Shadow Price of Water Under Varying Risk Behavior

Sankalp Sharma (University of Nebraska – Lincoln)

Tajamul Haque (Council for Social Development/LANDESA)

Rifat Mansur (North Carolina State University)

Anil Giri (University of Central Missouri)

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What is the problem?

- Acute groundwater shortage in India being worsened by disproportionate irrigation use.
- Water rights tied to land acquisition.
- Owner of a land parcel can freely access groundwater.
- Rice, the second-most prominent crop is inefficiently produced in several states.
- For example, Chhattisgarh, Odisha, Haryana, U.P and Punjab use 35% or more water than West Bengal.
- Punjab is the most inefficient at 51.2 percent. 5337 litres/ kg rice production.

Causes of inefficiency

- More irrigation increases land productivity.

- Punjab produces 58.5 qtl/ha, whereas West Bengal produces 41.2 qtl/ha.

- Power subsidies provided to farmers in most states.

- Incentivizes them to pump more groundwater.

Government scheme

- Water ceiling proposed for non-domestic uses.
- If a producer uses less water, then cash-subsidy is provided.

Better solution required

- Cash incentive scheme creates burden on tax-payers.
- Water ceiling restricts a producer’s optimal choices.
- They may require more water.
- The government scheme will be better supported by a water market.
- Producers must be given choice of buying/selling water from the government and/or neighboring producers.

Question & Objectives

"Is there an amount that producers are willing to pay above the government mandated cap for irrigation, under varying producer risk aversion?"

Objectives:

- We determine the willingness-to-pay for water in the agricultural context (India).
- Our model incorporates producer behavior under risk aversion and yield uncertainty.

Theoretical model

Let π be the profit function of a producer.

$$\pi = \sum_{j=1}^{n} p_j y_j(x_i, I) - wI - \sum_{i=1}^{K} r_i x_i$$

Where,

p_j is the output price, y_j is the uncertain yield, n is the number of crops

I is irrigation use, w is the energy price required to pump water.

x_i are the non-irrigation inputs (fertilizer, chemical, etc.), K is the total number of inputs, r_i are the input prices.

Theoretical model: continued

Re-writing the profit function as:

$$\pi = Y - wI - \sum_{i=1}^{K} r_i x_i$$

Where,

$Y = \sum_{j=1}^{n} y_j$, is the random aggregate yield per-acre.

$Y = f(\boldsymbol{x}) + h(\boldsymbol{x})\theta$, is the Just-Pope production function.

$f(\boldsymbol{x}) = A\Pi_{i=1}^{K} x_i^{\beta_i} I^{\beta_I}$, $\mathrm{h}(\boldsymbol{x}) = B\Pi_{i=1}^{K} x_i^{\beta_i'} I^{\beta_I'} \theta$ (both f and h are Cobb-Douglas)

$\theta \sim B(s_1, s_2)$, where s_1 and s_2 are the shape parameters.

Producers are Expected Utility maximizers

Two types of producers: **Risk Neutral** and **Risk Averse**

Let each producer have a concave utility function $U(\pi)$.

Standard assumptions apply: $U'(\pi) \geq 0$ and $U''(\pi) \leq 0$

Functional form, if producer is **risk neutral**:

$$U(\pi) = \alpha\pi$$

Functional form, if producer is **risk averse**:

$$U(\pi) = -\exp(-\xi\pi)$$

Optimization problem

Producer’s optimization problem can be written as:

$$\underset{I, x_i}{Max}\ U(\pi)$$

Subject to,

$$I \leq \bar{I}$$

Where, $\bar{I}$ is the government mandated cap on irrigation.

Lagrangian under **risk neutrality**:

$$L(x_i, I, \lambda) = \int (\alpha\pi(\boldsymbol{x}, \theta, \beta, \beta'))\gamma(\theta) - wI - \sum_{i=1}^{K} r_i x_i - \lambda(I - \bar{I})$$

Lagrangian under **risk aversion:**

$$L(x_i, I, \lambda) = \int (-\exp(-\xi\pi(\boldsymbol{x}, \theta, \beta, \beta'))\gamma(\theta)) - wI - \sum_{i=1}^{K} r_i x_i - \lambda(I - \bar{I})$$

Solutions

Optimal λ for **risk neutrality** comes from the first-order condition w.r.t. I.

$$\lambda^* = A\beta_I x_1^{\beta_1} x_2^{\beta_2} I^{\beta_I - 1} + B^{\frac{1}{2}} \frac{\frac{\beta_I'}{2} x_1^{\frac{\beta_1'}{2}} x_2^{\frac{\beta_2'}{2}} I^{\frac{\beta_I - 1}{2}}}{s_1 + s_2} - w$$

For a producer to have positive willingness-to-pay for irrigation above the endowment, we need:

1. $I^\star = \bar{I}$ (from the Kuhn tucker conditions: if $I^* < \bar{I}$ then $\lambda = 0$, has to be true.

