrow \delta$ and $\uparrow \frac{K_0}{Y_0}$ all make growth harder via capital accumulation

• Users can choose preloaded data source & time horizon via dropdown menu – and compare in “data summary” tab
Solving the Model - required assumptions (future)

- Needed for all submodels:
  - Growth rate of TFP ($g_{A,t}$); Human Capital per worker ($g_{h,t}$);
  - Demographics: Population ($g_N$) & Working–age–pop ratio ($g_\omega$)
  - Participation rates ($g_\rho,t$)

- Submodel 1: Choose Investment share of GDP ($I_t/Y_t$)
  - Model calculates returns the growth rate of GDP per capita (or GDP per worker)

- Submodel 2: Choose Growth rate of GDP per capita ($g_{y,PC,t+1}$)
  - Model calculates the investment share of GDP

- Submodel 3: Choose Savings share of GDP ($S_t/Y_t$) and CAB/Y or Ext. Debt/Y & FDI/Y
  - Model calculates the growth rate of GDP per capita (or GDP per worker)
Poverty and Growth

• 2030 Goals to eliminate extreme poverty & halve poverty (at national lines)
  • But what growth rates are required? How do current growth paths affect poverty?

• Based on Log-Normal approx. of the income distribution
  • Can analyze in Excel simply using preloaded data (no microdata required)
  • Automatically produces a Growth Elasticity of Poverty (GEP) (or users can add their own)

• Can assume constant inequality or reduced inequality (income Gini)
  • Lower inequality: (i) reduces poverty directly & (ii) increases effect of growth on poverty

• “Shared prosperity premium” where income of B40 grows faster
  • Translate this into path for Gini coefficient and examines effect on poverty rates

• Caveat: the “type” of growth doesn’t impact poverty (eg which sector grows)
How poverty model works

• Assume a constant Gini coefficient over time.

• Growth increases everyone’s income or consumption by the same percentage
  • Shifts the log distribution to the right

• Effect on poverty varies by how many people are near the poverty line
  • Larger ppt fall in poverty when poverty rate is close to 50%
  • Varies by country, poverty line & time

• In more equal countries (lower Gini coeff)
  → more compressed distribution
  → more ppl near poverty line
  → larger effect of growth on poverty

A2: Spreadsheet Tutorial
(Hands-on demonstration)

Download LTGM spreadsheet from [www.worldbank.org/LTGM](http://www.worldbank.org/LTGM)
LTGM Spreadsheet Structure

• **Yellow** - user can change/edit (dropdown menu or text box)

• *InputDataA GeneralAssumptions* – Assumptions/parameters that affect all simulations (country, start year, TFP growth, K/Y, poverty etc)

• *GraphsA* plots all general assumptions in *InputDataA*.

• *InputDataB ModelSpecAssumptions* – Assumptions for specific models
  • Model 1: Investment share of GDP → GDP Growth
  • Model 2: Growth rate → Investment share of GDP
  • Model 3: Savings share of GDP → GDP Growth

• *GraphsB* plots **results** of each model (and assumptions from *InputDataB*).

• Submodel 1/1s/2/2s/3/3s -- see the formulas here (no macros)

• *DataSummary* – overview of historical data and parameters from different sources
Baseline

Tab InputdataA: Choose general parameters
• Country=Peru [Cell B3; the Default]
• Initial year=2019 [Cell D/E7; the Default]
• Labor share ($\beta$) = 53.8%
  [in cell C15 select PWT 9 LS. 3 Avg Wage]
• Depreciation rate ($\delta$) = 4.2% [the default]
• Initial ($K_0/Y_0$) = 2.62 [Cells I/J13; default]
• 0.004 (0.4%) Human capital growth initial & target [Cell D/E22; the default]
• 0.008 (0.8%) TFP growth – initial & target [Cell D/E31; the default]

Tab InputDataB: Choose the investment-to-GDP Ratio = 21% [Cell D/E6; the default]

Should get this graph:
• 2020 growth of 3.6%
• 2050 growth of 2.5%
(i) Submodel 1: Investment → Growth

- Tab InputDataB: Submodel 1 specific assumptions

- Baseline – no change (target=initial)

- Scenario – 0.30 I/Y [Cell E9] by 2030 (set as target) [Cell E10]

- Tab GraphsB - check get these graph for investment and growth

- Growth should peak at 4.3% in 2031 in scenario
(ii) Submodel 1: effect of growth on poverty

Start with Model 1 simulation in (i)

InputdataA: Choose general parameters

- National Poverty line (default) [Cell C130]
- Initial poverty rate at 0.207 (default) [Cell D/E125]
- Set the Growth Elasticity of Poverty (GEP) on Automatic (the default) [Cell D/E133]
- Make sure the Gini coefficient option is chosen [Cell D/E140, the default]
- Initial Gini coefficient of income 0.438
  - Constant (initial= target) [Cell D/E137, default]

GraphsB - make sure you get this poverty →

By 2050 Results (Submodel 1/1s sheet bottom)

- Poverty: 4.8% (baseline) [Cell AJ 65 Submodel1] vs 2.9% (scenario) [Cell AJ 65 Submodel1s]
(iii) Submodel 1: Effect of ↓inequality on poverty

**Goal:** Lower inequality 43.8% → 40% by 2030
(in addition to effects of higher investment)

*InputDataA:* Make sure the Gini coefficient option is chosen [Cell D/E140, the default]

Start with Model 1 simulation in (ii)

*InputDataA:* parameters for poverty

- Scenario target Gini of 0.40 by 2030 [change Cell E144 in InputdataA]

*GraphsB - make sure you get this poverty →*

By 2050 Results *(Submodel 1/1s sheet bottom)*

- Poverty: 4.8% *(baseline)* [Cell AJ 65 Submodel1]
  vs 1.4% *(scenario)* [Cell AJ 65 Submodel1s]
(iv) Submodel 2: Growth → Investment (w/ TFP)

Tab InputDataB: target GDP growth rate is 3.5% in baseline and scenario [Cell D/E13 type 0.035]

InputdataA parameters for TFP growth
• Initial TFP growth of 0.008 (0.8% TFP growth) [Default Cell D/E29]
• Scenario: 0.02 (2%) [Cell E34] TFP growth target by 2030

GraphsB - check get this graph for investment →

By 2049: Required investment (Submodel 2/2s sheet)
• 0.327 (baseline) [Cell AI18] vs 0.124 (scenario) [Cell AI18]

Explanation: Declining marginal product of capital
(v) Submodel 3: Savings + CAB → Growth (ASSUMPTIONS)

Reset TFP growth rate in scenario: type “=E31” in Cell E34 tab InputdataA

*InputdataA: External Balance (previously didn’t matter)*

- Current Account Balance Constraint (dropdown menu) [Cell D/E60; the default]

- CAB target in Scenario: 6% GDP (0.06 CAB/GDP [Cell E69]) by 2030

- *GraphsA - check get this graph (CAB red/blue only)* →
(v) Submodel 3: Savings + CAB → Growth (RESULTS)

InputDataB - check savings is constant 19% of GDP [Cell D26 & E26]

GraphsB - check get GDP Growth → (Headline GDP Growth, not per capita)

By 2030: GDP Growth result:
• 3.1% (baseline) [Cell P32 Submodel 3]
• 2.1% (scenario) [Cell P32 Submodel 3s]
Part B. Public Capital Extension (LTGM-PC) and other Extensions

Download LTGM-PC spreadsheet from www.worldbank.org/LTGM
LTGM Extensions

1. **Public Capital extension** (Devadas & Pennings 2018)
   - TODAY Private & Public investment (quantity/quality) → growth

2. **TFP Extension** (Kim & Loayza 2018) - download at www.worldbank.org/LTGM
   - Uses cross country regression to calculate path for TFP growth based on country’s scores for:
     - innovation, education, market efficiency, infrastructure, and institutions

3. **Human Capital Extension** (beta)
   - Adapts the World Bank HCI for growth analysis (schooling quantity/quality, health)
   - Based on population cohorts – reforms affecting today’s children affect growth in the future.
   - Example from Malaysia box: increase HCI components to median of high income country
FIGURE 21
Simulations for Malaysia’s rate of human capital growth

Human Capital, y/y, Selected Simulations

- Quality of Education + ASR + Stunting
- Quality of Education + ASR
- Quality of Education
- Baseline

2020 2025 2030 2035 2040 2045 2050
0.0% 0.2% 0.4% 0.6% 0.8%

Source: Malaysia Economic Monitor (June 2019)

FIGURE 22
Simulations for Malaysia’s long-term rate of GDP growth

GDP, y/y, Selected Simulations

- Quality of Education + ASR + Stunting
- Quality of Education + ASR
- Quality of Education
- Baseline