2. $A\beta_I x_1^{\beta_1} x_2^{\beta_2} I^{\beta_I - 1} + B^{\frac{1}{2}} \frac{\frac{\beta_I'}{2} x_1^{\frac{\beta_1'}{2}} x_2^{\frac{\beta_2'}{2}} I^{\frac{\beta_I - 1}{2}}}{s_1 + s_2} > w$

Optimal λ for **risk aversion**, follows similarly from the second Lagrangian

Simulation Analysis

We require the following parameters to determine λ for both risk neutrality and aversion.

1. **Production parameters**: A, β_i, β_I (mean elasticities) and B, β_i', β_I' (risk elasticities)
2. **Optimal inputs:** $x_i^\star$ $(i = 1,2)$ and $I^\star (= \bar{I})$.
3. **Mean** (μ) and **variance** (σ) of yield.
4. **Shape parameters of the yield distribution:** s_1 and s_2.
5. **Cost of energy**: w

Simulation Analysis: Input use

- We use Nitrogen (N) and Phosphorous Pentoxide (P_2O_5) as our two non-irrigation inputs.
- According to the World Bank database, Indian producers used 150 kg/ha fertilizers in 2015.
- Endowment limit for $\bar{I} = 2616$ litres/kg, stress level $\bar{I} = 2000$ litres /kg.

Simulation Analysis: cost of energy

- Indian producers pay 13% of the total electricity cost of pumping due to the high subsidy rates.
- Energy required to lift a feet of groundwater: weight of water × feet of lift.
- Lifting 1233420 ft-kgs of water approximately requires 1.02 kWh of power.
- Producer requires 8.26-e7 kWh of energy to pump an additional liter of water.
- Cost of electricity for non-domestic use in Punjab is stated as Rs. 6.75 per kWh.
- Marginal cost of groundwater is 8.26e-7 × 6.75 = 5.57-e6 per-kg (liter).