2020 2025 2030 2035 2040 2045 2050
2.0% 2.5% 3.0% 3.5% 4.0% 4.5% 5.0%
4. **Natural Resource extension** (beta)
   - Commodity export sector calibrated to commodity exporters all over the world
   - Analyze the effect of long-run growth of
     - Commodity price changes
     - Commodity discoveries
   - Analyze different fiscal rules for commodity revenues:
     - Balance Budget Rule, Structural Surplus Rule, Hartwick’s Rule

5. **Advanced Sectoral Growth Tool** (in progress)
   - 3 sectors - agriculture, manufacturing and services
   - Structural transformation
   - Distortions in the allocation of factors of production across sectors
LTGM-Public Capital Extension - Overview

1. Disaggregate total investment into public $I_t^G$ (infrastructure) and private $I_t^P$
   • What is effect on growth of 1ppt GDP ↑public investment? And when?
   • Is that larger or smaller than the same size effect of private investment?

2. Introduce new quality/efficiency of public capital $\theta_t$ (between 0 and 1)
   → Measured as new Infrastructure Efficiency Index (IEI) based on:
     - Water leaks, Power transmission losses, Unpaved roads
   • What is the effect of efficiency/quality of public investment on growth?
The LTGM-PC production function

- New/changed parts in the LTGM-PC in red:

\[ Y_t(GDP) = A_t(\theta_t K_t^G)\phi (K_t^P)^{1-\beta-\phi} (h_t L_t)^\beta \]

- Log linear approximation:

\[ g_{Y,t+1} \approx \tilde{g}_{t+1} + \phi \left( \frac{\theta_t^N - \theta_t}{\theta_t} \right) \frac{I_t^G}{Y_t} + \phi \left[ \frac{I_t^G}{Y_t} \frac{K_t^G}{Y_t} - \delta^G \right] + (1 - \beta - \phi) \left( \frac{I_t^P}{Y_t} \frac{K_t^P}{Y_t} - \delta^P \right) \]

- Growth due to private Investment
- Growth due to quantity of public K (more \( K_t^G \))
- Growth due to quality of public K (more \( \theta \))
# Calibration of LTGM-PC to Malaysia

<table>
<thead>
<tr>
<th>Variable</th>
<th>Baseline Value</th>
<th>Source/Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor share income</td>
<td>$\beta = 0.5$</td>
<td>Penn World Tables v9 (2014)</td>
</tr>
<tr>
<td>Deprecation rate</td>
<td>$\delta = 5.8%$</td>
<td>Penn World Tables v9 (2014)</td>
</tr>
<tr>
<td>Total factor productivity (TFP) growth</td>
<td>$g_A = 0.9%$ initially 0.6% by 2050</td>
<td>Similar to PWT v9: 30 years median; 15 year average</td>
</tr>
<tr>
<td>Human capital (HC) growth</td>
<td>$g_h = 0.6%$ initially 0.1% by 2040</td>
<td>LTGM-Human Capital Extension Similar to 2011-14 average from PWT 9</td>
</tr>
<tr>
<td>Total investment rates</td>
<td>$I/Y = 24%$</td>
<td>IMF Article IV (2019) -2020-23 average</td>
</tr>
<tr>
<td>Public Investment rate</td>
<td>$I^G/Y = 6%$</td>
<td></td>
</tr>
<tr>
<td>Private Investment rate</td>
<td>$I^P/Y = 18%$</td>
<td></td>
</tr>
<tr>
<td>Total Capital-to-output ratio</td>
<td>$K/Y = 2.25$</td>
<td>Steady state $K/Y=(I/Y)/(g_Y + \delta)$</td>
</tr>
<tr>
<td>Private capital-to-output ratio</td>
<td>$K_G/Y = 1.11$</td>
<td></td>
</tr>
<tr>
<td>Population growth (2019-2050)</td>
<td>$g_N=1.3% \rightarrow 0.4%$</td>
<td>UN Population projections (via WB HDN)</td>
</tr>
<tr>
<td>Headline GDP growth in 2020</td>
<td>$g_Y=4.5%$</td>
<td>World Bank MTI forecasts (2019-2021)</td>
</tr>
</tbody>
</table>