Simulation Analysis: Results

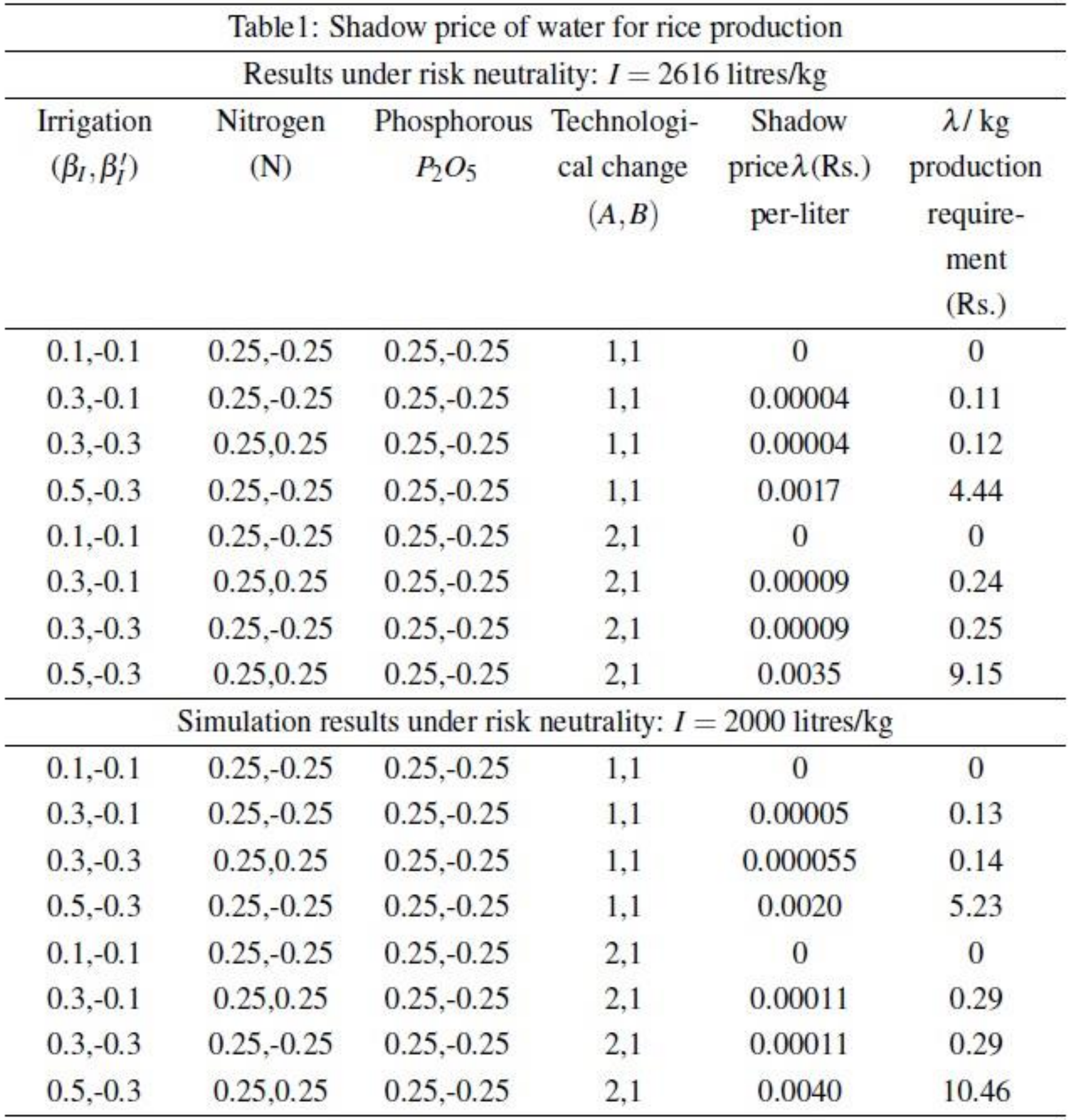

Table1: Shadow price of water for rice production

Results under risk neutrality: $I = 2616$ litres/kg

Irrigation (β_I, β_I')	Nitrogen (N)	Phosphorous P_2O_5	Technological change (A, B)	Shadow price λ(Rs.) per-liter	λ/ kg production requirement (Rs.)
0.1,-0.1	0.25,-0.25	0.25,-0.25	1,1	0	0
0.3,-0.1	0.25,-0.25	0.25,-0.25	1,1	0.00004	0.11
0.3,-0.3	0.25,0.25	0.25,-0.25	1,1	0.00004	0.12
0.5,-0.3	0.25,-0.25	0.25,-0.25	1,1	0.0017	4.44
0.1,-0.1	0.25,-0.25	0.25,-0.25	2,1	0	0
0.3,-0.1	0.25,0.25	0.25,-0.25	2,1	0.00009	0.24
0.3,-0.3	0.25,-0.25	0.25,-0.25	2,1	0.00009	0.25
0.5,-0.3	0.25,0.25	0.25,-0.25	2,1	0.0035	9.15
Simulation results under risk neutrality: $I = 2000$ litres/kg					
0.1,-0.1	0.25,-0.25	0.25,-0.25	1,1	0	0
0.3,-0.1	0.25,-0.25	0.25,-0.25	1,1	0.00005	0.13
0.3,-0.3	0.25,0.25	0.25,-0.25	1,1	0.000055	0.14
0.5,-0.3	0.25,-0.25	0.25,-0.25	1,1	0.0020	5.23
0.1,-0.1	0.25,-0.25	0.25,-0.25	2,1	0	0
0.3,-0.1	0.25,0.25	0.25,-0.25	2,1	0.00011	0.29
0.3,-0.3	0.25,-0.25	0.25,-0.25	2,1	0.00011	0.29
0.5,-0.3	0.25,0.25	0.25,-0.25	2,1	0.0040	10.46

Simulation Analysis: Results

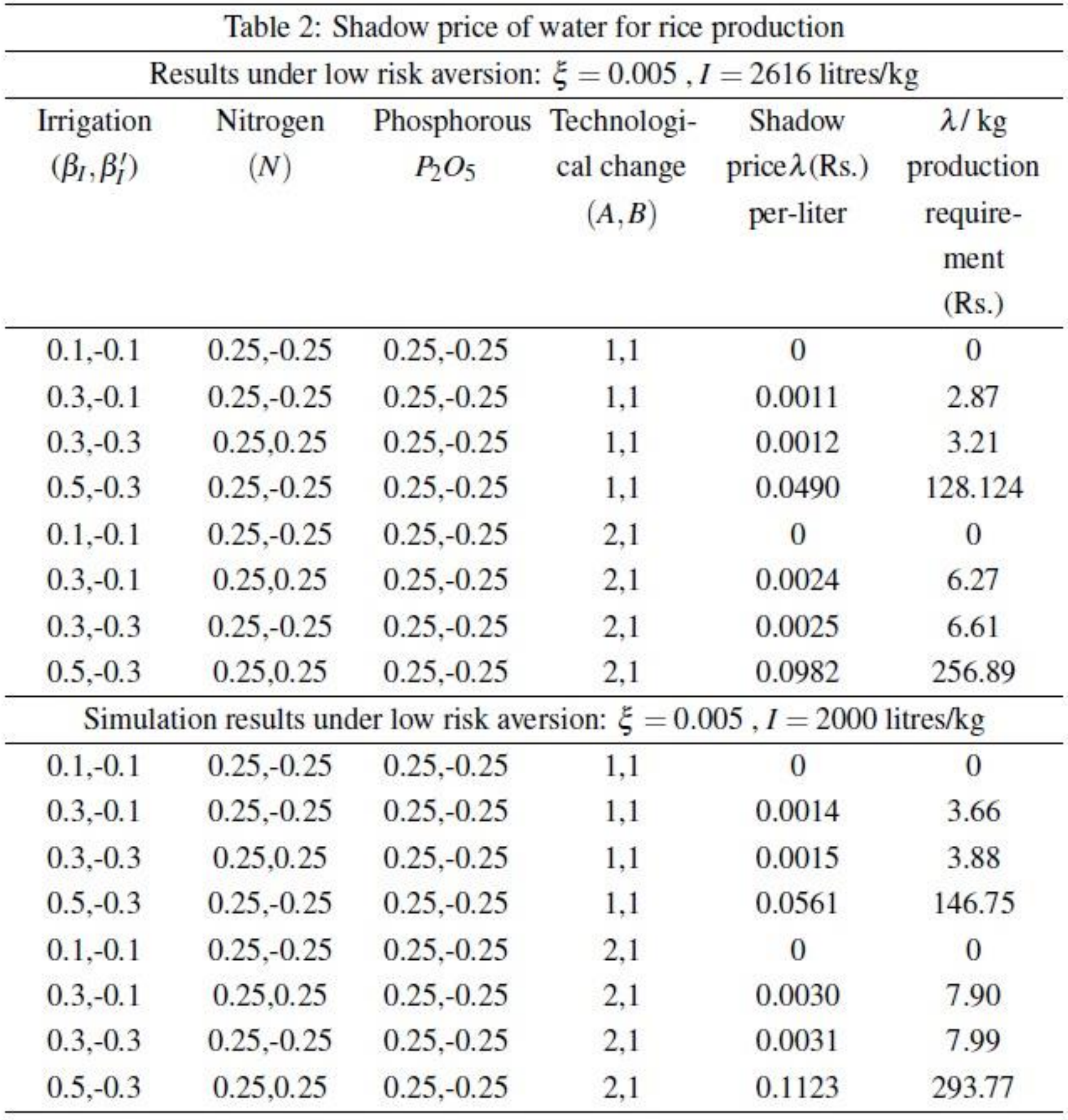

Table 2: Shadow price of water for rice production

Results under low risk aversion: $\xi = 0.005$, $I = 2616$ litres/kg

Irrigation (β_I, β_I')	Nitrogen (N)	Phosphorous P_2O_5	Technological change (A,B)	Shadow priceλ(Rs.) per-liter	λ/ kg production requirement (Rs.)
0.1,-0.1	0.25,-0.25	0.25,-0.25	1,1	0	0
0.3,-0.1	0.25,-0.25	0.25,-0.25	1,1	0.0011	2.87
0.3,-0.3	0.25,0.25	0.25,-0.25	1,1	0.0012	3.21
0.5,-0.3	0.25,-0.25	0.25,-0.25	1,1	0.0490	128.124
0.1,-0.1	0.25,-0.25	0.25,-0.25	2,1	0	0
0.3,-0.1	0.25,0.25	0.25,-0.25	2,1	0.0024	6.27
0.3,-0.3	0.25,-0.25	0.25,-0.25	2,1	0.0025	6.61
0.5,-0.3	0.25,0.25	0.25,-0.25	2,1	0.0982	256.89
Simulation results under low risk aversion: $\xi = 0.005$, $I = 2000$ litres/kg					
0.1,-0.1	0.25,-0.25	0.25,-0.25	1,1	0	0
0.3,-0.1	0.25,-0.25	0.25,-0.25	1,1	0.0014	3.66
0.3,-0.3	0.25,0.25	0.25,-0.25	1,1	0.0015	3.88
0.5,-0.3	0.25,-0.25	0.25,-0.25	1,1	0.0561	146.75
0.1,-0.1	0.25,-0.25	0.25,-0.25	2,1	0	0
0.3,-0.1	0.25,0.25	0.25,-0.25	2,1	0.0030	7.90
0.3,-0.3	0.25,-0.25	0.25,-0.25	2,1	0.0031	7.99
0.5,-0.3	0.25,0.25	0.25,-0.25	2,1	0.1123	293.77

Summary & Further Work

- If yield's response to irrigation is low, then producer willingness to pay for water is 0.

- If risk elasticity of irrigation increases, producer's are willing to pay a higher price.

- Under stress irrigation levels, producers are clearly willing to pay more.

Further work:

- Precipitation deficit unaccounted for in current paper.

- Use real-world data to compute elasticities.

- Incorporate water table values in the optimization problem.

Thank You!

Questions