Note: also assume that quality $\theta$ is constant
Baseline for the LTGM-PC

Tab InputdataA: Choose general parameters

• Country=Malaysia [Cell B3]
• Initial year=2019 [Cell D/E7; default]
• Labor share ($\beta$) = 50% [Cell D16 & E16]
• Deprec. rate ($\delta$) = 5.8% [Cell D/E13; default]
• Initial ($K_0/Y_0$) = 2.25 [Cell I13 & J13]
• Human capital growth
  • Initial 0.006 (0.6%) [Cell D32 & E32]
  • Target: 0.001 (0.1%) [Cell D37 & E37] by 2040
• TFP growth
  • Initial 0.009 (0.9%) [Cell D41 & E41]
  • Target 0.006 (0.6%) [Cell D46 & E46]
    • by 2050 [Cell D47 & E47]

Tab InputDataB:

• Choose the public investment-to-GDP Ratio = 6% [Cell D8 & E8]
• Choose the private investment-to-GDP Ratio = 18% [Cell D18 & E18]

Should get this graph:

• 2020 GDP growth of 4.5%
• 2050 GDP growth of 1.9%
(vi) Submodel 1: Public Investment → Growth

- Public Investment Shock (InputDataB):
  - Permanent 9% (0.09) of GDP [Cell E13] by 2020 [Cell E14]

- GDP Growth results in 2030:
  - 3.5% (baseline) [Cell P38 in Submodel 1]
  - 3.8% (scenario) [Cell P38 in Submodel 1s]

Graphs - make sure you get these graphs for Public Investment & GDP Growth →
(vii) Submodel 1: Private Investment $\rightarrow$ Growth

- Reset Public Investment: 6% (0.06) of GDP target [Cell E13 in InputDataB]

- Private Investment Shock:
  - permanent 20% (0.20) of GDP [Cell E23]
  - by 2020 [Cell E24] in InputDataB

**GDP Growth results in 2030:**
- 3.5% (*baseline*) [Cell P38 in Submodel 1]
- 3.7% (*scenario*) [Cell P38 in Submodel 1s]

*GraphsB - check get this graph for Public Investment & GDP Growth →*
Thank you!
Questions/comments/suggestions

• Latest version available at www.worldbank.org/LTGM (or http://LTGM on intranet)
• Always trying to improve the LTGM -- your comments and suggestions are welcome
• Please contact us if you would like to use the model in your country:
  • Steven Pennings (spennings@worldbank.org),
  • Norman Loayza (nloayza@worldbank.org), or
  • Jorge Guzmán (jguzman.correa@worldbank.org)
• We can also provide help with analysis, presentations, training etc
## Input/Output: Three Versions of the Model

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Submodel 1</th>
<th>Submodel 2</th>
<th>Submodel 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Growth</strong> given <strong>Investment</strong></td>
<td></td>
<td><strong>Investment</strong> given <strong>Output/Growth Target</strong></td>
<td><strong>Investment/Growth</strong> given <strong>Savings</strong></td>
</tr>
<tr>
<td><strong>Inputs:</strong></td>
<td>Investment rate ( \left( \frac{I_t}{Y_t} \right) )</td>
<td>Growth rate of GDP ( (g_Y) ) OR Growth rate of GDP per capita ( (g_Y^{pc}) ) OR Time path of GDP ( (GDP_t) ) OR Poverty</td>
<td>Savings rate ( \left( \frac{S_t}{I_t} \right) )</td>
</tr>
<tr>
<td><strong>Outputs:</strong></td>
<td>Growth rate of GDP ( (g_Y) ), Growth rate of GDP per capita ( (g_Y^{pc}) ), Level of GDP ( (GDP_t) ), Poverty rate</td>
<td>Other three of the four measures</td>
<td>Growth rate of GDP ( (g_Y) ), Growth rate of GDP per capita ( (g_Y^{pc}) ), Level of GDP ( (GDP_t) ), Poverty rate</td>
</tr>
<tr>
<td><strong>Savings/Investment</strong></td>
<td>Savings rate ( \left( \frac{S_t}{Y_t} \right) )</td>
<td>Investment rate ( \left( \frac{I_t}{Y_t} \right) ), Savings rate ( \left( \frac{S_t}{Y_t} \right) )</td>
<td>Investment rate ( \left( \frac{I_t}{Y_t} \right) )</td>
</tr>
<tr>
<td><strong>External Sector</strong></td>
<td>CAB to GDP ( \left( \frac{CAB_t}{Y_t} \right) ) OR External Debt to GDP ( \left( \frac{D_t}{Y_t} \right) )</td>
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